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MAGNETOTHERMOELASTIC STRESSES INDUCED BY A TRANSIENT MAGNETIC FIELD IN AN INFINITE CONDUCTING PLATE

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We investigate the dynamic and quasistatic behavior of magnetothermoelastic stresses induced by a transient magnetic field in an infinite conducting plate. A transient magnetic field defined by an arbitrary function of time acts on both surfaces of the infinite plate and parallel to them. The fundamental equations of one-dimensional electromagnetic, temperature and elastic fields are formulated, and solutions for the magnetic field, eddy current, temperature change and dynamic and quasistatic solutions for stresses and deformations are analytically derived, in terms of the excitation function. The stress solutions are determined to be sums of a thermal stress component caused by eddy current loss and a magnetic stress component caused by the Lorentz force. The case of a magnetic field defined by a smoothed ramp function with a sine-function profile is examined in particular, and the dynamic and quasistatic behavior of the stresses are numerically calculated.

1. Introduction

Mechanical structures that are activated when a magnetic field is applied has been of increasing interest in recent years. When a time-dependent magnetic field acts on a conducting medium, an eddy current is induced, which generates heat; this is the eddy current energy loss due to the Joule effect. The conducting medium is also subjected to a Lorentz force. Thus, two kinds of stress arise: thermal stress caused by eddy current loss magnetic stress caused by the Lorentz force.

In the field of magnetoelasticity or magnetothermoelasticity, many studies have employed an analytical treatment of the interaction between elastic, electromagnetic and temperature fields; see, for instance, [Kaliski and Nowacki 1962; Kaliski and Michalec 1963; Paria 1967; Wauer 1996; Banerjee and Roychoudhuri 1997; Wang et al. 2002; 2003; Librescu et al. 2003; Ezzat and Youssef 2005; Zheng et al. 2005]. However, there have been only a few analytical studies of thermal stresses induced by time-dependent magnetic fields [Moon and Chattopadhyay 1974; Chian and Moon 1981; Wauer 1995]. Moon and Chattopadhyay [1974] have studied thermal stresses and magnetic stresses in a conducting half-space caused by an applied jump in tangential magnetic field at the boundary. Chian and Moon [1981] have extended that work, investigating the same stresses in a hollow cylindrical conductor caused by a pulsed magnetic field at the cavity. Wauer [1995] has studied the dynamic behavior of a magnetothermoelastic plate layer whose surfaces are subjected to a magnetic field composed of a constant and a harmonically oscillating part in the direction parallel to the surfaces. He has mentioned the stability of the plate due to the external magnetic field. Pantelyat and Féliachi [2002] have studied the mechanical behavior of metals

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in induction heating devices by using of finite element method. They have calculated thermoelastic-plastic stresses induced by an alternating magnetic field, taking into account the temperature dependence of the material properties.

Here we investigate the dynamic and quasistatic behavior of magnetothermoelastic stresses induced by a transient magnetic field on an infinite conducting plate made of a nonferromagnetic metal such as copper or aluminum. Assuming an applied magnetic field defined by an arbitrary function of time, acting on both sides of the plate and parallel to it, we formulate the fundamental equations of the one-dimensional electromagnetic, temperature and elastic fields. We then solve for the electromagnetic field, temperature change and dynamic and quasistatic solutions of stresses and displacements, analytically deriving expressions for these fields in terms of the arbitrary excitation function. The stresses solutions are determined to be the sums of a thermal stress and a magnetic stress component.

We then focus on the case of an excitation given by a smoothed ramp function with sine-function profile, studying numerically the dynamic and quasistatic behavior of the induced thermal and magnetic stresses.

2. Fundamental equations

2.1. ***Electromagnetic field.*** Figure 1 shows an infinite conducting plate of thickness $2b$ with a Cartesian coordinate system, subject to a time-dependent magnetic field $H_0\phi(t)$ that is uniformly distributed along the x and z directions and acts on both side surfaces of the infinite plate in the z direction, starting at time $t = 0$. Here H_0 is a reference magnetic field strength and $\phi(t)$ is an arbitrary function of time.

Let the magnetic field be $\boldsymbol{H} = (0, 0, H_z(y, t))$ in the infinite plate, and let the electric field vector be $\boldsymbol{E} = (E_x(y, t), 0, 0)$. Disregarding the displacement current, the governing equations and the constitutive relations of electromagnetics reduce to (see [Stoll 1974; Moon and Chattopadhyay 1974])

$$\begin{gathered} -\frac{\partial E_x}{\partial y} + \frac{\partial B_z}{\partial t} = 0, \quad \frac{\partial H_z}{\partial y} = J_x, \\ \sigma\left(E_x + B_z \frac{\partial v}{\partial t}\right) = J_x, \quad B_z = \mu H_z, \end{gathered} \tag{1}$$

where B_z is the magnetic flux in the z direction, J_x is the electric current density in the x direction, v is the displacement in the y direction (as discussed later, no displacement is considered in the x and z directions), and σ and μ are the electric conductivity and the magnetic permeability in the infinite plate.

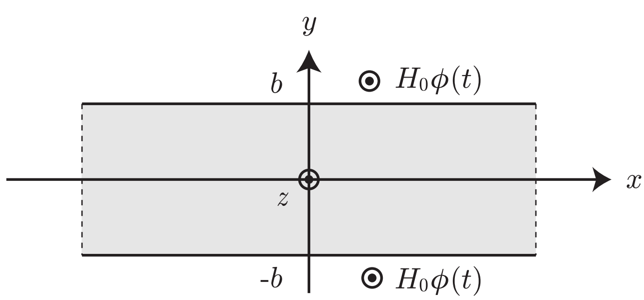

Figure 1. Conditions and coordinate system of infinite plate.

This leads to the fundamental equation of magnetic field [Moon and Chattopadhyay 1974]:

$$\frac{\partial^2 H_z}{\partial y^2} = \mu\sigma \frac{\partial H_z}{\partial t} + \mu\sigma \frac{\partial}{\partial y}\left(H_z \frac{\partial v}{\partial t}\right), \tag{2}$$

where the second term on the right is a nonlinear coupling term with elastic field. This coupling term is small compared with the first term $\mu\sigma\,\partial H_z/\partial t$, as shown in [Moon and Chattopadhyay 1974; Chian and Moon 1981]. Therefore, the coupled equation (2) with the elastic field reduces to the uncoupled equation

$$\frac{\partial^2 H_z}{\partial y^2} = \mu\sigma \frac{\partial H_z}{\partial t}. \tag{3}$$

The boundary conditions and initial condition are

$$\begin{aligned} &\text{at } y = \pm b: \quad H_z = H_0 \phi(t), \\ &\text{at } t = 0: \qquad H_z = 0. \end{aligned} \tag{4}$$

The current density $J_x = \partial H_z/\partial y$ induced by the variation of the magnetic field is called the eddy current.

2.2. ***Temperature field.*** The eddy current J_x generates Joule heat, giving rise to the so-called eddy current loss $w(y, t)$. The eddy current loss per unit time per unit volume is given by (see [Moon and Chattopadhyay 1974])

$$w(y, t) = \sigma^{-1} J_x(y, t)^2. \tag{5}$$

We assume that the infinite plate with zero initial temperature change is heated by the eddy current loss $w(y, t)$ from time $t = 0$, and that both side surfaces are insulated, or subjected to surrounding media at temperature 0, with relative heat transfer coefficients h.

The one-dimensional heat conduction equation taking into account the eddy current loss [Moon and Chattopadhyay 1974] is then given by

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2} + \frac{w}{C\rho}, \tag{6}$$

with boundary conditions and initial condition

$$\begin{aligned} &\text{at } y = \pm b: \quad \frac{\partial T}{\partial y} \pm hT = 0, \\ &\text{at } t = 0: \qquad T(y, 0) = 0, \end{aligned} \tag{7}$$

where $T = T(y, t)$ is temperature change and κ, C and ρ denote the thermal conductivity, the specific heat and the mass density. If both surfaces are insulated, then h in (7) becomes zero. In (6), the coupling term with strain is neglected because the coupling effect mainly occurs at large times [Boley and Tolins 1962; Moon and Chattopadhyay 1974].

2.3. ***Elastic field.*** Besides the temperature change arising from the eddy current loss, the plate is subjected to a Lorentz force $\boldsymbol{f}$, given by (see [Moon and Chattopadhyay 1974])

$$\boldsymbol{f} = \boldsymbol{J} \times \boldsymbol{B} = \begin{pmatrix} \dfrac{\partial H_z}{\partial y} \\ 0 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ \mu H_z \end{pmatrix} = \begin{pmatrix} 0 \\ -\dfrac{\mu}{2}\dfrac{\partial}{\partial y}\left[H_z(y,t)\right]^2 \\ 0 \end{pmatrix} \tag{8}$$

Thus the Lorentz force only has a y component:

$$f_y(y,t) = -\frac{\mu}{2}\frac{\partial}{\partial y}\big(H_z(y,t)\big)^2 \tag{9}$$

Because the temperature change and Lorentz force depend only on y and t, the displacement components are assumed to be $(0, v(y,t), 0)$. Thus the stress-displacement relations taking into account temperature change reduce to (see [Sternberg and Chakravorty 1959])

$$\begin{aligned} \sigma_{xx}(y,t) = \sigma_{zz}(y,t) &= \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\left(\frac{\nu}{1-\nu}\frac{\partial v}{\partial y} - \frac{1+\nu}{1-\nu}\alpha T\right), \\ \sigma_{yy}(y,t) &= \frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\left(\frac{\partial v}{\partial y} - \frac{1+\nu}{1-\nu}\alpha T\right), \end{aligned} \tag{10}$$

where $(\sigma_{xx}, \sigma_{yy}, \sigma_{zz})$ are the stress components and ν, E and α denote the Poisson ratio, the Young's modulus and the coefficient of linear thermal expansion. The equation of motion in the y direction, taking into account Lorentz force, is given by (see [Moon and Chattopadhyay 1974])

$$\frac{\partial \sigma_{yy}}{\partial y} + f_y = \rho\frac{\partial^2 v}{\partial t^2}. \tag{11}$$

Substitution of (9) and $\bar{\sigma}_{yy}$ from (10) into (11) leads to the displacement equation of motion

$$\frac{\partial^2 v}{\partial y^2} = \frac{1}{C_L^2}\frac{\partial^2 v}{\partial t^2} + \frac{1+\nu}{1-\nu}\alpha\frac{\partial T}{\partial y} + \frac{(1+\nu)(1-2\nu)}{(1-\nu)E}\frac{\mu}{2}\frac{\partial}{\partial y}\big(H_z\big)^2, \tag{12}$$

where

$$C_L = \sqrt{\frac{(1-\nu)E}{(1+\nu)(1-2\nu)\rho}}. \tag{13}$$

is the velocity of longitudinal wave. The infinite plate is at rest before $t = 0$ and we suppose that the surfaces are traction-free ($\sigma_{yy} = 0$). Thus the mechanical boundary conditions and initial conditions are

$$\begin{aligned} &\text{at } y = \pm b: \qquad \frac{\partial v}{\partial y} = \frac{1+\nu}{1-\nu}\alpha T, \\ &\text{at } t = 0: \qquad v = \frac{\partial v}{\partial t} = 0. \end{aligned} \tag{14}$$

2.4. *Dimensionless quantities.* We define the dimensionless quantities

$$\bar{y}=\frac{y}{b},\quad \bar{H}_z=\frac{H_z}{H_0},\quad \tau=\frac{t}{\mu\sigma b^2},\quad \bar{J}_x=\frac{bJ_x}{H_0},\quad \bar{w}=\frac{\sigma b^2 w}{H_0^2},\quad \bar{T}=\frac{C\gamma T}{\mu H_0^2},\quad \bar{h}=bh,$$
$$\bar{f}_y=\frac{bf_y}{\mu H_0^2},\quad (\bar{\sigma}_{xx},\bar{\sigma}_{yy},\bar{\sigma}_{zz})=\frac{(\sigma_{xx},\sigma_{yy},\sigma_{zz})}{\frac{\mu H_0^2}{2}},\quad \bar{v}=\frac{(1-\nu)E}{(1+\nu)(1-2\nu)}\frac{2}{b\mu H_0^2}v \tag{15}$$

and

$$\chi_1=\mu\sigma\kappa,\quad \chi_2=\mu\sigma bC_L,\quad \chi_3=\frac{2\alpha E}{(1-2\nu)C\rho}. \tag{16}$$

In terms of these dimensionless quantities, the equality $J_x=\partial H_z/\partial y$ and Equations (3)–(7), (9), (10), (12), (14) become:

(1) *Electromagnetic field:*

Equation system:
$$\frac{\partial^2\bar{H}_z}{\partial\bar{y}^2}=\frac{\partial\bar{H}_z}{\partial\tau} \tag{17}$$

with conditions
$$\begin{aligned}&\text{at } \bar{y}=\pm 1: && \bar{H}_z=\phi(\tau)\\ &\text{at } \tau=0: && \bar{H}_z=0\end{aligned} \tag{18}$$

Eddy current:
$$\bar{J}_x(\bar{y},\tau)=\frac{\partial\bar{H}_z(\bar{y},\tau)}{\partial\bar{y}} \tag{19}$$

(2) *Temperature field:*

Eddy current loss:
$$\bar{w}(\bar{y},\tau)=\big(\bar{J}_x(\bar{y},\tau)\big)^2 \tag{20}$$

Equation system:
$$\frac{\partial\bar{T}}{\partial\tau}=\chi_1\frac{\partial^2\bar{T}}{\partial\bar{y}^2}+\bar{w} \tag{21}$$

with conditions
$$\begin{aligned}&\text{at } \bar{y}=\pm 1: && \frac{\partial\bar{T}}{\partial\bar{y}}\pm\bar{h}\bar{T}=0\\ &\text{at } \tau=0: && \bar{T}=0\end{aligned} \tag{22}$$

(3) *Elastic field:*

Lorentz force:
$$\bar{f}_y(\bar{y},\tau)=-\frac{1}{2}\frac{\partial}{\partial\bar{y}}\big(\bar{H}_z(\bar{y},\tau)\big)^2 \tag{23}$$

Stress-displacement relations:
$$\begin{aligned}\bar{\sigma}_{xx}(\bar{y},\tau)=\bar{\sigma}_{zz}(\bar{y},\tau)&=\frac{\nu}{1-\nu}\frac{\partial\bar{v}}{\partial\bar{y}}-\chi_3\bar{T}\\ \bar{\sigma}_{yy}(\bar{y},\tau)&=\frac{\partial\bar{v}}{\partial\bar{y}}-\chi_3\bar{T}\end{aligned} \tag{24}$$

Equation system:
$$\frac{\partial^2 \bar{v}}{\partial \bar{y}^2} = \frac{1}{\chi_2^2}\frac{\partial^2 \bar{v}}{\partial \tau^2} + \chi_3 \frac{\partial \bar{T}}{\partial \bar{y}} + \frac{\partial}{\partial \bar{y}}\left(\bar{H}_z\right)^2 \tag{25}$$

with conditions
$$\begin{aligned} &\text{at } \bar{y} = \pm 1: \quad \frac{\partial \bar{v}}{\partial \bar{y}} = \chi_3 \bar{T} \\ &\text{at } \tau = 0: \qquad \bar{u} = \frac{\partial \bar{u}}{\partial \tau} = 0 \end{aligned} \tag{26}$$

3. Solutions

3.1. ***Magnetic field.*** To transform the inhomogeneous boundary condition $\bar{H}_z = \phi(\tau)$ from (18) into a homogeneous one, we assume that the solution of (17) is given by

$$\bar{H}_z(\bar{y}, \tau) = h_z(\bar{y}, \tau) + \phi(\tau). \tag{27}$$

By substitution of (27) into (17)–(18), the equation system with respect to h_z becomes

$$\frac{\partial^2 h_z}{\partial \bar{y}^2} = \frac{\partial h_z}{\partial \tau} + \frac{\partial \phi(\tau)}{\partial \tau} \tag{28}$$

with boundary and initial conditions

$$\begin{aligned} &\text{at} \bar{y} = \pm 1: \quad h_z = 0, \\ &\text{at } \tau = 0: \qquad h_z = -\phi(0). \end{aligned} \tag{29}$$

By separation of variables, the solution of (28) will be assumed to be of the form

$$h_z(\bar{y}, \tau) = \sum_{n=1}^{\infty} a_n(\tau) \cos(k_n \bar{y}), \tag{30}$$

where the $a_n(\tau)$ are unknown functions of τ and the k_n are the positive roots of the eigenequation

$$\cos(k_n) = 0 \quad \therefore \quad k_n = \frac{(2n-1)\pi}{2} \quad (n = 1, 2, \dots) \tag{31}$$

The solution $h_z(\bar{y}, \tau)$ in (30) clearly satisfies the homogeneous boundary conditions in (29).

Substitution of (30) into (28) gives

$$-\sum_{n=1}^{\infty} k_n^2 a_n(\tau) \cos(k_n \bar{y}) = \sum_{n=1}^{\infty} \frac{\mathrm{d}a_n(\tau)}{\mathrm{d}\tau} \cos(k_n \bar{y}) + \frac{\mathrm{d}\phi(\tau)}{\mathrm{d}\tau} \tag{32}$$

Multiplying both sides by $\cos(k_m \bar{y})$ and integrating it from -1 to 1, we obtain

$$\begin{aligned} -\sum_{n=1}^{\infty} k_n^2 a_n(\tau) &\int_{-1}^{1} \cos(k_n \bar{y}) \cos(k_m \bar{y})\, \mathrm{d}\bar{y} \\ &= \sum_{n=1}^{\infty} \frac{\mathrm{d}a_n(\tau)}{\mathrm{d}\tau} \int_{-1}^{1} \cos(k_n \bar{y}) \cos(k_m \bar{y})\, \mathrm{d}\bar{y} + \int_{-1}^{1} \frac{\mathrm{d}\phi(\tau)}{\mathrm{d}\tau} \cos(k_m \bar{y})\, \mathrm{d}\bar{y}. \end{aligned} \tag{33}$$

By virtue of the orthogonal property of trigonometric functions, we obtain

$$\int_{-1}^{1} \cos(k_n \bar{y}) \cos(k_m \bar{y})\, \mathrm{d}\bar{y} = \begin{cases} 1 & (m = n), \\ 0 & (m \neq n). \end{cases} \tag{34}$$

Substituting (34) into (33) gives

$$\frac{\mathrm{d}a_n(\tau)}{\mathrm{d}\tau} + k_n a_n(\tau) = -\int_{-1}^{1} \frac{\mathrm{d}\phi(\tau)}{\mathrm{d}\tau} \cos(k_n \bar{y})\, \mathrm{d}\bar{y}. \tag{35}$$

By use of the initial condition in (29), the solutions of (35) are determined to be

$$a_n(\tau) = \frac{2(-1)^n}{k_n} \hat{a}_n(\tau). \tag{36}$$

where the $\hat{a}_n(\tau)$ are determined by the function $\phi(\tau)$:

$$\hat{a}_n(\tau) = \int_0^{\tau} \mathrm{e}^{-k_n^2(\tau-\tau')} \frac{\mathrm{d}\phi(\tau')}{\mathrm{d}\tau'}\, \mathrm{d}\tau'. \tag{37}$$

From (27), (30) and (36), the magnetic field $\bar{H}_z$ is written as

$$\bar{H}_z(\bar{y}, \tau) = \phi(\tau) + 2\sum_{n=1}^{\infty} \frac{(-1)^n}{k_n} \cos(k_n \bar{y}) \hat{a}_n(\tau). \tag{38}$$

Substitution of (38) into (19) gives the eddy current $\bar{J}_x$ as follows:

$$\bar{J}_x(\bar{y}, \tau) = 2\sum_{n=1}^{\infty} (-1)^{n+1} \sin(k_n \bar{y}) \hat{a}_n(\tau). \tag{39}$$

3.2. ***Temperature field.*** By separation of variables, the solution of (21) will be assumed to be of the form

$$\bar{T}(\bar{y}, \tau) = \sum_{j=\beta}^{\infty} b_j(\tau) \cos(p_j \bar{y}), \qquad \beta = \begin{cases} 0 & \text{for} \quad \bar{h} = 0, \\ 1 & \text{for} \quad \bar{h} > 0, \end{cases} \tag{40}$$

where the $b_j(\tau)$ are unknown functions of τ, and the p_j are the nonnegative roots of the eigenequations

$$\begin{aligned} &\sin p_j = 0 \qquad (p_j \geq 0 \text{ for } j = 0, 1, 2, \ldots) \qquad \text{if } \bar{h} = 0, \\ &\tan p_j = \frac{h}{p_j} \quad (p_j > 0 \text{ for } j = 1, 2, 3, \ldots) \qquad \text{if } \bar{h} > 0. \end{aligned} \tag{41}$$

The solution $\bar{T}(\bar{y}, \tau)$ in (40) clearly satisfies the boundary conditions in (22).

By virtue of the orthogonal property of trigonometric functions, we have

$$\int_{-1}^{1} \cos(p_j \bar{y}) \cos(p_l \bar{y})\, \mathrm{d}\bar{y} = \begin{cases} M_j & (l = j), \\ 0 & (l \neq j), \end{cases} \tag{42}$$

where
$$M_j = \begin{cases} 2 & (j=0) \\ \left.\begin{array}{ll} 1 & \text{for} \quad \bar{h}=0 \\ \dfrac{\bar{h}+\bar{h}^2+p_j^2}{\bar{h}^2+p_j^2} & \text{for} \quad \bar{h}>0 \end{array}\right\} & (j>0) \end{cases} \tag{43}$$

Substituting (40) into (21) and using (42), we obtain

$$\frac{\mathrm{d}b_j(\tau)}{\mathrm{d}\tau} + \chi_1 p_j^2 b_j(\tau) = \frac{1}{M_j}\int_{-1}^{1} \bar{w}(\bar{y},\tau)\cos(p_j\bar{y})\,\mathrm{d}\bar{y} \tag{44}$$

Through the use of the initial condition in (21), the solutions of (44) are determined to be

$$b_j(\tau) = \begin{cases} \dfrac{1}{2}\displaystyle\int_{-1}^{1}\left(\int_0^{\tau} \bar{w}(\bar{y},\tau')\,\mathrm{d}\tau'\right)\mathrm{d}\bar{y} & (j=0) \\ \dfrac{1}{M_j}\displaystyle\int_{-1}^{1}\left(\int_0^{\tau} \mathrm{e}^{-\chi_1 p_j^2(\tau-\tau')}\bar{w}(\bar{y},\tau')\,\mathrm{d}\tau'\right)\cos(p_j\bar{y})\,\mathrm{d}\bar{y} & (j>0) \end{cases} \tag{45}$$

Substituting (39) into (20), we obtain the eddy current loss:

$$\bar{w}(\bar{y},\tau) = 4\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}(-1)^{m+n}\sin(k_m\bar{y})\sin(k_n\bar{y})\hat{a}_m(\tau)\hat{a}_n(\tau) \tag{46}$$

Substitution of (46) into (45) gives

$$b_j(\tau) = \begin{cases} 2\displaystyle\sum_{n=1}^{\infty}\hat{b}_n^{(0)}(\tau) & (j=0), \\ \dfrac{4}{M_j}\displaystyle\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} I_{1jmn}\hat{b}_{jmn}(\tau) & (j>0), \end{cases} \tag{47}$$

where

$$\begin{aligned} I_{1jmn} &= (-1)^{m+n}\int_{-1}^{1}\sin k_m\bar{y}\sin k_n\bar{y}\cos p_j\bar{y}\,\mathrm{d}\bar{y} \\ &= \begin{cases} \left.\begin{array}{ll} -\frac{1}{2}(-1)^{m+n} & (m+n=j+1) \\ \frac{1}{2}(-1)^{m+n} & (|m-n|=j) \\ 0 & (\text{otherwise}) \end{array}\right\} & \text{if } \bar{h}=0, \\ \bar{h}\cos(p_j)\dfrac{2(k_m^2+k_n^2-p_j^2)}{(2k_mk_n)^2-(k_m^2+k_n^2-p_j^2)^2} & \text{if } \bar{h}>0, \end{cases} \end{aligned} \tag{48}$$

and where the $\hat{b}_n^{(0)}(\tau)$ and $\hat{b}_{jmn}(\tau)$ are determined by the function $\phi(\tau)$:

$$\begin{aligned} \hat{b}_n^{(0)}(\tau) &= \int_0^{\tau} \left(\hat{a}_n(\tau')\right)^2 \, \mathrm{d}\tau', \\ \hat{b}_{jmn}(\tau) &= \int_0^{\tau} \mathrm{e}^{-\chi_1 p_j^2(\tau-\tau')} \hat{a}_m(\tau')\hat{a}_n(\tau') \, \mathrm{d}\tau'. \end{aligned} \tag{49}$$

Substituting (47) into (40), we obtain for the temperature change

$$\bar{T}(\bar{y}, \tau) = 2\sum_{n=1}^{\infty} \hat{b}_n^{(0)}(\tau) + 4\sum_{j=1}^{\infty} \frac{1}{M_j} \cos(p_j \bar{y}) \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} I_{1jmn} \hat{b}_{jmn}(\tau) \tag{50}$$

where, for $\bar{h} > 0$, the first term on the right-hand side in (50) is ignored.

3.3. *Elastic field.*

3.3.1. *Dynamic solutions.* To transform the inhomogeneous boundary condition $\frac{\partial \bar{v}}{\partial \bar{y}} = \chi_3 \bar{T}$ from (26) into a homogeneous one, we assume that the displacement $\bar{v}(\bar{y}, \tau)$ is given by

$$\bar{v}(\bar{y}, \tau) = v_1(\bar{y}, \tau) + v_2(\bar{y}, \tau), \tag{51}$$

where $v_1(\bar{y}, \tau)$ satisfies

$$\frac{\partial^2 v_1}{\partial \bar{y}^2} = 0 \qquad \text{with boundary condition } \frac{\partial v_1}{\partial \bar{y}} = \chi_3 \bar{T} \text{ at } \bar{y} = \pm 1. \tag{52}$$

The solution of (52) is

$$v_1 = \chi_3 \bar{T}(\pm 1, \tau)\bar{y}, \tag{53}$$

where $\bar{T}(1, \tau) = \bar{T}(-1, \tau)$ from (50).

Substitution of (51) with (52) into (25)–(26) gives the equation system with respect to v_2 as

$$\frac{\partial^2 v_2}{\partial \bar{y}^2} = \frac{1}{\chi_2^2}\frac{\partial^2 v_2}{\partial \tau^2} + \frac{1}{\chi_2^2}\frac{\partial^2 v_1}{\partial \tau^2} + \chi_3 \frac{\partial \bar{T}}{\partial \bar{y}} + \frac{\partial}{\partial \bar{y}}(\bar{H}_z)^2 \tag{54}$$

with conditions

$$\begin{aligned} &\text{at } \bar{y} = \pm 1: \quad \frac{\partial v_2}{\partial \bar{y}} = 0, \\ &\text{at } \tau = 0: \quad \frac{\partial v_2}{\partial \tau} = -\frac{\partial v_1}{\partial \tau}, \quad v_2 = -v_1. \end{aligned} \tag{55}$$

By separation of variables, the solution of (54) will be assumed to be of the form

$$v_2(\bar{y}, \tau) = \sum_{i=1}^{\infty} c_i(\tau) \sin(\eta_i \bar{y}), \tag{56}$$

where the $c_i(\tau)$ are unknown functions of τ and the η_i are the positive roots of the eigenequation

$$\cos(\eta_i) = 0 \quad \therefore \quad \eta_i = \frac{(2i-1)\pi}{2} \quad (i = 1, 2, \ldots). \tag{57}$$

The solution $v_2(\bar{y}, \tau)$ in (56) clearly satisfies the homogeneous boundary conditions in (55).

By the orthogonality of trigonometric functions, we obtain

$$\int_{-1}^{1} \sin(\eta_i \bar{y}) \sin(\eta_q \bar{y})\, \mathrm{d}\bar{y} = \begin{cases} 1 & (q = i), \\ 0 & (q \neq i). \end{cases} \tag{58}$$

Substituting (56) into (54), and using (58) we obtain

$$\frac{\partial^2 c_i(\tau)}{\partial \tau^2} + \Omega_i^2 c_i(\tau) = -\int_{-1}^{1} \frac{\partial^2 v_1}{\partial \tau^2} \sin(\eta_i \bar{y})\, \mathrm{d}\bar{y} - \chi_3 \chi_2^2 \int_{-1}^{1} \frac{\partial \bar{T}}{\partial \bar{y}} \sin(\eta_i \bar{y})\, \mathrm{d}\bar{y} - \chi_2^2 \int_{-1}^{1} \frac{\partial}{\partial \bar{y}} \left(\bar{H}_z\right)^2 \sin(\eta_i \bar{y})\, \mathrm{d}\bar{y}, \tag{59}$$

where the Ω_i are the natural angular frequencies of the i-th mode in dimensionless form:

$$\Omega_i = \chi_2 \eta_i. \tag{60}$$

By the use of the initial condition in (55), the solutions of (59) are determined to be

$$\begin{aligned} c_i(\tau) = {} & \int_{-1}^{1} \left(-v_1(\bar{y}, \tau) + \Omega_i \int_0^{\tau} \sin \Omega_i (\tau - \tau') v_1(\bar{y}, \tau')\, \mathrm{d}\tau' \right) \sin(\eta_i \bar{y})\, \mathrm{d}\bar{y} \\ & - \chi_3 \frac{\chi_2}{\eta_i} \int_{-1}^{1} \frac{\partial}{\partial \bar{y}} \left(\int_0^{\tau} \sin \Omega_i (\tau - \tau') \bar{T}(\bar{y}, \tau')\, \mathrm{d}\tau' \right) \sin(\eta_i \bar{y})\, \mathrm{d}\bar{y} \\ & - \frac{\chi_2}{\eta_i} \int_{-1}^{1} \frac{\partial}{\partial \bar{y}} \left(\int_0^{\tau} \sin \Omega_i (\tau - \tau') \left(\bar{H}_z(\bar{y}, \tau')\right)^2 \mathrm{d}\tau' \right) \sin(\eta_i \bar{y})\, \mathrm{d}\bar{y}. \end{aligned} \tag{61}$$

Substitution of (38), (50) and (53) into (61) gives

$$c_i(\tau) = c_i^T(\tau) + c_i^M(\tau), \tag{62}$$

where

$$\begin{aligned} c_i^T(\tau) = {} & \frac{4(-1)^i}{\eta_i^2} \chi_3 \Bigg(\frac{1}{2} \sum_{n=1}^{\infty} \left(\hat{b}_n^{(0)}(\tau) - \Omega_i \hat{c}_{in}^{T(0)}(\tau) \right) \\ & + \sum_{j=1}^{\infty} \frac{\cos(p_j)}{M_j} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} I_{1jmn} \left(\hat{b}_{jmn}(\tau) + \frac{\eta_i^2}{p_j^2 - \eta_i^2} \Omega_i \hat{c}_{ijmn}^T(\tau) \right) \Bigg) \end{aligned} \tag{63}$$

is the contribution from the temperature change (the first term on the right-hand side being ignored, for $\bar{h} = 0$), and

$$c_i^M(\tau) = \frac{4(-1)^i \chi_2}{\eta_i} \left(\hat{c}_{ii}^{M(1)}(\tau) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{4\eta_i^2}{(k_m^2 + k_n^2 - \eta_i^2)^2 - (2k_m k_n^2)^2} \hat{c}_{imn}^{M(2)}(\tau) \right) \tag{64}$$

is the contribution from the Lorentz force. In these expressions $\hat{c}_{in}^{T(0)}(\tau)$, $\hat{c}_{ijmn}^{T}(\tau)$, $\hat{c}_{ii}^{M(1)}(\tau)$ and $\hat{c}_{imn}^{M(2)}(\tau)$ are defined in terms of the excitation function $\phi(\tau)$ by

$$\begin{aligned}
\hat{c}_{in}^{T(0)}(\tau) &= \int_0^\tau \sin \Omega_i(\tau - \tau')\hat{b}_n^{(0)}(\tau')\,\mathrm{d}\tau', \\
\hat{c}_{ijmn}^{T}(\tau) &= \int_0^\tau \sin \Omega_i(\tau - \tau')\hat{b}_{jmn}(\tau')\,\mathrm{d}\tau', \\
\hat{c}_{ii}^{M(1)}(\tau) &= \int_0^\tau \sin \Omega_i(\tau - \tau')\phi(\tau')\hat{a}_i(\tau')\,\mathrm{d}\tau', \\
\hat{c}_{imn}^{M(2)}(\tau) &= \int_0^\tau \sin \Omega_i(\tau - \tau')\hat{a}_m(\tau')\hat{a}_n(\tau')\,\mathrm{d}\tau'.
\end{aligned} \tag{65}$$

From (51), (53), (56), and (62), we have

$$\begin{aligned}
\bar{v}^T(\bar{y}, \tau) &= \chi_3 \bar{T}(\pm 1, \tau)\bar{y} + \sum_{i=1}^{\infty} c_i^T(\tau)\sin(\eta_i \bar{y}), \\
\bar{v}^M(\bar{y}, \tau) &= \sum_{i=1}^{\infty} c_i^M(\tau)\sin(\eta_i \bar{y}),
\end{aligned} \tag{66}$$

where $\bar{v}^T(\bar{y}, \tau)$ and $\bar{v}^M(\bar{y}, \tau)$ are the radial displacements due to temperature change and due to Lorentz force, respectively, and satisfy

$$\bar{v}(\bar{y}, \tau) = \bar{v}^T(\bar{y}, \tau) + \bar{v}^M(\bar{y}, \tau). \tag{67}$$

Substituting (67) with (66) into (24), we obtain the dynamic solutions for the stress components:

$$\begin{aligned}
\bar{\sigma}_{xx}^T(\bar{y}, \tau) = \bar{\sigma}_{zz}^T(\bar{y}, \tau) &= \frac{\nu}{1-\nu}\left(\chi_3 \bar{T}(\pm 1, \tau) + \sum_{i=1}^{\infty} c_i^T(\tau)\eta_i \sin(\eta_i \bar{y})\right) - \chi_3 \bar{T}(\bar{y}, \tau), \\
\bar{\sigma}_{yy}^T(\bar{y}, \tau) &= \chi_3\big(\bar{T}(\pm 1, \tau) - \bar{T}(\bar{y}, \tau)\big) + \sum_{i=1}^{\infty} c_i^T(\tau)\eta_i \sin(\eta_i \bar{y}),
\end{aligned} \tag{68}$$

$$\begin{aligned}
\bar{\sigma}_{xx}^M(\bar{y}, \tau) = \bar{\sigma}_{zz}^M(\bar{y}, \tau) &= \frac{\nu}{1-\nu}\sum_{i=1}^{\infty} c_i^M(\tau)\eta_i \sin(\eta_i \bar{y}), \\
\bar{\sigma}_{yy}^M(\bar{y}, \tau) &= \sum_{i=1}^{\infty} c_i^M(\tau)\eta_i \sin(\eta_i \bar{y}),
\end{aligned} \tag{69}$$

where $(\bar{\sigma}_{xx}^T, \bar{\sigma}_{yy}^T, \bar{\sigma}_{zz}^T)$ and $(\bar{\sigma}_{xx}^M, \bar{\sigma}_{yy}^M, \bar{\sigma}_{zz}^M)$ are the thermal and magnetic stress components, satisfying

$$\bar{\sigma}_{xx} = \bar{\sigma}_{xx}^T + \bar{\sigma}_{xx}^M, \quad \bar{\sigma}_{yy} = \bar{\sigma}_{yy}^T + \bar{\sigma}_{yy}^M, \quad \bar{\sigma}_{zz} = \bar{\sigma}_{zz}^T + \bar{\sigma}_{zz}^M. \tag{70}$$

3.3.2. *Quasistatic solutions.* We now derive the quasistatic solutions of the displacements and stresses. Neglect of the inertia term on the right-hand side of (25) gives the equilibrium equation

$$\frac{\mathrm{d}^2\bar{v}}{\mathrm{d}\bar{y}^2} = \chi_3 \frac{\mathrm{d}\bar{T}}{\mathrm{d}\bar{y}} + \frac{\mathrm{d}}{\mathrm{d}\bar{y}}\big(\bar{H}_z\big)^2 \tag{71}$$

Solving (71) with the boundary condition in (26), we obtain the quasistatic solutions of the displacements due to temperature change and due to Lorentz force:

$$\bar{v}^T(\bar{y},\tau) = \chi_3 \int \bar{T}(\bar{y},\tau)\,\mathrm{d}\bar{y}, \qquad \bar{v}^M(\bar{y},\tau) = \int \bar{H}_z(\bar{y},\tau)^2\,\mathrm{d}\bar{y} - \phi(\tau)^2. \tag{72}$$

Substituting (72) with the relation of (67) into (24), we obtain the quasistatic solutions of the stress components as follows:

$$\bar{\sigma}^T_{xx}(\bar{y},\tau) = \bar{\sigma}^T_{zz}(\bar{y},\tau) = -\frac{1-2\nu}{1-\nu}\chi_3 \bar{T}(\bar{y},\tau), \qquad \bar{\sigma}^T_{yy}(\bar{y},\tau) = 0, \tag{73}$$

$$\bar{\sigma}^M_{xx}(\bar{y},\tau) = \bar{\sigma}^M_{zz}(\bar{y},\tau) = \frac{\nu}{1-\nu}\big(\bar{H}_z(\bar{y},\tau)^2 - \phi(\tau)^2\big), \qquad \bar{\sigma}^M_{yy}(\bar{y},\tau) = \big(\bar{H}_z(\bar{y},\tau)\big)^2 - \big(\phi(\tau)\big)^2, \tag{74}$$

These quasistatic thermal stresses and magnetic stresses satisfy the relations in (70).

4. Numerical results and discussion

So far we have assume the excitation $\phi(\tau)$ to be arbitrary. Now we specialize to the case of a smoothed ramp function with a sine-function profile:

$$\phi(\tau) = \begin{cases} \sin\left(\dfrac{\pi}{2\tau_0}\tau\right) & (\tau < \tau_0), \\ 1 & (\tau \geq \tau_0), \end{cases} \tag{75}$$

where τ_0 is the (nondimensional) rise time. The particular expressions for the various functions of τ in (37), (49) and (65)—$\hat{a}_n(\tau)$, $\hat{b}_n^{(0)}(\tau)$, $\hat{b}_{jmn}(\tau)$, $\hat{c}_{in}^{T(0)}(\tau)$, $\hat{c}^T_{ijmn}(\tau)$, $\hat{c}_{ii}^{M(1)}(\tau)$, and $\hat{c}_{imn}^{M(2)}(\tau)$—will not be spelled out because they can be easily derived.

We carried out numerical calculations corresponding to the analytical results above in the case of aluminum, whose material properties are

$$\mu = 4\pi \times 10^{-7}\,[\mathrm{H/m}], \quad \sigma = 3.42 \times 10^7\,[\mathrm{S/m}], \quad C = 2.7 \times 10^3\,[\mathrm{J/kgK}], \quad \rho = 0.9 \times 10^3\,[\mathrm{kg/m^3}],$$
$$\kappa = 92.6 \times 10^{-6}\,[\mathrm{m^2/sec}], \quad \nu = 0.33, \quad E = 70\,[\mathrm{GPa}], \quad \alpha = 24 \times 10^{-6}\,[1/\mathrm{K}].$$

In addition, since the nondimensional variable χ_2 in (16) includes the half-thickness b, this dimension needs to be fixed. We chose $b = 1.0 \times 10^{-4}$ [m] to ensure the convergence of the solutions. The rise time τ_0 is given by

$$\tau_0 = \varepsilon \frac{1}{\chi_2},$$

where ε is a dimensionless parameter and $1/\chi_2$ is the nondimensional time needed by the stress waves created at the surfaces to arrive at the middle of the infinite plate.

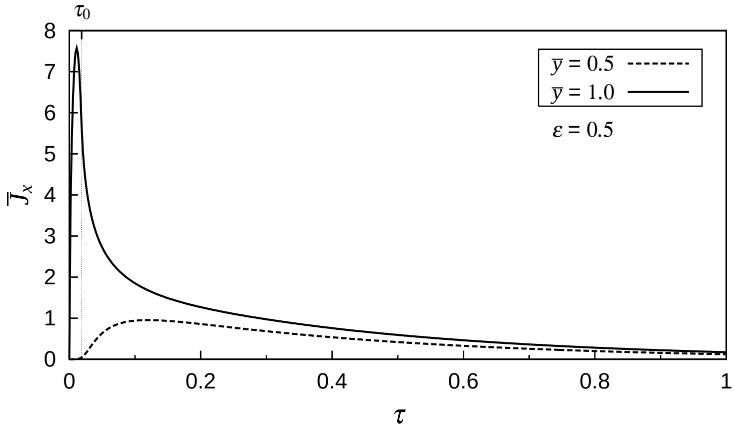

Figure 2. Time evolution of the eddy current $\bar{J}_x$, for $\varepsilon = 0.5$.

We first present the numerical results for $\varepsilon = 0.5$. Figure 2 shows the time evolution of eddy current $\bar{J}_x$. It shows a peak ahead of $\tau = \tau_0$ at the surface ($\bar{y} = 1.0$), then it decays slowly with time. Figure 3 shows the time evolution of the temperature change $\bar{T}$ for $\bar{h} = 0.0$ and 1.0 until they attain steady state. It can be seen from that figure that temperature changes take a long time to attain a steady state, in comparison with the eddy current $\bar{J}_x$. This is because the value of χ_1 in (16), which is the ratio of the diffusion coefficient of temperature field κ to that of magnetic field $(\mu\sigma)^{-1}$, is very small: $\chi_1 = \kappa\mu\sigma \cong 3.98 \times 10^{-3}$ for aluminum. In the case of $\bar{h} = 1.0$, the temperature change converges to zero, whereas in the case of $\bar{h} = 0.0$ (insulated plate), it converges to a value that can be determined from (45):

$$\bar{T} = \frac{1}{2}\int_{-1}^{1}\left(\int_{0}^{\infty} \bar{w}(\bar{y}, \tau')\,\mathrm{d}\tau'\right)\mathrm{d}\bar{y}. \tag{76}$$

However, as shown in Figure 4 (short-term time evolution of temperature), there is not a large difference between the insulated and noninsulated cases: the temperature changes always propagate from the surface more slowly than the eddy current. Therefore numerical results on the thermal stresses are shown only for the case of $\bar{h} = 0.0$. Figure 5 shows the dynamic and quasistatic behaviors of the

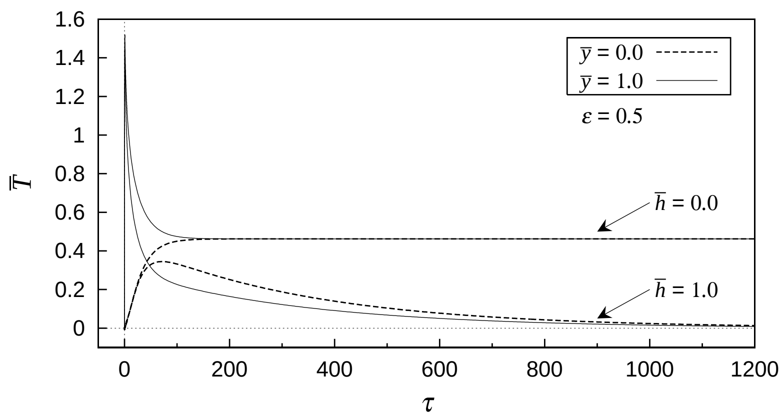

Figure 3. Time evolution of temperature changes $\bar{T}$, for $\bar{h} = 0.0$ and 1.0, $\varepsilon = 0.5$.

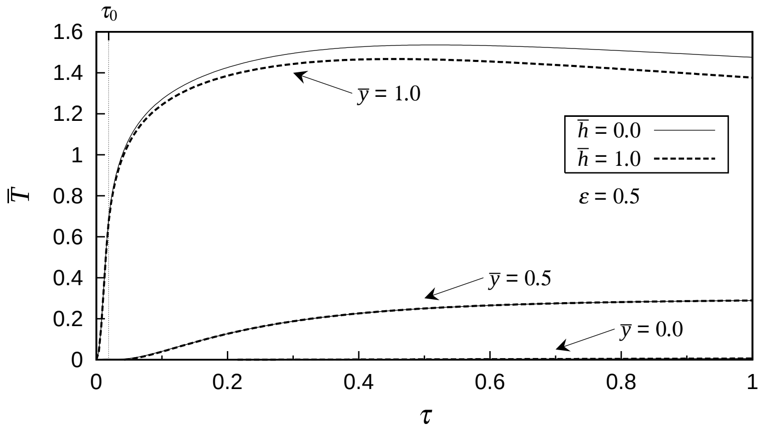

Figure 4. Short-term evolution of temperature change $\bar{T}$, for $\bar{h} = 0.0$ and 1.0, $\varepsilon = 0.5$.

thermal stress $\bar{\sigma}_{xx}^T (= \bar{\sigma}_{zz}^T)$ at the middle ($\bar{y} = 0.0$) and the surface ($\bar{y} = 1.0$) of the infinite plate versus nondimensional time τ. Note that the dynamic solution of $\bar{\sigma}_{xx}^T$ corresponds to the quasistatic one at the surface (see (68) and (73)), and that the quasistatic solution is proportional to temperature change $\bar{T}$ with the negative constant $-(1-2\nu)\chi_3/(1-\nu) \cong -2.06$ for aluminum. Therefore, the thermal stress $\bar{\sigma}_{xx}^T$ is compressive at the surface. However, the dynamic stress at the middle shows different behavior from the quasistatic one as shown in Figure 5. The dynamic behavior of the thermal stress $\bar{\sigma}_{yy}^T$ is shown in Figure 6. Consequently, the quasistatic one is identically zero as shown by the second relation in (73). In Figure 6, a new nondimensional time τ_E is introduced for convenience. The nondimensional time τ_E is based on the longitudinal wave velocity C_L, defined as

$$\tau_E = \frac{\tau}{\chi_2} = \frac{C_L}{b} t.$$

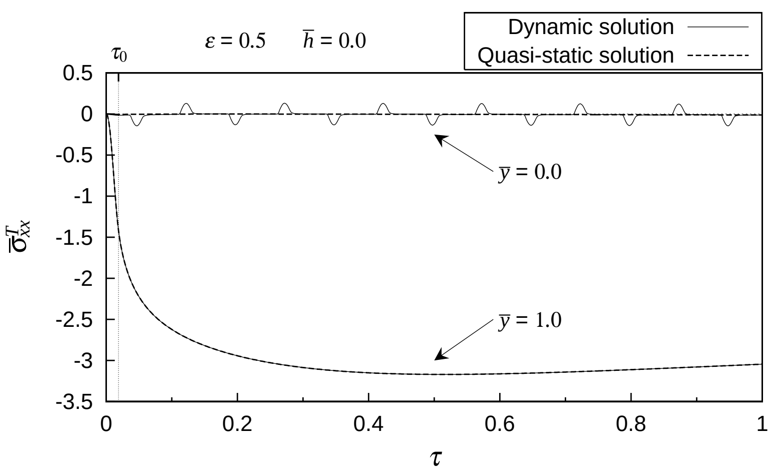

Figure 5. Dynamic and quasistatic behaviors of the thermal stress $\bar{\sigma}_{xx}^T$ versus nondimensional time τ, for $\bar{h} = 0.0$, $\varepsilon = 0.5$.

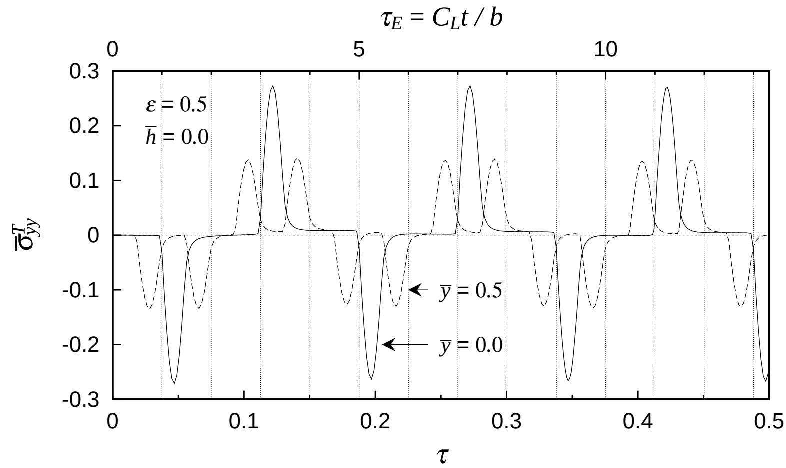

Figure 6. Dynamic behavior of the thermal stress $\bar{\sigma}^T_{yy}$ versus nondimensional time τ and τ_E, for $\bar{h} = 0.0$, $\varepsilon = 0.5$.

We see from Figure 6 that a pulsed stress wave is induced by the rapid surface temperature rise in Figure 4. As shown in Figure 7, the stress waves created on both surfaces propagate to the middle, and then interfere with each other at the middle. Therefore, the absolute value of the stress at the middle becomes about twice of those in distant positions from the middle as shown in Figures 6 and 7.

Figure 8 shows the dynamic and quasistatic behaviors of the magnetic stress $\bar{\sigma}^M_{yy}$ at the middle versus nondimensional time τ. The magnetic stress components $\bar{\sigma}^M_{xx}$ and $\bar{\sigma}^M_{zz}$ are omitted here because those components are proportional to the component $\bar{\sigma}^M_{yy}$ with $\nu/(1-\nu) \cong 0.49$ as shown by (69) and (74). Although the maximum absolute value of the quasistatic stress $\bar{\sigma}^M_{yy}$ is less than 1, as shown by the second relation in (74), the absolute value of the dynamic stress can exceed 1, as shown in Figure 8. The variation in the y direction of the dynamic magnetic stress $\bar{\sigma}^M_{yy}$ is shown in Figure 9.

The stress waves, whose maximum absolute value is 1, created at both surfaces due to the Lorentz force propagate into the middle, and then get superimposed. Therefore, the maximum absolute value of the dynamic stress becomes 2 except near the surfaces, as shown in Figure 9. Comparing the magnetic

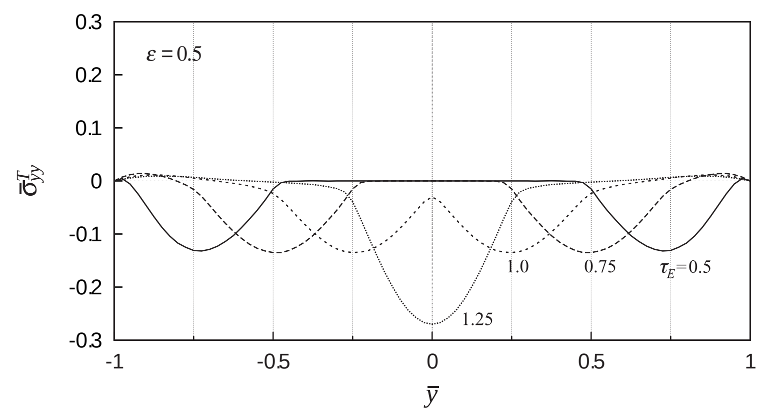

Figure 7. Variation in the y direction of the dynamic thermal stress $\bar{\sigma}^T_{yy}$.

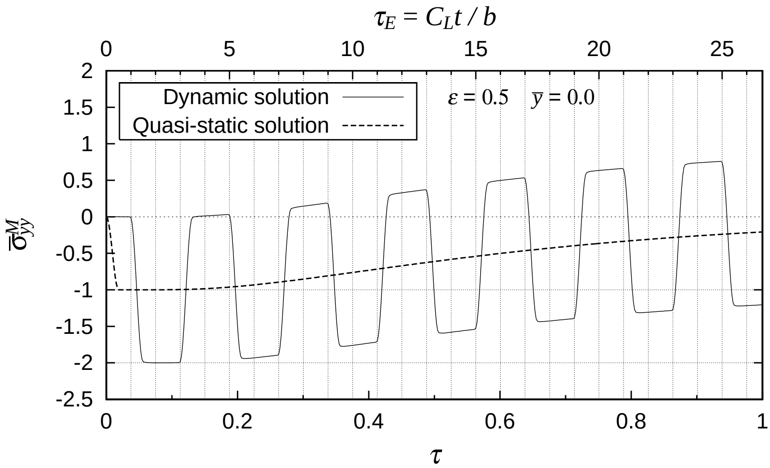

Figure 8. Dynamic and quasistatic behavior of the magnetic stress $\bar{\sigma}^M_{yy}$ versus nondimensional time τ and τ_E, for $\varepsilon = 0.5$.

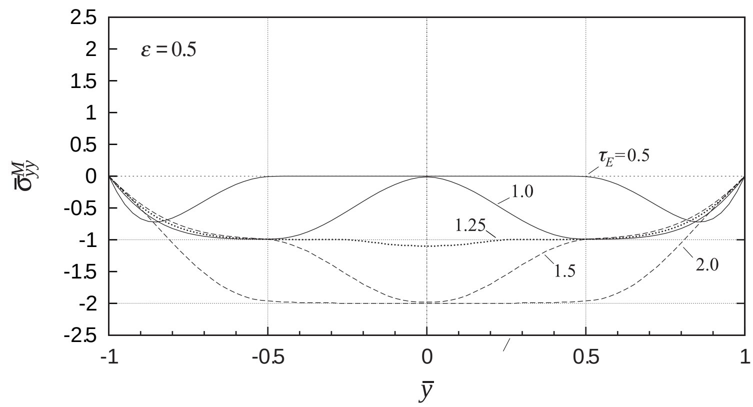

Figure 9. Variation in the y direction of the dynamic magnetic stress $\bar{\sigma}^M_{yy}$.

stress $\bar{\sigma}^M_{yy}$ in Figure 8 with the thermal stress $\bar{\sigma}^T_{yy}$ in Figure 6, we see that the former is the dominant stress component in the y direction.

Numerical results for $\varepsilon = 10.0$ — eddy current, thermal stress and magnetic stress — are presented in Figures 10–12. It can be seen from Figure 10 that the eddy current $\bar{J}_x$ is small and varies slowly in comparison with the case $\varepsilon = 0.5$. Therefore, the maximum absolute values of both the thermal stress $\bar{\sigma}^T_{xx}$ and the magnetic stress $\bar{\sigma}^M_{yy}$ are smaller and there is almost no difference in behavior between the dynamic and quasistatic solutions.

References

[Banerjee and Roychoudhuri 1997] S. Banerjee and S. K. Roychoudhuri, "Magneto-thermo-elastic interactions in an infinite isotropic elastic cylinder subjected to a periodic loading", *Int. J. Eng. Sci.* **35**:4 (1997), 437–444.

[Boley and Tolins 1962] B. A. Boley and I. S. Tolins, "Transient coupled thermoelastic boundary value problems in the half-space", *Trans. ASME Ser. E. J. Appl. Mech.* **29** (1962), 637–646. MR 26 #4570

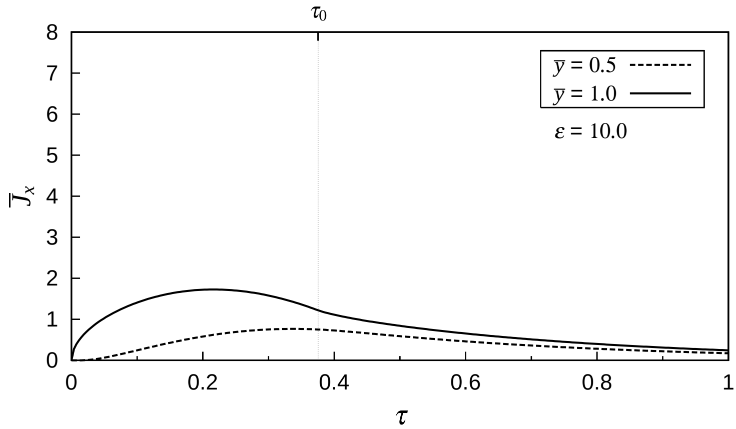

Figure 10. Time evolution of the eddy current $\bar{J}_x$ for $\varepsilon = 10.0$.

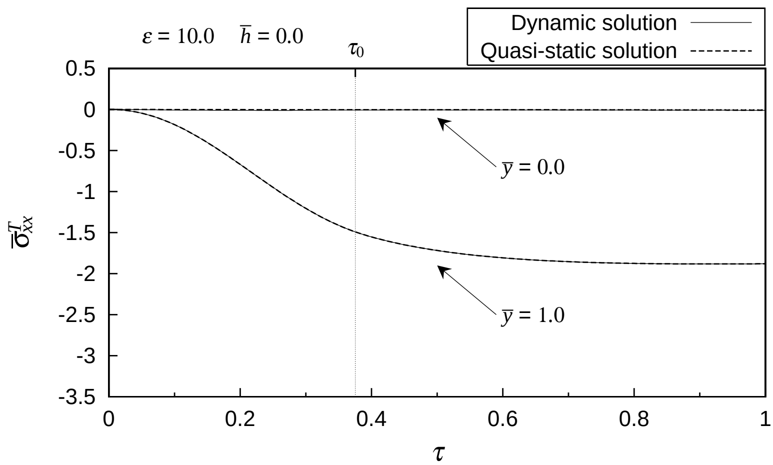

Figure 11. Dynamic and quasistatic behavior of the thermal stress $\bar{\sigma}_{xx}^T$ for $\varepsilon = 10.0$, $\bar{h} = 0.0$.

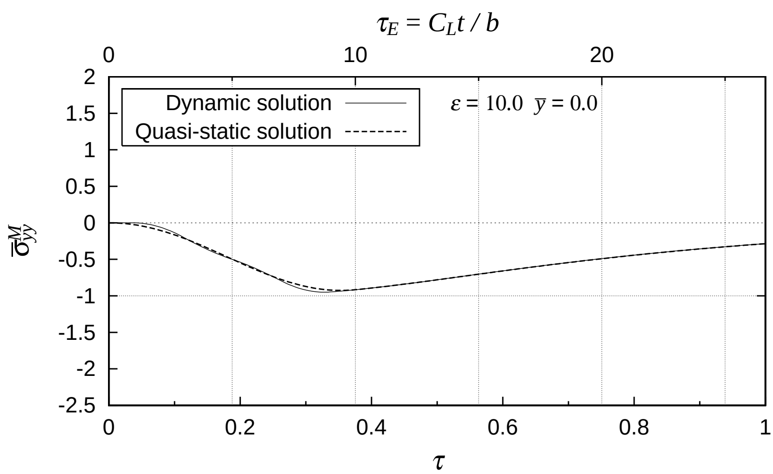

Figure 12. Dynamic and quasistatic behavior of the magnetic stress $\bar{\sigma}_{yy}^M$ for $\varepsilon = 10.0$.

[Chian and Moon 1981] C. T. Chian and F. C. Moon, "Magnetically induced cylindrical stress waves in a thermoelastic conductor", *Int. J. Solids Struct.* **17**:11 (1981), 1021–1035.

[Ezzat and Youssef 2005] M. A. Ezzat and H. M. Youssef, "Generalized magneto-thermoelasticity in a perfectly conducting medium", *Int. J. Solids Struct.* **42** (2005), 6319–6334.

[Kaliski and Michalec 1963] S. Kaliski and J. Michalec, "The resonance amplification of a magnetoelastic wave radiated from a cylindrical cavity", *Proc. Vibr. Prob.* **4**:1 (1963), 7–15.

[Kaliski and Nowacki 1962] S. Kaliski and W. Nowacki, "Combined elastic and electromagnetic waves produced by thermal shock in the case of a medium of finite electric conductivity", *Bull. Acad. Polon. Sci. Sér. Sci. Tech.* **10** (1962), 213–223.

[Librescu et al. 2003] L. Librescu, D. Hansanyan, Z. Qin, and D. R. Ambur, "Nonlinear magnetothermoelasticity of anisotropic plates immersed in a magnetic field", *J. Therm. Stresses* **26** (2003), 1277–1304.

[Moon and Chattopadhyay 1974] F. C. Moon and S. Chattopadhyay, "Magnetically induced stress waves in a conducting solid: Theory and experiment", *J. Appl. Mech.* (*ASME*) **41** (1974), 641–647.

[Pantelyat and Féliachi 2002] M. G. Pantelyat and M. Féliachi, "Magneto-thermo-elastic-plastic simulation of inductive heating of metals", *Eur. Phys. J. Appl. Phys.* **17** (2002), 29–33.

[Paria 1967] G. Paria, "Magneto-elasticity and magneto-thermo-elasticity", *Adv. Appl. Mech.* **10** (1967), 73–112.

[Sternberg and Chakravorty 1959] E. Sternberg and J. G. Chakravorty, "On inertia effects in a transient thermoelastic problem", *J. Appl. Mech.* **26** (1959), 503–509. MR 22 #8877

[Stoll 1974] R. L. Stoll, *The analysis of eddy currents*, Clarendon Press, Oxford, 1974.

[Wang et al. 2002] X. Wang, Y.-H. Zhou, and X. Zheng, "A generalized variational model of magneto-thermo-elasticity for nonlinearly magnetized ferroelastic bodies", *Internat. J. Engrg. Sci.* **40**:17 (2002), 1957–1973. MR 2003i:74015

[Wang et al. 2003] X. Wang, J. S. Lee, and X. Zheng, "Magneto-thermo-elastic instability of ferromagnetic plates in thermal and magnetic fields", *Int. J. Solids Struct.* **40** (2003), 6125–6142.

[Wauer 1995] J. Wauer, "Parametric vibrations in a magneto-thermo-elastic layer of finite thickness", pp. 407–414 in *Proceedings of the 1995 Design Engineering Technical Conferences*, vol. 3A, *Vibration of nonlinear, random, and time-varying systems*, edited by S. C. Sinha, Design Engineering Division **84-1**, ASME, New York, 1995.

[Wauer 1996] J. Wauer, "Free and forced magneto-thermo-elastic vibrations in a conducting plate layer", *J. Therm. Stresses* **19** (1996), 671–691.

[Zheng et al. 2005] X. Zheng, J. Zhang, and Y. Zhou, "Dynamic stability of a cantilever conductive plate in transverse impulsive magnetic field", *Int. J. Solids Struct.* **42** (2005), 2417–2430.

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