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3D GREEN'S FUNCTIONS FOR A STEADY POINT HEAT SOURCE INTERACTING WITH A HOMOGENEOUS IMPERFECT INTERFACE

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The image method is applied to derive the three-dimensional temperature field induced by a steady point heat source interacting with a homogeneous imperfect interface. Our approach is a straightforward extension of that of Sommerfeld who addressed the half-space Green's function for a steady point heat source at the beginning of the last century. Both weakly and highly conducting type imperfect interface conditions are considered. It is found that the temperature field for both types of imperfect interface is only dependent on the two-phase conductivity parameter and another parameter measuring the interface "rigidity". As an application, we discuss the Coulomb force on a static point electric charge due to its interaction with the imperfect interface. It is possible to find an equilibrium position for the electric charge interacting with an imperfect interface. In addition, the equilibrium position is stable provided the interface is weakly conducting whereas the equilibrium position is unstable if the interface is highly conducting.

1. Introduction

Recently, Ang et al. [2004] calculated the steady state two-dimensional temperature field in a thermally isotropic bimaterial with a homogeneous imperfect interface. They derived a special Green's function for a steady line heat source in two bonded half-planes with an imperfect interface so as to devise a boundary element method (BEM) which does not require the interface to be discretized. As discussed in [Ang et al. 2004], the problem is two-dimensional, in that the temperature is independent of a certain coordinate (say, the z-coordinate) and the imperfect interface is a weakly conducting one. The weakly conducting interface is based on the assumption that the normal component of heat flux is continuous but that the temperature across the interface is discontinuous. More precisely, the jump in temperature is proportional to the normal component of heat flux. Discussions on weakly conducting interface can also be found in the works of [Benveniste and Miloh 1986; Ru and Schiavone 1997; Chen 2001], among others. For a highly conducting interface, the temperature is continuous across the interface, whereas the normal component of the heat flux has a discontinuity across the interface which is proportional to a certain differential expression of the temperature (see [Miloh and Benveniste 1999; Chen 2001; Benveniste 2006] among others).

At the beginning of the last century, [Sommerfeld 1926; 1978] derived the half-space Green's function for a steady point heat source by using the image method. As stated by Ochmann [2004], "Sommerfeld [1978] treated the half-space problem by writing the total thermal field as a superposition of an original

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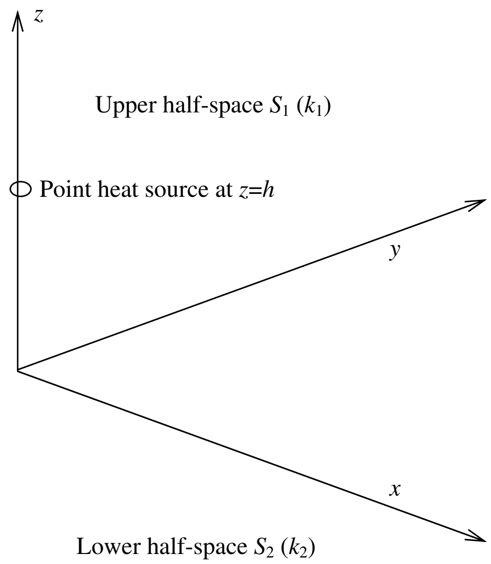

Figure 1. A steady point heat source in two imperfectly bonded half-spaces.

heat source, a mirror source and a line integral combined of single thermal sources placed at the z-axis below the mirror source". We note that Sommerfeld's technique has been recently extended to address the sound field caused by a monopole source above an impedance plane by means of the complex image method [Ochmann 2004; Taraldsen 2005b; Taraldsen 2005a]. Sommerfeld's method can guide us on the matter of how to conceive the Green's functions for a steady heat source in two imperfectly bonded half-spaces. Here we write the total temperature field in the upper half-space, in which the heat source is located at $(0, 0, h)$, $(h > 0)$, as a superposition of the original heat source, a mirror source, and a line integral combined of single thermal sources placed at the z-axis below the mirror source. On the other hand, we write the temperature field in the lower half-space as a line integral combined of single thermal sources placed at the z-axis above the location of the original heat source. We find that by using this method, we can arrive at the Green's function for both a weakly conducting interface and a highly conducting one.

The objective of the present work is to seek the possibility of deriving the corresponding three-dimensional Green's functions for a steady point heat source in two bonded half-spaces with a weakly or highly conducting interface. The expressions of the derived Green's functions should be as simple as possible in order to conveniently incorporate them in BEM.

2. Preliminaries

In a fixed Cartesian coordinate system (x, y, z), we consider the upper and the lower half-spaces, $S_1 : z \geq 0$ and $S_2 : z \leq 0$, in which the conductivity of each phase is denoted by k_1 and k_2, as shown in Figure 1.

The two half-spaces are separated by the imperfect interface $z = 0$. Let T be the temperature field and the heat fluxes are given by $q_x = -kT_{,x}$, $q_y = -kT_{,y}$, $q_z = -kT_{,z}$. A steady point heat source of strength H is located at the point $(0, 0, h)$, $(h > 0)$ in the upper half-space. Under steady state conditions, the temperature obeys the three-dimensional inhomogeneous Laplace equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = -\frac{H}{k_1}\delta(x)\delta(y)\delta(z-h), \text{ in } S_1 \text{ and } S_2, \tag{1}$$

where $\delta(*)$ is the Dirac delta function.

For a weakly conducting interface, the normal heat flux is continuous, whereas the temperature field undergoes a discontinuity which is proportional to the normal heat flux as

$$k_1 \frac{\partial T_1}{\partial z} = k_2 \frac{\partial T_2}{\partial z} = \alpha(T_1 - T_2), \quad (z = 0), \tag{2}$$

where the nonnegative interface parameter α is defined by

$$\alpha = \lim_{\substack{t\to 0\\ k_0\to 0}} \frac{k_0}{t}, \tag{3}$$

and where k_0 and t are respectively the interphase conductivity and its thickness. The case where $\alpha \to \infty$ corresponds to a perfectly bonded interface whereas $\alpha = 0$ stands for adiabatic contact. In this work we assume that α is constant (that is, the imperfection is uniformly distributed over the interface).

For a highly conducting interface, the temperature field is continuous whereas the normal heat flux undergoes a discontinuity of the type

$$T_1 = T_2, \qquad k_2 \frac{\partial T_2}{\partial z} - k_1 \frac{\partial T_1}{\partial z} = \beta \Delta_s T_1 = \beta \Delta_s T_2, \quad (z = 0), \tag{4}$$

where

$$\Delta_s T_1 = \frac{\partial^2 T_1}{\partial x^2} + \frac{\partial^2 T_1}{\partial y^2}$$

is the operator of the surface Laplacian and the nonnegative interface parameter β is defined by

$$\beta = \lim_{\substack{t\to 0\\ k_0\to \infty}} k_0 t. \tag{5}$$

The case where $\beta = 0$ corresponds to a perfectly bonded interface, whereas $\beta \to \infty$ describes contact with a medium of infinite conductivity. In writing Equation (4), it has been assumed that β is a constant.

Following the idea of [Sommerfeld 1978] who treated half-space problems of heat conduction, let us write the total temperature field in the upper half-space as a superposition of the original heat source at $(0, 0, h)$, a mirror source at $(0, 0, -h)$ and a line integral combined of single thermal sources placed at the z-axis below the mirror source. By contrast, let us write the temperature field in the lower half-space as a line integral combined of single thermal sources placed at the z-axis above the location of the original heat source. Thus, the distribution of temperature in the upper and lower half-spaces can be expressed as

$$T_1 = \frac{H}{4\pi k_1}\left(\frac{1}{\sqrt{x^2+y^2+(z-h)^2}} + \frac{A}{\sqrt{x^2+y^2+(z+h)^2}} - B\int_0^{+\infty} \frac{\exp(-\gamma\eta)}{\sqrt{x^2+y^2+(z+h+\eta)^2}} d\eta\right), \quad (z \geq 0), \quad (6)$$

$$T_2 = \frac{HC}{4\pi k_1}\int_0^{+\infty} \frac{\exp(-\gamma\eta)}{\sqrt{x^2+y^2+(z-h-\eta)^2}} d\eta, \quad (z \leq 0), \quad (7)$$

where A, B, C and γ are unknowns to be determined.

3. A point heat source interacting with weakly conducting interface

Let us first consider a point heat source interacting with a weakly conducting interface described by Equation (2). Inserting Equations (6) and (7) into the interface condition (2) for a weakly conducting interface and using the following relations

$$\begin{aligned}
\int_0^{+\infty} \frac{\exp(-\gamma\eta)}{\sqrt{x^2+y^2+(z+h+\eta)^2}} d\eta &= \exp[\gamma(z+h)] \int_{z+h}^{+\infty} \frac{\exp(-\gamma q)}{\sqrt{x^2+y^2+q^2}} dq, \\
\int_0^{+\infty} \frac{\exp(-\gamma\eta)}{\sqrt{x^2+y^2+(z-h-\eta)^2}} d\eta &= -\exp[-\gamma(z-h)] \int_{z-h}^{-\infty} \frac{\exp(\gamma q)}{\sqrt{x^2+y^2+q^2}} dq,
\end{aligned} \quad (8)$$

we arrive at the following set of linear algebraic equations

$$\begin{aligned}
A &= 1, \\
k_1 B &= k_2 C, \\
k_1 B &= 2\alpha, \\
k_1 B\gamma &= \alpha(B+C).
\end{aligned} \quad (9)$$

Consequently, the unknowns A, B, C and γ can be uniquely determined and given by

$$\begin{aligned}
A &= 1, \\
B &= \frac{2\alpha}{k_1}, \\
C &= \frac{2\alpha}{k_2}, \\
\gamma &= \alpha\frac{k_1+k_2}{k_1 k_2}.
\end{aligned} \quad (10)$$

Thus, the explicit expressions for the temperature field in the two half-spaces are

$$T_1 = \frac{H}{4\pi k_1}\left(\frac{1}{\sqrt{x^2+y^2+(z-h)^2}} + \frac{1}{\sqrt{x^2+y^2+(z+h)^2}} - \frac{2\alpha}{k_1}\int_0^{+\infty} \frac{\exp\left(-\alpha\frac{k_1+k_2}{k_1k_2}\eta\right)}{\sqrt{x^2+y^2+(z+h+\eta)^2}}\,d\eta\right), \quad (z \geq 0), \tag{11}$$

$$T_2 = \frac{\alpha H}{2\pi k_1 k_2}\int_0^{+\infty} \frac{\exp\left(-\alpha\frac{k_1+k_2}{k_1k_2}\eta\right)}{\sqrt{x^2+y^2+(z-h-\eta)^2}}\,d\eta, \quad (z \leq 0). \tag{12}$$

Here it should be mentioned that the line integrals in Equations (11) and (12) are convergent due to the fact that

$$\gamma = \alpha\frac{k_1+k_2}{k_1k_2} > 0.$$

The distribution of temperature along the z-axis can be concisely given as

$$\tilde{T} = \begin{cases} \dfrac{1}{|\tilde{z}-1|} + \dfrac{1}{\tilde{z}+1} - \dfrac{2-\Gamma}{\tilde{z}+1} f\left(\lambda_1\dfrac{\tilde{z}+1}{2}\right), & (\tilde{z} \geq 0), \\ \dfrac{\Gamma}{1-\tilde{z}} f\left(\lambda_1\dfrac{1-\tilde{z}}{2}\right), & (\tilde{z} \leq 0), \end{cases} \tag{13}$$

where

$$\begin{aligned} \tilde{T} &= \frac{4\pi h k_1}{H}T, \\ \tilde{z} &= \frac{z}{h}, \\ \Gamma &= \frac{2k_1}{k_1+k_2}, \\ \lambda_1 &= \alpha h\frac{k_1+k_2}{k_1k_2}, \end{aligned} \tag{14}$$

and $f(\lambda)$, which falls in the range between 0 and 1, is defined by [Fan and Wang 2003]

$$f(\lambda) = 2\lambda\exp(2\lambda)E_1(2\lambda), \tag{15}$$

with $E_1(2\lambda)$ being the exponential integral function defined as follows [Abramovitz and Stegun 1972]:

$$E_1(2\lambda) = \int_{2\lambda}^{\infty} \frac{\exp(-t)}{t}\,dt. \tag{16}$$

Expression (13) indicates that the distribution of temperature along the z-axis is totally reliant on the two-phase conductivity parameter Γ, $(0 \leq \Gamma \leq 2)$ and λ_1, which measures the interface "rigidity" [Fan and Wang 2003]. Figure 2 demonstrates the distribution of temperature along the z-axis for a weakly conducting interface under various values of λ_1 with $\Gamma = 1.2$. It is observed that temperature is continuous across the interface $z = 0$ only when $\lambda_1 = \infty$ for a perfect interface, otherwise the temperature will be discontinuous across the weakly conducting interface. The influence of the interface imperfections on the temperature distribution is especially apparent for those points very close to the interface. The

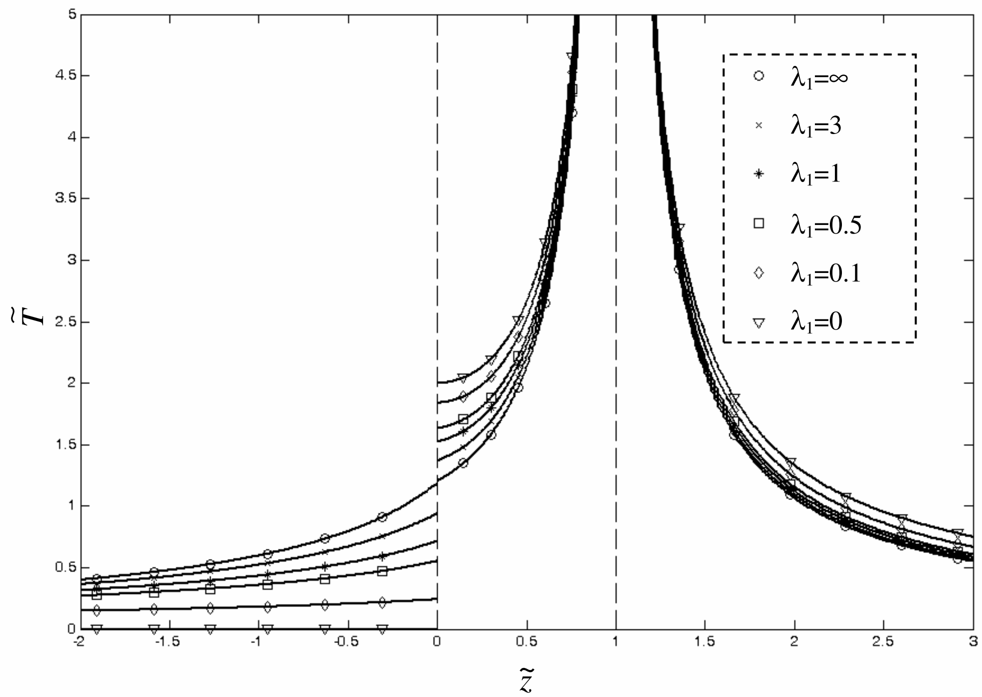

Figure 2. Distribution of the temperature along the z-axis for a weakly conducting interface with $\Gamma = 1.2$.

temperature in the positive z-axis is always higher than the corresponding one for a perfect interface, while the temperature in the negative z-axis is always lower than the corresponding one for a perfect interface.

4. A point heat source interacting with highly conducting interface

In this section, let us consider a point heat source interacting with a highly conducting interface, as described by Equation (4). Because the temperature field satisfies Laplace's Equation (2), the interface conditions for a highly conducting interface can be equivalently expressed as

$$T_1 = T_2, \qquad k_1\frac{\partial T_1}{\partial z} - k_2\frac{\partial T_2}{\partial z} = \beta\frac{\partial^2 T_1}{\partial z^2} = \beta\frac{\partial^2 T_2}{\partial z^2}, \quad (z = 0). \tag{17}$$

Inserting Equations (6) and (7) into the above interface conditions and using Equation (8), we arrive at the following set of linear algebraic equations

$$\begin{aligned} A &= -1, \\ B + C &= 0, \\ 2k_1 &= -\beta B, \\ k_1 + k_2 &= \beta\gamma. \end{aligned} \tag{18}$$

Consequently, the unknowns A, B, C and γ can be uniquely determined and given by

$$A = -1, \qquad B = -\frac{2k_1}{\beta}, \qquad C = \frac{2k_1}{\beta}, \qquad \gamma = \frac{k_1 + k_2}{\beta}. \tag{19}$$

Thus, the explicit expressions for the temperature field in the two half-spaces are

$$T_1 = \frac{H}{4\pi k_1}\left(\frac{1}{\sqrt{x^2+y^2+(z-h)^2}} - \frac{1}{\sqrt{x^2+y^2+(z+h)^2}} + \frac{2k_1}{\beta}\int_0^{+\infty} \frac{\exp\left(-\frac{k_1+k_2}{\beta}\eta\right)}{\sqrt{x^2+y^2+(z+h+\eta)^2}}\, d\eta\right), \quad (z \ge 0), \tag{20}$$

$$T_2 = \frac{H}{2\pi\beta}\int_0^{+\infty} \frac{\exp\left(-\frac{k_1+k_2}{\beta}\eta\right)}{\sqrt{x^2+y^2+(z-h-\eta)^2}}\, d\eta, \quad (z \le 0). \tag{21}$$

Here it should mentioned that the line integrals in Equations (20) and (21) are convergent due to the fact that

$$\gamma = \frac{k_1 + k_2}{\beta} > 0.$$

The distribution of temperature along the z-axis can also be concisely given by

$$\tilde{\boldsymbol{T}} = \begin{cases} \dfrac{1}{|\tilde{z}-1|} - \dfrac{1}{\tilde{z}+1} + \dfrac{\Gamma}{\tilde{z}+1} f\left(\lambda_2 \dfrac{\tilde{z}+1}{2}\right), & (\tilde{z} \ge 0), \\ \dfrac{\Gamma}{1-\tilde{z}} f\left(\lambda_2 \dfrac{1-\tilde{z}}{2}\right), & (\tilde{z} \le 0), \end{cases} \tag{22}$$

where $\tilde{T}$, $\tilde{z}$, Γ have been defined by Equation (14) and

$$\lambda_2 = \frac{h(k_1 + k_2)}{\beta}. \tag{23}$$

Expression (22) indicates that the distribution of temperature along the z-axis is totally reliant on Γ and λ_2 which also measures the interface "rigidity" as λ_1. Figure 3 illustrates the distribution of temperature along the z-axis for a highly conducting interface under various values of λ_2 with $\Gamma = 1.2$. It is observed that temperature is always continuous across the highly conducting interface $z = 0$. The temperature along the total z-axis is always lower than the corresponding one for a perfect interface. By comparing Figures 2 and 3 we observe that the distributions of the temperature in the negative z-axis ($z \le 0$) are the same for the two kinds of imperfect interface conditions when $\lambda_1 = \lambda_2$. In fact, by comparing Equation (12) with Equation (21), we find that the distribution of the temperature in the lower half-space is always

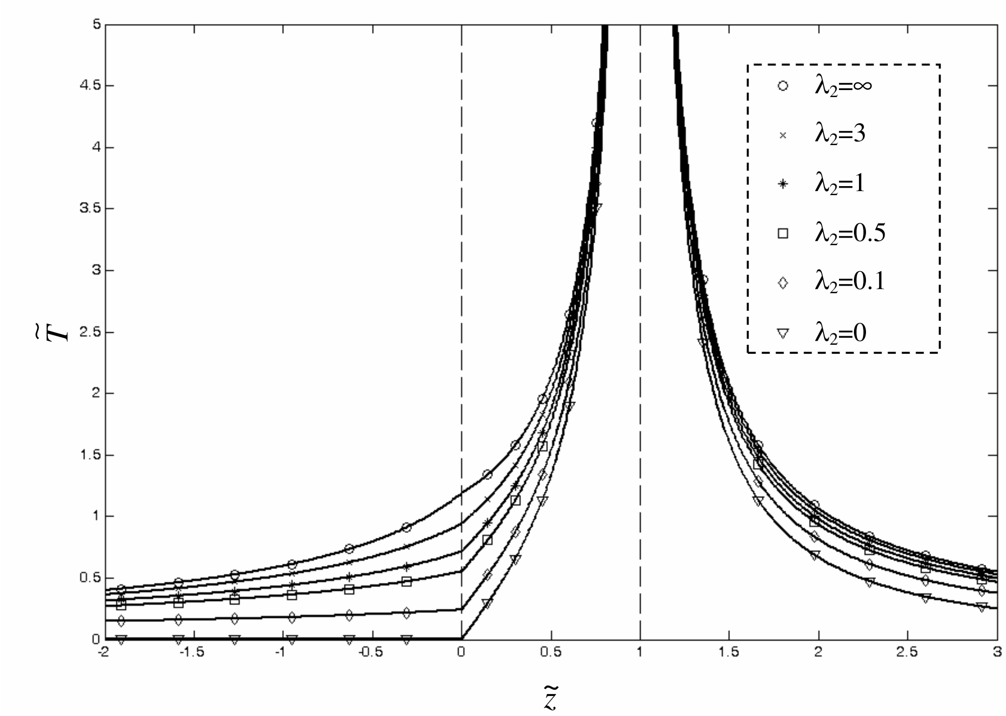

Figure 3. Distribution of the temperature along the z-axis for a highly conducting interface with $\Gamma = 1.2$.

exactly the same for the two types of imperfect interface conditions when $\lambda_1 = \lambda_2$, or equivalently when $\alpha\beta = k_1 k_2$.

5. An application

Besides the incorporation of the present solution in BEM which does not require the discretization of the imperfect interface, another interesting application is that of a static point electric charge Q located at $(0, 0, h)$, $(h > 0)$ in two imperfectly bonded half-spaces with dielectric constants ϵ_1 and ϵ_2. (Since the differential equations for the electrostatic problem and for the heat conduction problem are identical, the results for the heat conduction problem obtained in the previous two sections can be applied directly to the electrostatic problem considered in this section). The Coulomb force F on the electric charge due to its interaction with a weakly conducting interface is

$$F = \frac{Q^2\Big(1 - 2\lambda_1(2-\Gamma)\big(1 - f(\lambda_1)\big)\Big)}{16\pi\epsilon_1 h^2}, \tag{24}$$

where

$$\Gamma = \frac{2\epsilon_1}{\epsilon_1+\epsilon_2} \quad \text{and} \quad \lambda_1 = \alpha h \frac{\epsilon_1+\epsilon_2}{\epsilon_1\epsilon_2}.$$

Here the boundary conditions on the weakly conducting interface are

$$\epsilon_1 \frac{\partial \phi_1}{\partial z} = \epsilon_2 \frac{\partial \phi_2}{\partial z} = \alpha(\phi_1 - \phi_2), \quad (z = 0), \tag{25}$$

where ϕ is the electric potential, α is a nonnegative constant interface parameter.

Similarly, the Coulomb force F on the electric charge due to its interaction with a highly conducting interface is

$$F = \frac{Q^2\Big(2\lambda_2\Gamma\big(1 - f(\lambda_2)\big) - 1\Big)}{16\pi\epsilon_1 h^2}, \tag{26}$$

where

$$\lambda_2 = \frac{h(\epsilon_1 + \epsilon_2)}{\beta}.$$

Here the boundary conditions on the highly conducting interface are

$$\phi_1 = \phi_2, \qquad \epsilon_2 \frac{\partial \phi_2}{\partial z} - \epsilon_1 \frac{\partial \phi_1}{\partial z} = \beta\Delta_s\phi_1 = \beta\Delta_s\phi_2, \quad (z = 0), \tag{27}$$

where β is a nonnegative constant interface parameter.

In Equations (24) and (26) the Coulomb force F is in the z-direction due to the fact that all the image charges are distributed at the z-axis. A positive value of the force means that the electric charge is repelled from the interface whereas a negative value means that the electric charge is attracted to the interface. By applying the following asymptotic expansion

$$1 - f(\eta) \cong \frac{1}{2\eta} - \frac{1}{2\eta^2} + o\left(\frac{1}{\eta^3}\right), \qquad \text{when } \eta \to \infty, \tag{28}$$

the Coulomb force F on the electric charge due to its interaction with a perfect interface ($\lambda_1, \lambda_2 \to \infty$) is

$$F = \frac{(\Gamma - 1)Q^2}{16\pi\epsilon_1 h^2}. \tag{29}$$

The above indicates that the electric charge will be repelled from the perfect interface when $\Gamma > 1$ and it will be attracted to the perfect interface when $\Gamma < 1$. The situation in which $F = 0$ occurs only when $\Gamma = 1$ or $\epsilon_1 = \epsilon_2$. In other words, there is no equilibrium position, $F = 0$, for an electric charge interacting with a perfect interface separating two half-spaces with different dielectric properties.

On the other hand, it follows from Equation (24) that it is possible to find a situation in which the Coulomb force F on the electric charge due to its interaction with a weakly conducting interface is zero, if the following condition is satisfied

$$f(\lambda_1) = 1 - \frac{1}{2\lambda_1(2 - \Gamma)}. \tag{30}$$

It can be easily observed from (24) that

$$F = \frac{Q^2}{16\pi\epsilon_1 h^2} > 0,$$

Γ	λ_1	Γ	λ_2
0	0.7798	2	0.7798
0.1	0.8788	1.9	0.8788
0.2	1.004	1.8	1.004
0.3	1.1671	1.7	1.1671
0.4	1.3873	1.6	1.3873
0.5	1.6999	1.5	1.6999
0.6	2.1754	1.4	2.1754
0.7	2.9792	1.3	2.9792
0.8	4.6095	1.2	4.6095
0.9	9.563	1.1	9.563
0.95	19.534	1.05	19.534
1	∞	1	∞

Table 1. The pairs of Γ and λ_1 (left side) that satisfy Equation (30). The pairs of Γ and λ_2 (right side) that satisfy Equation (31).

if $h \to 0$ (or equivalently $\lambda_1 \to 0$) and

$$F = \frac{Q^2(\Gamma - 1)}{16\pi\epsilon_1 h^2} < 0,$$

if $h \to \infty$ (or equivalently $\lambda_1 \to \infty$) and $\Gamma < 1$. Consequently, the equilibrium position determined by Equation (30) is a stable one. Table 1 (left side) presents the pairs of Γ and λ_1 that satisfy (30). We find that only when $\Gamma < 1$ (or equivalently $\epsilon_1 < \epsilon_2$, when the upper half-space is less conducting than the lower half-space) does an equilibrium position for the electric charge exist.

Similarly, it follows from Equation (26) that it is possible to find a situation in which the Coulomb force F on the electric charge due to its interaction with a highly conducting interface is zero if the following condition is satisfied

$$f(\lambda_2) = 1 - \frac{1}{2\lambda_2\Gamma}. \tag{31}$$

It can also be easily observed from Equation (26) that

$$F = -\frac{Q^2}{16\pi\epsilon_1 h^2} < 0,$$

if $h \to 0$ (or equivalently $\lambda_2 \to 0$) and

$$F = \frac{Q^2(\Gamma - 1)}{16\pi\epsilon_1 h^2} > 0,$$

if $h \to \infty$ (or equivalently $\lambda_2 \to \infty$) and $\Gamma > 1$. Consequently, the equilibrium position determined by Equation (31) is an unstable one. Table 1 (right side) presents the pairs of Γ and λ_2 that satisfy (31). We find that only when $\Gamma > 1$ (or equivalently $\epsilon_1 > \epsilon_2$, the upper half-space is more conducting than the lower half-space) does an equilibrium position for the electric charge exist.

The physical implication of the phenomenon of the existence of an equilibrium position for the electric charge interacting with the imperfect interface is that a properly chosen imperfect interface "shields" the charge located in the upper half-space from the interference of the lower half-space which has a different dielectric constant than the one in which it is embedded. The shielding is achieved by a compensation effect that the imperfect interface introduces. This, in a sense, is similar to the so-called neutral inhomogeneities which may be rendered "invisible" through some properly chosen imperfect interface (see [Benveniste and Miloh 1999], and [Milton 2003, Section 7.11]). In the present case, the lower half-space is the neutral inhomogeneity. In other words, as far as the point charge is concerned, its effect has been made neutral through the presence of a suitably chosen imperfect interface.

6. Conclusions

We have presented in Equations (11) and (12) the Green's function for a steady heat source interacting with a weakly conducting interface. Similarly, Equations (20) and (21) present the Green's function for a steady heat source interacting with a highly conducting interface. In particular, the temperature along the z-axis for both kinds of imperfect interface conditions can be concisely expressed in terms of the exponential integral function. We have also tried to derive the Green's function for a steady point heat source interacting with the following interface model of [Bövik 1994]

$$\left.\begin{aligned} T_1 - T_2 &= \frac{t}{2}\left(\frac{k_1}{k_0}-1\right)\frac{\partial T_1}{\partial z} + \frac{t}{2}\left(\frac{k_2}{k_0}-1\right)\frac{\partial T_2}{\partial z}, \\ k_2\frac{\partial T_2}{\partial z} - k_1\frac{\partial T_1}{\partial z} &= \frac{t}{2}(k_0-k_1)\Delta_s T_1 + \frac{t}{2}(k_0-k_2)\Delta_s T_2, \end{aligned}\right\} \quad (z=0), \tag{32}$$

which can reduce to a weakly conducting interface, Equation (2), by letting $t \to 0$ and $k_0 \to 0$. Also (32) can be reduced to a highly conducting one, (4), by letting $t \to 0$ and $k_0 \to \infty$. Unfortunately, the image method adopted here is invalid in treating this more general kind of imperfect interface. More specifically, the assumption of Equation (6) and (7) with four undetermined constants A, B, C and γ for the temperature field in the two half-spaces is not sufficient to satisfy Equation (32).

Acknowledgments

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