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EFFECT OF THE ORDER OF PLATES ON THE BALLISTIC RESISTANCE OF DUCTILE LAYERED SHIELDS PERFORATED BY NONCONICAL IMPACTORS

G. BEN-DOR, A. DUBINSKY AND T. ELPERIN

In our previous studies using the two-term impactor-shield localized interaction model, we derived the rule determining the order of the plates with different mechanical properties in a multilayer shield that yields a maximum ballistic limit velocity against conical impactors. In the present study we show that this rule is valid also for ogive-shaped, nonconical impactors.

1. Introduction

Several topics associated with the investigation of layering and spacing of the shields are extensively covered in the literature on high-speed penetration mechanics. Many studies have compared ballistic characteristics of monolithic shields with those of the shields composed of several plates with the same total thickness and manufactured from the same material. The plates may be in contact or there may be air gaps between them. Therefore, as alternatives to the monolithic shield, many types of shields with different numbers of plates and different thicknesses of the plates and of the air gaps are feasible. Analyses of the effect of the order of plates manufactured from different materials on the ballistic characteristics of the shield have attracted particular interest. The simplest case of this problem is interchanging the plates in a two-layered shield. In the general case, the number of plates may vary and they may be manufactured from different materials. The combined effects of changing the order of plates and of using air gaps on the ballistic performance of the shield and various problems of optimization of the structure of the shield have also been studied in a number of investigations.

A brief survey of the state of the art presented below (mainly on penetration in metal shields) supports the assessment of [Radin and Goldsmith 1988] that "only limited results for multiple target materials exist in the literature..., and the results obtained cannot easily be correlated since different target and projectile materials, nose shapes, impact geometries and striker speeds were used". Clearly, the latter assessment is not related to the problem of selecting the best shield among the given set of shields against the impactor with a given shape. This problem can be often solved experimentally, and the obtained results can be explained using relatively simple physical reasoning. The problem is to determine a more or less general law that will enable predicting the change of the ballistic characteristics of the shield by varying the structure of the shield. This problem has not been solved as yet, although a number of experimental and theoretical studies have been performed in this direction.

Honda et al. [1930] investigated experimentally the impact of steel plates by conical-nosed projectiles. It was found that a shield composed of thin plates had a lower ballistic resistance than a monolithic shield

Keywords: impact, layered shield, ballistic limit velocity, optimization, plate.

with the same thickness. However, a spaced shield with the thicknesses of the plates equal to the half-thickness of a monolithic shield performed better than a monolithic shield. Marom and Bodner [1979] conducted a combined analytical and experimental comparative study of monolithic, layered and spaced thin aluminum shields. They found that the ballistic resistance of a monolithic shield is higher than that of a multilayered shield with the plates in contact and lower than the ballistic resistance of a spaced shield. The study by [Radin and Goldsmith 1988] was also based on semiempirical models and experimental investigations. They found a monolithic aluminum shield to be superior to a layered shield with the same total thickness for conical-nose and blunt projectiles, while the spaced shields were less effective. Corran et al. [1983b; 1983a], using experimental results on penetration of mild steel plates by impactors having "increasingly rounded nose shape", plotted the curve of perforation energy versus plate thickness for all considered variants of the shield and found a "kink" in the curve "at about 3.5 mm total thickness". The occurrence of the kink was explained by the change of character of energy absorption. The authors arrived at the conclusions that the order of unequal plate thickness is important. No advantage was found in using multilayered targets below the kink. Above this point the best combinations may approach the best-fit line to the single layer tests below the kink. It was found that there is an advantage to placing the layers in contact.

Nixdorff [1984a; 1984b; 1987] compared the ballistic performance of a monolithic metal shield with a shield manufactured from the same material, having the same total thickness, and consisting of several plates in contact. Using the theory developed by Awerbuch and Bodner [1974], Nixdorff showed that separation of a homogeneous shield into several layers implies a reduction of the ballistic limit velocity (BLV) of the shield.

Zukas [1996] and Zukas and Scheffler [2001] found, on the basis of numerical simulations with metallic shields, that layering dramatically weakens thin [$b/(2R) < 1$] and intermediate thickness [$3 < b/(2R) < 10$] shields, while thick shields [$b/(2R) > 10$] show small changes in projectile residual properties [residual mass and residual velocity] when compared to their monolithic equivalent. Here b and R are the thickness of the shield and the shank radius of the impactor, respectively.

Madhu et al. [2003] conducted experiments with aluminum plates impacted normally and concluded that there is no significant change in the ballistic performance due to layering of such intermediate thickness of plates. They compared a monolithic shield with two- and three-layered shields of the same thickness. Gupta and Madhu [1997], using experimental results obtained for aluminum and steel plates, arrived at the same conclusion with respect to relatively thick plates. For thin shields, the layered combinations in contact yielded higher residual velocity as compared with a monolithic shield manufactured from either aluminum or steel. It was also found that for a spaced shield the residual velocity was higher than in the case of plates in contact, for the same impact velocity.

Weidemaier et al. [1993] conducted experiments and numerical simulations on the perforation of steel barriers by spherical impactors with a diameter of 17 mm. They studied a monolithic shield with a thickness of 43 mm and shields composed of plates in contact having the same total thickness. It was found that the ballistic characteristics of layered shields depended strongly on the order of the plates having different thicknesses and that layering could improve or impair the ballistic performance of the shield.

Almohandes et al. [1996] conducted a comprehensive experimental study on the perforation of mild steel by standard 7.62 mm bullets. They investigated shields with total thickness in the range of 8–14 mm that were layered in contact, spaced and monolithic. It was found that single shields were more effective

than laminated shields of the same total thickness, regardless of the configuration or striking velocity, and that the difference in performance diminished as the striking velocity increased. Moreover, the effectiveness of laminated targets—whether in contact or spaced—increased as the number of plates comprising each target decreased. Ballistic performance of laminated shields is further enhanced by using the thickest lamina as the rear lamina. The authors also studied shields with different structures in which fiberglass-reinforced polyester was used as the filler material, and showed that these shields performed better than weight-equivalent steel targets. The experimental results of Almohandes et al. [1996] were used by Liang et al. [2005] for validating their approximate penetration model. This model was used for comparative analysis of shields with different structures. It was concluded that the ballistic performance was the best for the double shield when the ratio of the thickness of the first layer to the total thickness was about 0.75, and the worst performance was obtained when this ratio was equal to 0.5. An air gap slightly influenced the resistance to perforation in multilayered targets.

Elek et al. [2005] developed a simple model to describe the perforation of monolithic and multilayered thin metallic plates by a flat-ended cylindrical impactor, and used their model for the analysis of the ballistic properties of multilayered spaced shields. The main results of this study may be summarized as follows. The suggested model predicted that the monolithic shield will have greater resistance than any other multilayered shield with standoff distance between the layers and equivalent total mass. The analysis of penetration in a two-layered shield showed that the maximum resistance could be obtained for very low ($< 20\%$ of total thickness) or very high ($> 80\%$ of total thickness) front-layer thickness. The increase of the number of the spaced layers in a multilayered shield, at constant total mass, caused a further decrease of the ballistic resistance. Deterioration of the ballistic performance of thin steel shields against flat-ended cylindrical impactors caused by layering had been noticed earlier by Zaid et al. [1973].

Shirai et al. [1997] investigated experimentally and numerically the impact resistance of reinforced concrete plates against projectile impact. They found that double-layered plates could be expected to have higher impact resistance than standard plates.

Park et al. [2005] suggested a multistage procedure for optimization of a two-layered shield. In the first stage, using numerical simulations to describe penetration into shields with different layer thicknesses $b^{(1)}$ and $b^{(2)}$, they determined the average temperature of a shield, T_{ave}, the average equivalent plastic strain ε_{ave} and the maximum equivalent plastic strain in a critical element of the shield $\varepsilon_{\max}$. In the second stage, the approximate functions describing the dependencies, T_{ave}, ε_{ave} and $\varepsilon_{\max}$ vs. $b^{(1)}$ and $b^{(2)}$, were determined. In the third stage, using a reduction to a single-criterion problem by a linear combination of criteria, they solved a two-objective optimization problem. The authors considered two variants of the optimization criterion, T_{ave} or ε_{ave} and the weight of a shield. The constraints included the upper bounds or $\varepsilon_{\max}$, and constraints on the thicknesses of the plates and the total thickness of a shield.

Aptukov [1985] and Aptukov et al. [1985], using Pontrjagin's maximum principle, determined the optimum distribution of the mechanical characteristics of a nonhomogeneous plate. The areal density of the shield along the trajectory of the impactor until it stopped was used as an optimization criterion, and cylindrical and cone-nosed impactors were considered. The two-term impactor-shield interaction model was employed, wherein the assumption about a linear dependence between the coefficients of the model was used. Using a cylindrical cavity expansion model, Aptukov et al. [1986] solved the discrete problem of optimization of a layered plate when the shield consisted of several layers of material and the material itself could be chosen from a given set of materials.

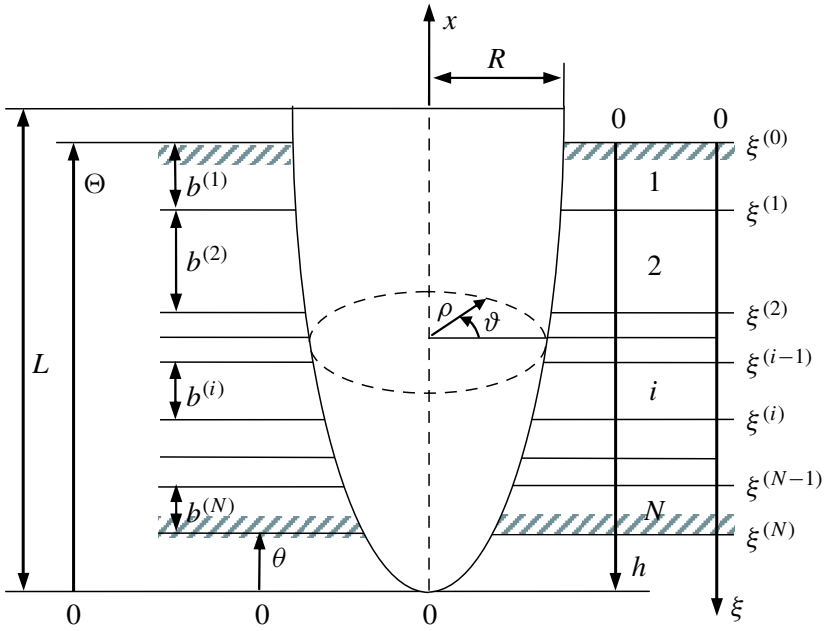

Figure 1. Notations.

Ben-Dor et al. [1998b; 1998a; 1999b; 1999a; 2000; 2006a] studied analytically the influence of air gaps between the plates and the order of plates on the BLV of a multilayered shield against conical-shaped impactors, and the results are summarized in [Ben-Dor. et al. 2006a]. They found that, for the wide class of impactor-shield interaction models, the ballistic performance of the shield is independent of the widths of the air gaps and of the sequence of plates in the shield and that it is determined only by the total thickness of the plates if the plates are manufactured from the same material. Using the two-term impactor-shield interaction model, they found the criterion (depending on mechanical properties of the materials of the plates) determining the order of plates in a multilayer shield that provides the maximum BLV. In the present study we showed that this criterion remains valid for the impactors with a shape different from conical.

2. Mathematical model and statement of problem

Consider a high speed normal penetration of a rigid sharp striker (a body of revolution) into a ductile layered shield with a finite thickness. We assume that the conditions of penetration are determined mainly by the "ductile hole enlargement" model [Backman and Goldsmith 1978]. The basic notations are shown in Figure 1; and we assume that only the nose part of the cylinder-shaped impactor interacts with the shield. The coordinate h, the current depth of penetration, is defined as the distance between the leading edge of the nose of the impactor and the rear surface of the shield. The coordinate ξ is associated with the shield. In cylindrical coordinates, x, ρ, ϑ, associated with the impactor the surface of the nose is described by the following equation:

$$r = \Phi(x, \theta), \quad 0 \le x \le L, \ 0 \le \theta \le 2\pi, \tag{1}$$

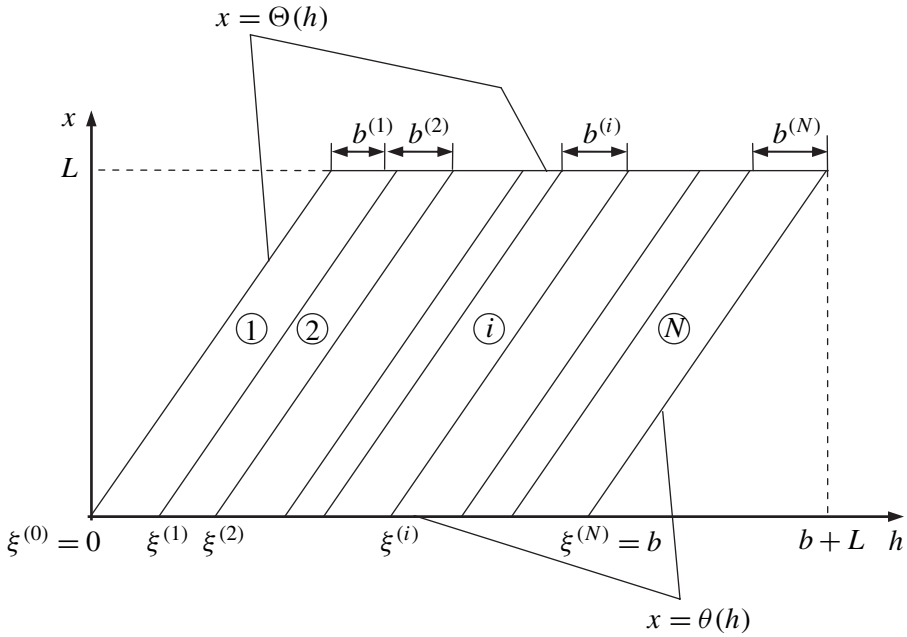

Figure 2. Model of the layered shield.

where L is the length of the impactor's nose, $\Phi(x)$ is an increasing, convex function. Assume that the shield consists of N layers (plates in-contact with different mechanical properties) with the thicknesses $b^{(1)}, b^{(2)}, \ldots, b^{(N)}$. The plate with a number i is located between the cross-sections $\xi = \xi^{(i-1)}$ and $\xi = \xi^{(i)}$, where $i = 1, 2, \ldots, N$ and $\xi^{(0)} = 0$. Let b be the total thickness of the shield that equals the sum of the thicknesses of all plates. It is assumed that the above parameters remain constant when the impactor penetrates into the shield. Then the part of the lateral surface of the impactor between the cross-sections $x = \theta(h)$ and $x = \Theta(h)$ (see Figure 1) interacts with some layers of the shield (see Figure 2) where

$$\theta(h) = \begin{cases} 0 & \text{if } 0 \le h \le b \\ h - b & \text{if } b \le h \le b + L \end{cases}, \qquad \Theta(h) = \begin{cases} h & \text{if } 0 \le h \le L \\ L & \text{if } h \ge L \end{cases}. \tag{2}$$

The equation of motion of the impactor, $m(d^2h/dt^2) = -D$, can be rewritten as follows:

$$mv(dv/dh) = -D, \tag{3}$$

where the velocity of the impactor v is considered to be a function of h, m is the mass of the impactor, and D is the resistance force. We consider the range of impact velocities v_{imp} when the projectile perforates the shield. Perforation occurs when the position of the striker is $h = b + L$ and its residual velocity is v_{res}. The BLV, v_{bl}, is defined as the impact velocity of the impactor required to emerge from the shield with zero residual velocity.

We assume that the impactor-shield interaction at a given location at the surface of the impactor that is in contact with i-th plate can be represented as follows:

$$d\vec{F} = \left[a_2^{(i)}\left(-\vec{v}^0 \cdot \vec{n}^0\right)^2 v^2 + a_0^{(i)}\right]\vec{n}^0 dS, \tag{4}$$

where $d\vec{F}$ is the force acting at the surface element dS of the impactor, $\vec{n}^0$ is the inner normal unit vector at a given location on the impactor's surface, $\vec{v}^0$ the unit vector of the impactor's velocity, the parameters $a_0^{(i)}$ and $a_2^{(i)}$ depend on the properties of the material of the shield, and hereafter the superscript in round brackets indicates the number of the layer. Equation (4) comprises most of the widely used phenomenological models for homogeneous shields (for details see [Ben-Dor. et al. 2005; 2006a] and [Recht 1990]. In particular, in the model proposed and validated in the comprehensive experimental study by [Vitman and Stepanov 1959], $a_2^{(i)}$ and $a_0^{(i)}$ are material density of the shield and "dynamical hardness", respectively. The values $a_0^{(i)}$ for some materials may be found in [Vitman and Ioffe 1948] (see also [Ben-Dor. et al. 2006b]).

In order to adapt Equation (4) for a layered shield let us define a step-functions ($\nu = 0, 2$):

$$a_\nu(\xi) = \begin{cases} a_\nu^{(1)} & \text{if } \xi^{(0)} \le \xi < \xi^{(1)} \\ \dots & \\ a_\nu^{(i)} & \text{if } \xi^{(i-1)} \le \xi < \xi^{(i)} \\ \dots & \\ a_\nu^{(N)} & \text{if } \xi^{(N-1)} \le \xi \le \xi^{(N)} \end{cases}. \tag{5}$$

Then Equation (4) can be rewritten as follows:

$$d\vec{F} = \left(a_2(\xi)\left(-\vec{v}^0 \cdot \vec{n}^0\right)^2 v^2 + a_0(\xi)\right)\vec{n}^0 dS. \tag{6}$$

The total force $\vec{F}$ acting on the impactor at some location inside the shield is found by integrating the local force over the impactor-shield contact surface area, that is, over the portion of the impactor's surface S that is determined by the inequalities $0 \le \upsilon \le 2\pi$ and $\theta(h) \le x \le \Theta(h)$. Taking into account the identity:

$$\xi = h - x, \tag{7}$$

and using the following formulas of differential geometry:

$$-\vec{v}^0 \cdot \vec{n}^0 = \Phi'/\sqrt{\Phi'^2 + 1}, \qquad dS = \sqrt{\Phi'^2 + 1}\,dx\,d\upsilon, \qquad \Phi' = d\Phi/dx, \tag{8}$$

we obtain the following expression for the drag force D:

$$\begin{aligned} D = \vec{F} \cdot (-\vec{v}^0) &= \iint_S \left(a_2(\xi)\left(-\vec{v}^0 \cdot \vec{n}^0\right)^2 v^2 + a_0(\xi)\right)\left(-\vec{v}^0 \cdot \vec{n}^0\right) dS \\ &= \frac{m}{2}[f_2(h)v^2 + f_0(h)], \end{aligned} \tag{9}$$

where

$$f_\nu(h) = \frac{4\pi}{m}\int_{\theta(h)}^{\Theta(h)} a_\nu(h-x)\Phi\psi_\nu(\Phi')dx, \qquad \psi_\nu(z) = z\left(\frac{z}{\sqrt{z^2+1}}\right)^\nu, \quad \nu = 0, 2. \tag{10}$$

Substituting D from Equation (9) into Equation (3), after some algebra we obtain an ordinary linear differential equation with respect to v^2:

$$dv^2/dh + f_2(h)v^2 + f_0(h) = 0. \tag{11}$$

The solution of Equation (11) with the initial condition $v(0) = v_{\text{imp}}$, which corresponds to the beginning of the motion of the impactor with the impact velocity v_{imp}, reads [Kamke 1959]:

$$v^2(h) = \frac{1}{q(h)}(v_{\text{imp}}^2 - g(h)), \tag{12}$$

where

$$q(h) = \exp\left(\int_0^h f_2(\eta)d\eta\right), \qquad g(h) = \int_0^h f_0(H)q(H)dH. \tag{13}$$

Equation (12) yields the following formulas for the residual velocity, $v_{\text{res}} = v(b+L)$, and the BLV, v_{bl}:

$$v_{\text{res}}^2 = \frac{1}{q(b+L)}\big(v_{\text{imp}}^2 - g(b+L)\big), \qquad v_{bl}^2 = g(b+L). \tag{14}$$

For further analysis it is convenient to rewrite the expression for v_{bl} using the dimensionless variables:

$$v_{bl}^2 = k\int_0^{\bar{b}+1} Q(\bar{h})d\bar{h}\int_{\bar{\theta}(\bar{h})}^{\bar{\Theta}(\bar{h})} \tilde{a}_0(\bar{h}-\bar{x})\bar{\Phi}\psi_0(\bar{\Phi}')d\bar{x}, \tag{15}$$

where L is chosen as a characteristic length, and

$$k = \frac{4\pi L^3}{m}, \qquad \bar{x} = \frac{x}{L}, \qquad \bar{\Phi} = \frac{\Phi}{L}, \qquad \bar{\Phi}' = \frac{d\bar{\Phi}}{d\bar{x}}, \qquad \bar{h} = \frac{h}{L}, \qquad \bar{b} = \frac{b}{L}, \tag{16}$$

$$Q(\bar{h}) = \exp\left(k\int_0^{\bar{h}} d\bar{H}\int_{\bar{\theta}(\bar{H})}^{\bar{\Theta}(\bar{H})} \tilde{a}_2(\bar{H}-\bar{x})\bar{\Phi}\psi_2(\bar{\Phi}')d\bar{x}\right), \tag{17}$$

$$\tilde{a}_\nu(\bar{\xi}) = a_\nu(L\bar{\xi}), \qquad \nu = 0, 2, \tag{18}$$

$$\bar{\theta}(\bar{h}) = \begin{cases} 0 & \text{if } 0 \le \bar{h} \le \bar{b} \\ \bar{h} - \bar{b} & \text{if } \bar{b} \le \bar{h} \le \bar{b}+1 \end{cases}, \qquad \bar{\Theta}(\bar{h}) = \begin{cases} \bar{h} & \text{if } 0 \le \bar{h} \le 1 \\ 1 & \text{if } \bar{h} \ge 1 \end{cases}. \tag{19}$$

It is shown by [Ben-Dor. et al. 1999b; Ben-Dor. et al. 1999a; Ben-Dor. et al. 2006a] that the maximum BLV of a layered shield against a conical impactor is attained if the plates are arranged in the increasing order of the parameter $\chi = a_0/a_2$. This means that if the plates are numbered, the shield must be constructed by successively adding the plates with the order numbers $i_1, i_2, \ldots, i_N$, that satisfy the condition $\chi^{(i_1)} \le \chi^{(i_2)} \le \cdots \le \chi^{(i_N)}$, where $\chi^{(i)} = a_0^{(i)}/a_2^{(i)}$. The main goal of this study is to validate the latter result for nonconical impactors.

3. Ogive-shaped generatrix

Since the main goal of this study is to investigate the effect of deviation from a conical shape on the ballistic properties of a multilayer shield, the most convenient impactors for this purpose are ogive-shaped impactors having a generatrix with a constant curvature. Therefore it is natural to employ curvature as a parameter that characterizes a deviation from a conical shape. Curvature, a reciprocal of the radius of the largest circle that is tangent to a curve (on its concave side) at a point, is equal zero for a straight line, that is, a generatrix of a conical impactor.

The equation of the circle having the radius $\bar{\rho}_*$ (in dimensionless coordinates) with a center in the point $(\bar{x}_*, \bar{\Phi}_*)$ reads (see Figure 3):

$$(\bar{x} - \bar{x}_*)^2 + (\bar{\Phi} - \bar{\Phi}_*)^2 = \bar{\rho}_*^2, \tag{20}$$

where the arc of the generatrix must pass through the points $(0, 0)$ and $(1, \tau)$, and the following requirements must be satisfied:

$$0 \le \bar{x} \le 1,\ 0 \le \bar{\Phi} \le \tau,\ \bar{\Phi}_* \le 0,\ \bar{x}_* \ge 1,\ \tau = R/L. \tag{21}$$

Omitting algebraic manipulations, let us write the equation of the generatrix in the form:

$$\bar{\Phi} = \frac{\tau}{2} - \frac{2\beta(\bar{x} - 0.5)^2 - 2\tau(\bar{x} - 0.5)\eta - 0.5\beta(\tau^2 + 1)}{\eta + \sqrt{\eta^2 + \beta^2(\tau^2 + 1) - 4\beta^2(\bar{x} - 0.5)^2 + 4\beta\tau\eta(\bar{x} - 0.5)}}, \tag{22}$$

where

$$\eta = \sqrt{\frac{4}{\tau^2 + 1} - \beta^2}, \qquad \beta = \frac{1}{\bar{\rho}_*}, \quad 0 \le \beta \le \frac{2\min(1, \tau)}{\tau^2 + 1}. \tag{23}$$

Equation (22) describes not only a circular arc but also a straight line, $\bar{\Phi} \le \tau\bar{x}$, for a conical impactor. This formula for generatrix allows us to avoid computational problems arising for small β.

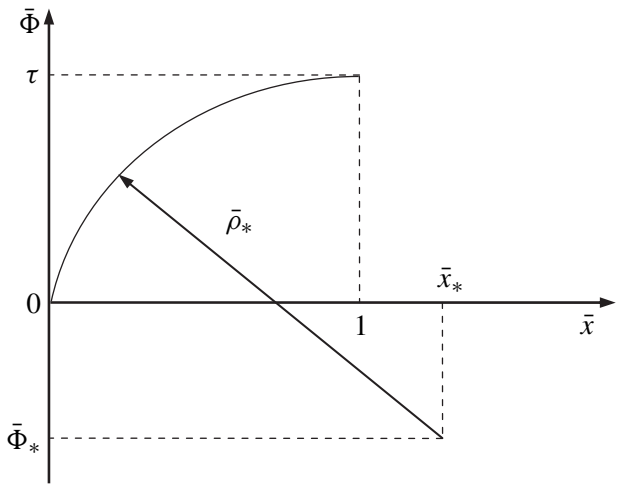

Figure 3. Ogive-shaped generatrix.

Equation (15) is quite involved and, consequently, it is not convenient for calculations and can introduce uncontrollable numerical error. All these problems can be avoided by using piecewise-linear approximation of the generatrix of a striker.

4. Piecewise-linear approximation of the generatrix

The equation of a piecewise linear generatrix can be written as follows (see Figure 4):

$$\bar{\Phi}(\bar{x}) = \begin{cases} \alpha_1 \bar{x} + \beta_1 & \text{if } 0 = \bar{x}_0 \le \bar{x} \le \bar{x}_1 \\ & \dots \\ \alpha_j \bar{x} + \beta_j & \text{if } \bar{x}_{j-1} \le \bar{x} \le \bar{x}_j \\ & \dots \\ \alpha_M \bar{x} + \beta_M & \text{if } \bar{x}_{M-1} \le \bar{x} \le \bar{x}_M = 1 \end{cases}, \tag{24}$$

where

$$\alpha_j = \frac{\bar{\Phi}_j - \bar{\Phi}_{j-1}}{\bar{x}_j - \bar{x}_{j-1}}, \qquad \beta_j = \frac{\bar{x}_j \bar{\Phi}_{j-1} - \bar{x}_{j-1} \bar{\Phi}_j}{\bar{x}_j - \bar{x}_{j-1}}, \quad j = 1, 2, \dots, M. \tag{25}$$

The domain determined by Equation (19) can be represented as a union of $N \times M$ sub-domains $S_j^{(i)}$ (see Figure 5). The parallelogram $A_j^{(i)} B_j^{(i)} C_j^{(i)} E_j^{(i)}$ with the vertices at the points $A_j^{(i)}(\bar{\xi}^{(i-1)} + \bar{x}_{j-1}, \bar{x}_{j-1})$, $B_j^{(i)}(\bar{\xi}^{(i-1)} + \bar{x}_j, \bar{x}_j)$, $C_j^{(i)}(\bar{\xi}^{(i)} + \bar{x}_j, \bar{x}_j)$ and $E_j^{(i)}(\bar{\xi}^{(i)} + \bar{x}_{j-1}, \bar{x}_{j-1})$, bounds the sub-domain $S_j^{(i)}$. The

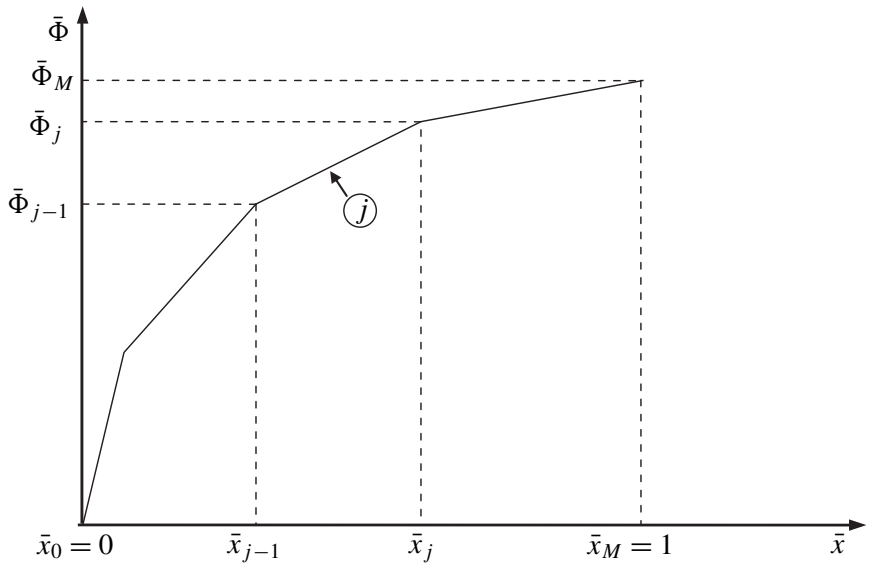

Figure 4. Piecewise linear generatrix.

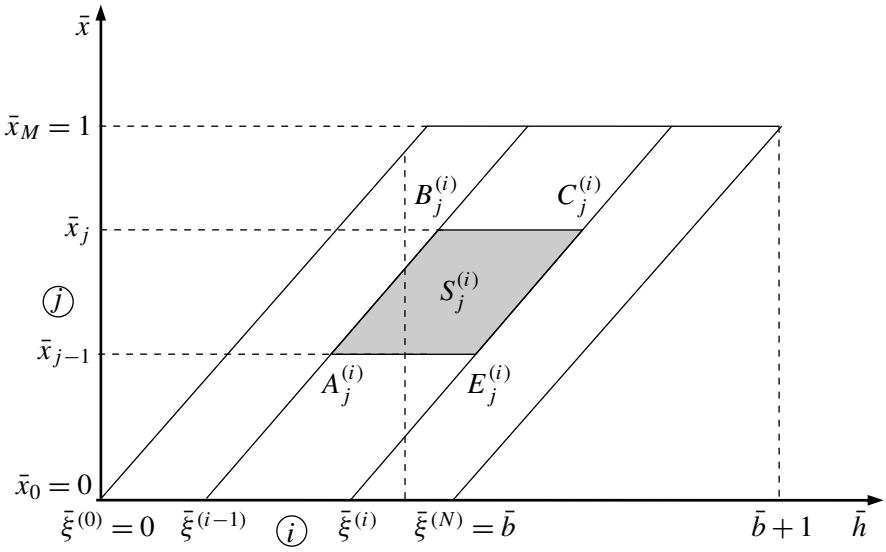

Figure 5. Domain of impactor/shield interaction for layered shield and impactor with linear generatrix.

sub-domains $S_j^{(i)}$ are determined as follows:

$$\begin{cases} \bar{\xi}^{(i-1)} + \bar{x}_{j-1} & \leq \bar{h} \leq \bar{\xi}^{(i)} + \bar{x}_j \\ \bar{\theta}_j^{(i)}(h) & \leq \bar{x} \leq \bar{\Theta}_j^{(i)}(h) \end{cases}, \tag{26}$$

where

$$\bar{\theta}_j^{(i)}(h) = \begin{cases} 0 & \text{if } \bar{h} \leq \bar{\xi}^{(i-1)} + \bar{x}_{j-1} \\ \bar{x}_{j-1} & \text{if } \bar{\xi}^{(i-1)} + \bar{x}_{j-1} \leq \bar{h} \leq \bar{\xi}^{(i)} + \bar{x}_{j-1} \\ \bar{x} - \xi^{(i)} & \text{if } \bar{\xi}^{(i)} + \bar{x}_{j-1} \leq \bar{h} \leq \bar{\xi}^{(i)} + \bar{x}_j \\ 0 & \text{if } \bar{h} \geq \bar{\xi}^{(i)} + \bar{x}_j \end{cases}, \tag{27}$$

$$\bar{\Theta}_j^{(i)}(h) = \begin{cases} 0 & \text{if } \bar{h} \leq \bar{\xi}^{(i-1)} + \bar{x}_{j-1} \\ \bar{x}_{j-1} - \bar{\xi}^{(i-1)} & \text{if } \bar{\xi}^{(i-1)} + \bar{x}_{j-1} \leq \bar{h} \leq \bar{\xi}^{(i-1)} + \bar{x}_j \\ \bar{x}_j & \text{if } \bar{\xi}^{(i-1)} + \bar{x}_j \leq \bar{h} \leq \bar{\xi}^{(i)} + \bar{x}_j \\ 0 & \text{if } \bar{h} \geq \bar{\xi}^{(i)} + \bar{x}_j \end{cases}. \tag{28}$$

Then the integral in Equation (17) can be represented as sum of integrals over sub-domains $S_j^{(i)}$:

$$\begin{aligned}\frac{1}{k}\ln[Q(\bar{h})] &= \sum_{\substack{1\le i\le N\\ 1\le j\le M}} \int_0^{\bar{h}} d\bar{H} \int_{\bar{\theta}_j^{(i)}(\bar{H})}^{\bar{\Theta}_j^{(i)}(\bar{H})} \tilde{a}_2(\bar{H}-\bar{x})\bar{\Phi}\psi_2(\bar{\Phi}')d\bar{x} \\ &= \sum_{\substack{1\le i\le N\\ 1\le j\le M}} \tilde{a}_2^{(i)}\psi_2(\alpha_j)\int_0^{\bar{h}} d\bar{H} \int_{\bar{\theta}_j^{(i)}(\bar{H})}^{\bar{\Theta}_j^{(i)}(\bar{H})} (\alpha_j\bar{x}+\beta_j)d\bar{x} \\ &= \sum_{\substack{1\le i\le N\\ 1\le j\le M}} \tilde{a}_2^{(i)}\psi_2(\alpha_j)\big[\Omega_j^{(i)}(\bar{h})-\omega_j^{(i)}(\bar{h})\big],\end{aligned} \tag{29}$$

where

$$\begin{aligned}\omega_j^{(i)}(\bar{h}) &= \int_{\bar{\xi}^{(i-1)}+\bar{x}_{j-1}}^{\bar{h}} \Big\{0.5\alpha_j\big[\bar{\theta}_j^{(i)}(\bar{H})\big]^2+\beta_j\bar{\theta}_j^{(i)}(\bar{H})\Big\}d\bar{H}, \\ \Omega_j^{(i)}(\bar{h}) &= \int_{\bar{\xi}^{(i-1)}+\bar{x}_{j-1}}^{\bar{h}} \Big\{0.5\alpha_j\big[\bar{\Theta}_j^{(i)}(\bar{H})\big]^2+\beta_j\bar{\Theta}_j^{(i)}(\bar{H})\Big\}d\bar{H}.\end{aligned} \tag{30}$$

The integrals in Equation (30) can be calculated taking into account the definition of functions $\bar{\theta}_j^{(i)}(h)$ and $\bar{\Theta}_j^{(i)}(h)$:

$$\omega_j^{(i)}(\bar{h}) = \begin{cases} 0 & \text{if } h \le \bar{\xi}^{(i-1)}+\bar{x}_{j-1}, \\ (0.5\alpha_j\bar{x}_{j-1}^2+\beta_j\bar{x}_{j-1})(\bar{h}-\bar{\xi}^{(i-1)}-\bar{x}_{j-1}) & \text{if } \bar{\xi}^{(i-1)}+\bar{x}_{j-1} \le h \le \bar{\xi}^{(i)}+\bar{x}_{j-1}, \\ (\alpha_j/6)\big((\bar{h}-\bar{\xi}^{(i)})^3-\bar{x}_{j-1}^3\big)+0.5\beta_j\big((\bar{h}-\bar{\xi}^{(i)})^2-\bar{x}_{j-1}^2\big) & \\ & \text{if } \bar{\xi}^{(i)}+\bar{x}_{j-1} \le h \le \bar{\xi}^{(i)}+\bar{x}_j, \\ \omega_j^{(i)}(\bar{\xi}^{(i)}+\bar{x}_j) & \text{if } h > \bar{\xi}^{(i)}+\bar{x}_j, \end{cases} \tag{31}$$

$$\Omega_j^{(i)}(\bar{h}) = \begin{cases} 0 & \text{if } h \le \bar{\xi}^{(i-1)}+\bar{x}_{j-1}, \\ (\alpha_j/6)\big((\bar{h}-\bar{\xi}^{(i-1)})^3-\bar{x}_{j-1}^3\big)+0.5\beta_j\big((\bar{h}-\bar{\xi}^{(i-1)})^2-\bar{x}_{j-1}^2\big) & \\ & \text{if } \bar{\xi}^{(i-1)}+\bar{x}_{j-1} \le h \le \bar{\xi}^{(i-1)}+\bar{x}_j, \\ (\alpha_j/6)(\bar{x}_j^3-\bar{x}_{j-1}^3)+0.5\beta_j(\bar{x}_j^2-\bar{x}_{j-1}^2)+\bar{x}_j(0.5\alpha_j\bar{x}_j+\beta_j)(\bar{h}-\bar{\xi}^{(i-1)}\bar{x}_j) & \\ & \text{if } \bar{\xi}^{(i-1)}+\bar{x}_j \le h \le \bar{\xi}^{(i)}+\bar{x}_j, \\ \Omega_j^{(i)}(\bar{\xi}^{(i)}+\bar{x}_j) & \text{if } h > \bar{\xi}^{(i)}+\bar{x}_j. \end{cases} \tag{32}$$

The expression for the BLV, Equation (15), can be rewritten similarly to Equation (29):

$$\begin{aligned}\frac{v_{bl}^2}{k} &= \sum_{\substack{1\le i\le N\\ 1\le j\le M}} \int_{\bar\xi^{(i-1)}+\bar x_{j-1}}^{\bar\xi^{(i)}+\bar x_j} Q(\bar H)d\bar H \int_{\bar\theta_j^{(i)}(\bar H)}^{\bar\Theta_j^{(i)}(\bar H)} \tilde a_0(\bar H-\bar x)\bar\Phi\psi_0(\bar\Phi')d\bar x \\ &= \sum_{\substack{1\le i\le N\\ 1\le j\le M}} \tilde a_0^{(i)}\psi_0(\alpha_j)\int_{\bar\xi^{(i-1)}+\bar x_{j-1}}^{\bar\xi^{(i)}+\bar x_j} Q(\bar h)\big[\Omega_j^{(i)}(\bar h)-\omega_j^{(i)}(\bar h)\big]d\bar h.\end{aligned} \tag{33}$$

Thus, determining the BLV is reduced to calculating one-dimensional integrals.

5. Result of numerical calculations and discussion

In numerical calculations we considered a two-layer shield where "the first" plate and "the second" plate are manufactured from the soft steel and aluminum, respectively. We used the model given by Equation (4) with $a_0^{(1)} = 1850\,\text{MPa}$, $a_2^{(1)} = 7830\,\text{kg}/\text{m}^3$, $a_0^{(2)} = 350\,\text{MPa}$ and $a_2^{(2)} = 2765\,\text{kg}/\text{m}^3$, where the values of the "dynamical hardness" $a_0^{(i)}$ are adopted from [Vitman and Ioffe 1948]. All calculations were performed for BLVs less than 1000 m/s. The latter constraint approximately determines the range of validity of this model.

The following numbers are assigned to the materials of the plates. The superscript $[1-2]$ means that the order of the plates in the shield is such that the plate manufactured from material #1 (soft steel) is perforated before the plate manufactured from material #2 (aluminum), the superscript $[2-1]$ indicates the reverse order of the plates. Since parameter $\chi = a_0/a_2$ decreases with the increase of the number of the material ($\chi^{(2)} = 0.127\cdot 10^6\,\text{m}^2/\text{s}^2$, $\chi^{(1)} = 0.236\cdot 10^6\,\text{m}^2/\text{s}^2$), then $v_{bl}^{[2-1]} \ge v_{bl}^{[1-2]}$ for conical-nosed impactors. The goal of our calculations was to estimate the effect of the parameter β, that characterizes the deviation from the conical shape on the index $\delta = v_{bl}^{[2-1]}/v_{bl}^{[1-2]}$, that quantifies the efficiency of changing the order of plates on the BLV. Typical results of these calculations are showed in Figures 6-7. It must be noted that $\beta = 0$ for a conical impactor.

In all these figures we showed the dependence δ versus β, and different curves correspond to different ratios of the thicknesses of the plates in a shield. The curves in Figure 6 are plotted for $\tau = 0.5$, for $\tau = 1$ in Figure 7. Clearly, in the second case and for relatively large values of β when the shape of the bluntness is close to spherical, the penetrator is not a sharp-shaped body, and the plots have only a formal meaning.

Figure 6a corresponds to the dimensionless total thickness of the shield $\bar b = 8.0$ and $k = 0.0004\,\text{m}^3/\text{kg}$. The curves of the dependencies δ vs. β are concave, i.e., at the beginning the increase of the curvature of the generatrix of a striker causes reduction of the index δ, while further increase of β is accompanied by the increase of the index δ. Notably, for relatively large β, the magnitude of the index δ may become larger than that for a cone-shaped impactor. When the shape of the impactor is specified (β is given) the effect of the change of the order of the plates (the magnitude of the index δ) depends essentially upon the ratio of the thicknesses of the plates. In the considered case for all ogive-shaped impactors this effect is maximal when $b^{(1)}/b \approx 0.3$.

In Figure 6b we showed two sets of plots, the first for $k = 0.0007\,\text{m}^3/kg$ and $\bar b = 8.0$, and the second for $k = 0.0004\,\text{m}^3/\text{kg}$ and $\bar b = 12.0$. The first set differs from that in Figure 6a by the increased value

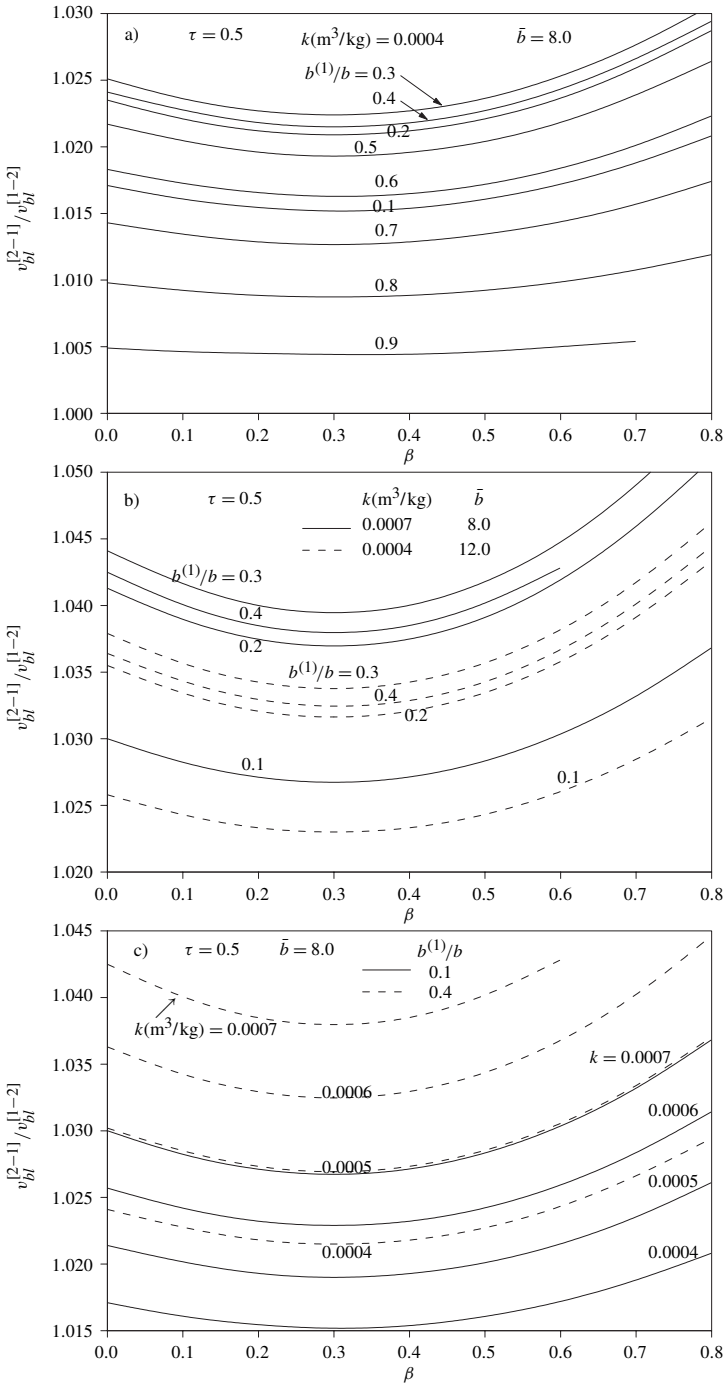

Figure 6. Influence of the curvature of the impactor's generatrix on the effect of rearranging the plates in the shield; $v_{bl}^{[1-2]}$ and $v_{bl}^{[2-1]}$ are the BLVs for the "direct" and the "reverse" order of the plates in the shield, respectively; β is the dimensionless curvature of the impactor's nose; $\tau = 0.5$.

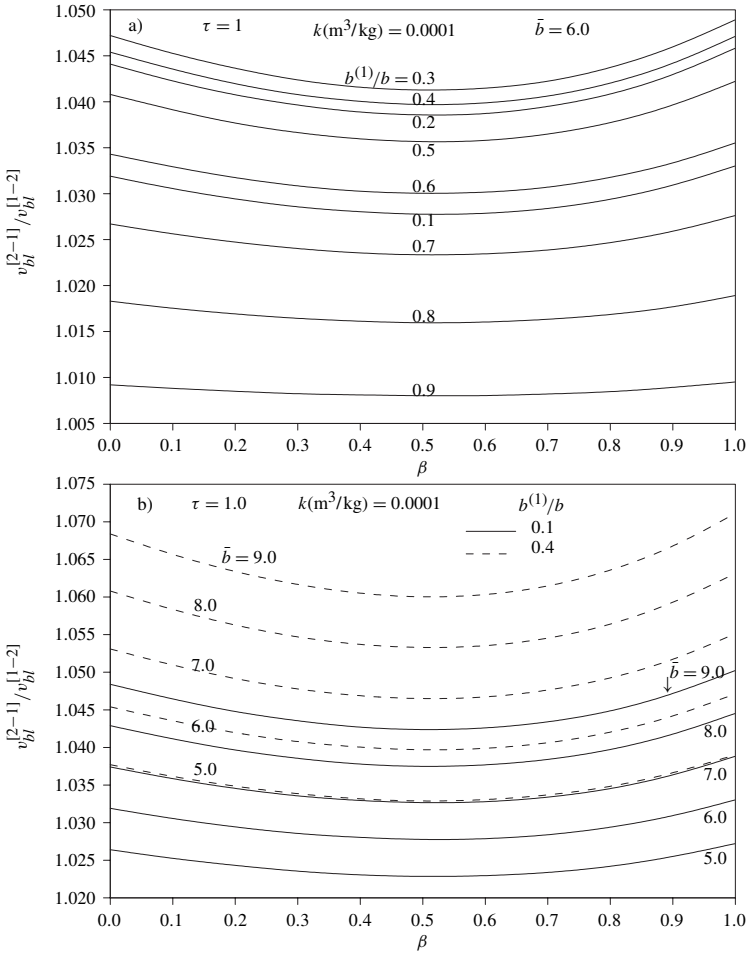

Figure 7. Influence of the curvature of the impactor's generatrix on the effect of rearranging the plates in the shield; $v_{bl}^{[1-2]}$ and $v_{bl}^{[2-1]}$ are the BLVs for the "direct" and the "reverse" order of the plates in the shield, respectively; β is the dimensionless curvature of the impactor's nose; $\tau = 1.0$

of k, while in the second set of curves we increased $\bar{b}$. Comparing the results showed in Figures 6a-6b demonstrates that increase of the magnitude of each of these two parameters, k and $\bar{b}$, results in the increase of the ratio of BLVs, δ, for every β. The effect of k on β is shown explicitly in Figure 6c.

Figure 7a for $\tau = 1$ demonstrates the same dependencies as Figure 6a. In Figure 7b we showed the effect of the total thickness of the shield for two values of the ratio $b^{(1)}/b$ (0.1 and 0.4) on the dependence $\delta = \delta(\beta)$. Inspection of these plots shows that the increase of the total width $\bar{b}$ causes the increase of the ratio δ.

Our calculations demonstrated that replacement of the conical head of the impactor by a convex ogive-shaped head can be accompanied either by the increase or decrease of the efficiency of changing the order

of the plates in a shield. However we did not encounter situations in which the optimal order of the plates in the shield is different for these two different impactor shapes.

6. Energy absorption

Using the relationships for the impact energy and residual energy of the impactor E_{imp} and E_{res}, respectively:

$$E_{\text{imp}} = 0.5\ \text{mv}_{\text{imp}}^2, \qquad E_{\text{res}} = 0.5\ \text{mv}_{\text{res}}^2 \tag{34}$$

and Equation (14) rewritten as

$$v_{\text{res}}^2 = \frac{1}{q_*}[v_{\text{imp}}^2 - v_{bl}^2], \qquad q_* = q(b+L) = Q(\bar{b}+1), \tag{35}$$

yields the following formula for the relative energy absorbed by a shield:

$$e_{\text{abs}} = \frac{E_{\text{imp}} - E_{\text{res}}}{E_{\text{imp}}} = \frac{q_* - 1}{q_*} + \frac{1}{q_*}\left(\frac{v_{bl}}{v_{\text{imp}}}\right)^2. \tag{36}$$

Assume that Equation (36) is written for the *initial* shield with a certain plate order. Consider also a *modified* shield with an altered plate order where the corresponding parameters in the modified shield are denoted by a tilde. Since q_* is independent of the order of the plates in the shield [Ben-Dor. et al. 1999b], we may write equation similar to Equation (36) for the modified shield:

$$\tilde{e}_{\text{abs}} = \frac{E_{\text{imp}} - \tilde{E}_{\text{res}}}{E_{\text{imp}}} = \frac{q_* - 1}{q_*} + \frac{1}{q_*}\left(\frac{\tilde{v}_{bl}}{v_{\text{imp}}}\right)^2. \tag{37}$$

Then

$$e_{\text{abs}} - \tilde{e}_{\text{abs}}^{=} \frac{1}{q_*}\left(\frac{\tilde{v}_{bl}}{v_{\text{imp}}}\right)^2 (\mu^2 - 1), \quad \mu = \frac{v_{bl}}{\tilde{v}_{bl}}. \tag{38}$$

Taking into account that

$$\mu^2 - 1 \equiv (\mu - 1)^2 + 2(\mu - 1) \approx 2(\mu - 1), \tag{39}$$

for $|\mu - 1| \ll 1$, Equation (38) can be rewritten as follows:

$$e_{\text{abs}} - \tilde{e}_{\text{abs}}^{\approx} \zeta(1 - \mu), \quad \zeta = \frac{2}{q_*}\left(\frac{\tilde{v}_{bl}}{v_{\text{imp}}}\right)^2. \tag{40}$$

Since $v_{bl}^0 \le v_{\text{imp}}$ and $q_* > 1$, then $\zeta < 2$. Therefore, the model employed in this study predicts that rearranging the plates in the shield causes a change in the relative magnitude of the absorbed energy that does not exceed the doubled ratio of the corresponding BLVs.

7. Concluding remarks

Using approximate model for ductile layered shields, we analyzed the effect of re-arranging plates in the shield against nonconical rigid impactors. We found that the criterion for the optimal arrangement of the plates in a shield, determined previously for conical impactors, is valid also for nonconical impactors. The theoretical results we obtained can be employed in further experimental studies on the optimization of impactors and shields.

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