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Employing Biot's theory of wave propagation in liquid saturated poroelastic media, the propagation of torsional vibrations in an infinite homogeneous, isotropic hollow poroelastic circular cylinder is investigated. Considering the boundaries to be stress free, the frequency equation of torsional vibrations is obtained in presence of dissipation. The frequency equation is discussed for the first two modes in the cases of a poroelastic thin shell, a poroelastic thick shell and a poroelastic solid cylinder. Phase velocity, group velocity and attenuation are determined and computed for the first mode of vibration for two different poroelastic materials as a function of frequency. These values are displayed graphically and then discussed.

1. Introduction

An understanding of the free vibrations of any beam is a prerequisite to the understanding of its response in forced vibrations. Propagation of elastic waves and vibrations in circular rods of uniform cross-section has been extensively studied [Love 1944; Kolsky 1963]. Armenàkas [1965] studied the torsional waves in composite infinite circular solid rods of two different materials. A study of inhomogeneous anisotropic hollow cylinders was presented by Stanisic and Osburn [1967].

The study of torsional vibrations of an elastic solid is important in several fields, for example, soil mechanics, transmission of power through shafts with flanges at the ends as integral parts of the shafts. It is now recognized that virtually no high-speed equipment can be properly designed without obtaining solution to what are essentially lateral or torsional vibration problems. Examples of torsional vibrations are vibrations in gear train and motor-pump shafts. Thus, from engineering point of view the study of torsional vibrations has great interest. Such vibrations, for example, are used in delay lines. Further, based on reflections and refractions during the propagation of a pulse, imperfections can be identified. The other use of torsional vibrations is the measurement of the shear modulus of a crystal.

The dynamic equations of a poroelastic solid are given in Biot [1956]. Biot's model consists of an elastic matrix permeated by a network of interconnected spaces called pores, saturated with liquid. Following Biot's theory of wave propagation, Tajuddin and Sarma [1980] studied torsional vibrations of poroelastic cylinders. Coussy et al. [1998] presented two different approaches for dealing with the mechanics of a deformable porous medium. Dynamic poroelasticity of thinly layered structures was studied by Gelinsky et al. [1998]. Degrande et al. [1998] studied the wave propagation in layered dry, saturated and unsaturated poroelastic media. Malla Reddy and Tajuddin [2000] studied the plane-strain vibrations of thick-walled hollow poroelastic cylinders. Wisse et al. [2002] presented the experimental

Keywords: Biot's theory, torsional vibrations, phase velocity, group velocity, attenuation.

results of guided wave modes in porous cylinders. The edge waves of poroelastic plates under plane-stress conditions were studied by Malla Reddy and Tajuddin [2003]. Chao et al. [2004] studied the shock-induced borehole waves in porous formations. Tajuddin and Ahmed Shah [2006] studied the circumferential waves of infinite hollow poroelastic cylinders in the presence of dissipation.

In the present analysis, the frequency equation of torsional vibrations of a homogeneous and isotropic poroelastic hollow circular cylinder of infinite extent is derived in the presence of dissipation and then discussed. Let the boundaries of the hollow poroelastic cylinder be free from stress. The frequency equation is discussed for the first mode in the case of a poroelastic thin shell, a poroelastic thick shell and a poroelastic solid cylinder. This progression is intended to describe the transition from the case of a plate — regarded as the limit of a curved thin shell as the thickness tends to zero — to the case of a poroelastic solid cylinder. Two values are considered for the ratio h/r_1 of wall thickness h to inner radius r_1. As this ratio tends to zero, the modes of an infinite poroelastic plate of thickness equivalent to wall thickness are obtained. All the modes of the thick-walled hollow poroelastic cylinder asymptotically approach the analogous modes for a poroelastic solid cylinder of radius h as the ratio r_1/h tends to zero. The expressions for nondimensional phase velocity, group velocity and attenuation are presented and then computed for the first mode as a function of nondimensional frequency for two types of poroelastic materials and then discussed.

2. Solution of the problem

Let (r, θ, z) be the cylindrical polar coordinates. Consider a homogeneous, isotropic hollow infinite poroelastic circular cylinder with inner and outer radii r_1 and r_2, respectively, whose axis is in the direction of z-axis. Then the thickness of the hollow poroelastic cylinder is $h[=(r_2-r_1)]>0$. Let the boundaries of the isotropic poroelastic cylinder be free from stress. The only nonzero displacement components of solid and liquid media are $\boldsymbol{u}(0, v, 0)$ and $\boldsymbol{U}(0, V, 0)$, respectively. These displacements are functions of r, z and time, t. Then the equations of motion [Biot 1956] reduce to

$$\left\{\begin{aligned} N\Big(\nabla^2-\frac{1}{r^2}\Big)v &= \frac{\partial^2}{\partial t^2}(\rho_{11}v+\rho_{12}V)+b\frac{\partial}{\partial t}(v-V),\\ 0 &= \frac{\partial^2}{\partial t^2}(\rho_{12}v+\rho_{22}V)-b\frac{\partial}{\partial t}(v-V),\end{aligned}\right. \tag{1}$$

where ρ_{11}, ρ_{12}, ρ_{22} are mass coefficients following Biot [1956], N is the shear modulus, b is the dissipation coefficient and ∇^2 is the well-known Laplacian operator. Let the propagation mode shapes of solid and liquid v and V be

$$v=f(r)e^{i(kz+\omega t)}, \qquad V=F(r)e^{i(kz+\omega t)}, \tag{2}$$

where k is the wavenumber, ω is the frequency of wave and i is complex unity or $i^2=-1$. Substitution of Equation (2) in (1) results in

$$\left\{\begin{aligned} N\Delta f &= -\omega^2(K_{11}f+K_{12}F),\\ 0 &= -\omega^2(K_{12}f+K_{22}F),\end{aligned}\right. \tag{3}$$

where

$$\Delta = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} - k^2,$$

$$K_{11} = \rho_{11} - \frac{ib}{\omega},$$

$$K_{12} = \rho_{12} + \frac{ib}{\omega},$$

$$K_{22} = \rho_{22} - \frac{ib}{\omega}.$$

The second equation in (3) gives

$$F = -\frac{K_{12}}{K_{22}} f. \tag{4}$$

Substituting Equation (4) into the first equation of (3), we obtain

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2} + \alpha_3^2\right) f = 0, \tag{5}$$

where V_3 is the shear wave velocity [Biot 1956] and α_3^2 is

$$\alpha_3^2 = \xi_3^2 - k^2, \qquad \xi_3^2 = \frac{\omega^2(K_{11}K_{22} - K_{12}^2)}{NK_{22}}, \qquad V_3^2 = \frac{NK_{22}}{K_{11}K_{22} - K_{12}^2}. \tag{6}$$

A solution of Equation (5) is

$$f(r) = C_1 J_1(\alpha_3 r) + C_2 Y_1(\alpha_3 r).$$

Thus the displacement of the solid is

$$v = \big(C_1 J_1(\alpha_3 r) + C_2 Y_1(\alpha_3 r)]e^{i(kz+\omega t)}, \quad (\alpha_3 \neq 0). \tag{7}$$

When $\alpha_3 = 0$, Equation (5) reduces to the form

$$\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \frac{1}{r^2}\right) f = 0, \tag{8}$$

and thus its bounded solution is

$$f(r) = C_1 r.$$

Therefore the propagation mode shapes are given by the displacement solutions

$$v = \begin{cases} \big(C_1 J_1(\alpha_3 r) + C_2 Y_1(\alpha_3 r)\big)e^{i(kz+\omega t)} & \alpha_3 \neq 0 \\ C_1 r & \alpha_3 = 0 \end{cases}. \tag{9}$$

Here C_1 and C_2 are constants.

From Equation (2), it can be seen that the normal strains e_{rr}, $e_{\theta\theta}$ and e_{zz} are all zero. Therefore the dilatations of solid and liquid media are both zero. Hence the liquid pressure s following Biot [1956] is identically zero. Accordingly for torsional vibrations no distinction between a pervious and an impervious surface is made. Considering the boundary to be stress free, the frequency equation obtained for torsional vibrations is the same for both pervious and impervious surfaces. Then the only nonzero computed stress, $\sigma_{r\theta}$ (see [Biot 1956]), is

$$\sigma_{r\theta} = -N\big(C_1 J_2(\alpha_3 r) + C_2 Y_2(\alpha_3 r)\big)e^{i(kz+\omega t)}. \tag{10}$$

3. Frequency equation

The stress-free boundary conditions for torsional vibrations at the inner and outer surfaces of the hollow poroelastic cylinder are at $r = r_1$ and $r = r_2$,

$$\sigma_{r\theta} = 0, \qquad s = 0, \qquad \frac{\partial s}{\partial r} = 0. \tag{11}$$

First two equations of (11) are to be satisfied for a pervious surface, while the first and third equations of (11) are to be satisfied for an impervious surface. Since the considered vibrations are shear vibrations, the dilatations of solid and liquid media are both zero, thus liquid pressure s developed in solid-liquid aggregate will be identically zero and no distinction between pervious and impervious surface is made. Thus the second and third equations of (11) are satisfied identically. Equations (11) together with Equation (10) yield a system of two homogeneous equations in two constants C_1 and C_2. By eliminating these constants, one can obtain

$$J_2(\alpha_3 r_1)Y_2(\alpha_3 r_2) - J_2(\alpha_3 r_2)Y_2(\alpha_3 r_1) = 0. \tag{12}$$

Equation (12) is the frequency equation of torsional vibrations of an infinite hollow poroelastic cylinder whether the surface is pervious or impervious. By eliminating liquid effects from (12), the results for a purely elastic solid [Gazis 1959, Equation (43)] are obtained as a special case. The roots will increase with increasing r_1 tending to infinity as r_1 tends to r_2. Two cases of special interest for limiting values of ratio of thickness to inner radius h/r_1 when these values are too small and too large are considered:

3.1. ***For thin poroelastic cylindrical shell.*** When $h/r_1 \ll 1$, under the verifiable assumption of nonzero $\alpha_3 h$ it is seen that $\alpha_3 r_1 \gg 1$ and $\alpha_3 r_2 \gg 1$. By using Hankel–Kirchhoff asymptotic approximations for Bessel functions [Abramowitz 1964]

$$\begin{cases} J_2(x) \approx -\sqrt{\dfrac{2}{\pi x}}\Big[\cos\Big(x - \dfrac{\pi}{4}\Big) - \dfrac{15}{8x}\sin\Big(x - \dfrac{\pi}{4}\Big)\Big], \\[2ex] Y_2(x) \approx -\sqrt{\dfrac{2}{\pi x}}\Big[\sin\Big(x - \dfrac{\pi}{4}\Big) + \dfrac{15}{8x}\cos\Big(x - \dfrac{\pi}{4}\Big)\Big], \end{cases}$$

the frequency equation of torsional vibrations, that is, Equation (12), reduces to

$$\sin\alpha_3 h - \frac{15\alpha_3 h}{8\alpha_3^2 r_1 r_2}\cos\alpha_3 h = 0. \tag{13}$$

Equation (13) is the frequency equation of vibrations of a thin poroelastic cylindrical shell. In the limiting case, when $\alpha_3 r_1 \to \infty$, $\alpha_3 r_2 \to \infty$, (13) simplifies to

$$\sin\alpha_3 h = 0, \tag{14}$$

and hence

$$\alpha_3 h = \pi q, \quad q = 1, 2, 3, \ldots$$

so that

$$\omega = V_3 \left(\frac{q^2 \pi^2}{h^2} + k^2 \right)^{\frac{1}{2}}, \quad q = 1, 2, 3, \ldots \tag{15}$$

which are the frequencies of poroelastic plate of thickness h. Moreover near the origin $h/r_1 = 0$, and substituting

$$\alpha_3 h = q\pi + \epsilon^*, \quad \epsilon^* \ll 1, \tag{16}$$

into the frequency equation of torsional vibrations of a thin poroelastic cylindrical shell, Equation (13), gives

$$\epsilon^* = \frac{15}{8(q\pi)} \left(\frac{h}{r_1} \right)^2, \quad q = 1, 2, 3, \ldots \tag{17}$$

Substituting Equation (17) into (16) gives the frequency values obtained from (15) in the form

$$\omega = \frac{V_3}{h} \left(q^2 \pi^2 \left[1 + \frac{15}{8(q\pi)^2} \left(\frac{h}{r_1} \right)^2 \right]^2 + k^2 h^2 \right)^{\frac{1}{2}}, \quad q = 1, 2, 3, \ldots. \tag{18}$$

These are the frequencies of torsional vibrations of a poroelastic plate of thickness h near the origin.

3.2. ***For poroelastic solid cylinder.*** When $h/r_1 \gg 1$, the frequency equation, (12), tends asymptotically to

$$J_2(\alpha_3 h) = 0, \tag{19}$$

which is the frequency equation of torsional vibrations of a poroelastic solid cylinder of radius h discussed in Tajuddin and Sarma [1980]. The limiting cases $h{r_1}^{-1} \ll 1$ and $h{r_1}^{-1} \gg 1$ cover the torsional vibrations of thick-walled poroelastic hollow cylinders in the entire range from 0 to ∞. Thus we are modeling the transition from plate (hence shell) vibrations to the vibrations of a poroelastic solid cylinder.

If the wave number k is zero, the problem reduces to the special case of axially symmetric shear vibrations studied in Malla Reddy and Tajuddin [2000, Sections 5.1.1 and 5.1.2], where a thin poroelastic cylindrical shell and a solid poroelastic cylinder are discussed in detail. Accordingly the case $k \neq 0$ is of special interest, and that is what we discuss below.

To analyze further the frequency equation, it is convenient to introduce the following nondimensional variables:

$$\begin{gathered} m_{11} = \rho_{11}\rho^{-1}, \qquad m_{12} = \rho_{12}\rho^{-1}, \qquad m_{22} = \rho_{22}\rho^{-1}, \qquad b_1 = bh(c_0\rho)^{-1}, \\ \Omega = \omega h {c_0}^{-1}, \qquad g = r_2 {r_1}^{-1}, \end{gathered} \tag{20}$$

so that $h{r_1}^{-1} = g - 1$, where b_1, Ω are nondimensional dissipation and frequency, and

$$\rho = \rho_{11} + 2\rho_{12} + \rho_{22}, \qquad {c_0}^2 = N\rho^{-1}.$$

Let

$${R_n}^2 = \alpha_3^2 h^2,$$

where α_3^2 is given in Equation (6). From these equations, we can write

$$\frac{N(R_n^2+k^2h^2)}{\rho\omega^2h^2} = E_r - iE_i, \tag{21}$$

where E_r and E_i are

$$E_r = \frac{\Omega^2 m_{22}(m_{11}m_{22}-m_{12}^2)+b_1^2}{\Omega^2 m_{22}^2+b_1^2}, \qquad E_i = \frac{b_1\Omega(m_{12}+m_{22})^2}{\Omega^2 m_{22}^2+b_1^2}. \tag{22}$$

To investigate the values of R_n, the frequency equation (12) in nondimensional form is

$$J_2\left(\frac{R_n}{g-1}\right)Y_2\left(\frac{R_n g}{g-1}\right) - J_2\left(\frac{R_n g}{g-1}\right)Y_2\left(\frac{R_n}{g-1}\right) = 0. \tag{23}$$

In (23) g is the ratio of outer to inner radius.

Three cases of physical interest have been considered, varying the g value: 1.034, 3, and infinity. These three cases represent a thin poroelastic shell, thick poroelastic shell and poroelastic solid cylinder, respectively. The phase and group velocities and attenuation can be determined for the first two modes, which have been computed from the frequency equation (23) for $\omega > |kV_3|$. The values for the said cases are 3.1423, 6.2835; 3.736, 6.6477; and 5.1356, 8.4172.

4. Phase velocity, group velocity and attenuation

Due to the dissipative nature of the medium, the wave number k is complex. The waves generated obey a diffusion process, and therefore get attenuated. Let $k = k_r + ik_i$; then the phase velocity c_p, group velocity c_g and attenuation x_h, respectively, are

$$c_p = \text{Real part } (\omega k^{-1}) = \frac{\omega}{|k_r|}, \qquad c_g = \frac{d\omega}{dk} \qquad \text{and} \qquad x_h = \frac{1}{|k_i|},$$

which in turn reduces to nondimensional form as

$$c_p c_0^{-1} = \sqrt{2}\Omega(B_1+B_2)^{-\frac{1}{2}}, \tag{24}$$

$$c_g c_0^{-1} = 2\sqrt{2}B_3^{-1}(B_1+B_2)^{\frac{1}{2}}, \tag{25}$$

and

$$x_h h^{-1} = \sqrt{2}(B_1-B_2)^{-\frac{1}{2}}. \tag{26}$$

In Equations (24)–(26), B_1, B_2 and B_3 are

$$\begin{cases} B_1 = \left(\Omega^4(E_r^2+E_i^2) - 2\Omega^2 E_r R_n^2 + R_n^4\right)^{\frac{1}{2}}, \\ B_2 = (\Omega^2 E_r - R_n^2), \\ B_3 = \Omega^2 G_1(1+\Omega^2 E_r B_1^{-1} - R_n^2 B_1^{-1}) \\ \qquad + 2\Omega E_r(1-R_n^2 B_1^{-1}) \\ \qquad + \Omega^3 B_1^{-1}\left(\Omega E_i G_2 + 2(E_r^2+E_i^2)\right), \end{cases} \tag{27}$$

Material Parameter	m_{11}	m_{12}	m_{22}
Material I	0.901	−0.001	0.101
Material II	0.877	0	0.123

Table 1. Properties of materials I and II.

where Ω is nondimensional frequency and R_n denotes modes of vibration, E_r and E_i are given in (22) while G_1 and G_2 are

$$G_1 = \frac{2b_1^2(E_r - 1)}{\Omega(\Omega^2 m_{22}^2 + b_1^2)}, \qquad G_2 = \frac{(b_1^2 - \Omega^2 m_{22}^2)E_i}{\Omega(\Omega^2 m_{22}^2 + b_1^2)}. \tag{28}$$

The nondimensional phase velocity, group velocity and attenuation equations of a poroelastic plate are similar to (24)–(26), respectively, wherein R_n is to be replaced by $q\pi$ $(q = 1, 2, 3, \ldots)$. Different values of q represent different modes of vibration. It is interesting to note that the first two modes of a poroelastic plate tally with the first two modes of a thin poroelastic shell.

5. Results and discussion

Two types of poroelastic materials are considered to carry out the computational work: sandstone saturated with kerosene, which we call Material I [Fatt 1959], and sandstone saturated with water, called Material II [Yew and Jogi 1976]. Their physical parameters are defined in Table 1.

For a given material, the nondimensional phase velocity, group velocity and attenuation are determined as a function of nondimensional frequency (Ω). The different dissipation parameters (b_1) chosen are 0.01, 0.1 and 1.

The phase velocity as a function of frequency is presented for first mode for the two materials in Figure 1 (top) for different dissipations in the case of a thin shell. The phase velocity has nearly the same shape when $b_1 = 0.01$ and 0.1 in material I. This is also true for material II. For $b_1 = 1$, the phase velocity is almost identical in both materials. The group velocity with respect to frequency is presented in Figure 1 (middle) for the first mode in case of a thin poroelastic shell. The results are true for all three dissipations considered, and are almost same as the phase velocity for both materials. The attenuation is presented in Figure 1 (bottom) for the first mode. When $b_1 = 0.01$, the attenuation is almost the same for both materials, and for $b_1 = 0.1$ and 1 it is virtually the same for both materials. Besides, it is clear that as b_1 increases from 0.01 to 1 the attenuation is decreasing. The nondimensional phase velocity and group velocity as a function of frequency is presented in Figure 2 for the two materials, for a thick poroelastic shell.

From Figure 2 (top) it is clear that for the first mode, the phase velocity is increasing in $0 < \Omega < 5$, and then decreasing in $5 \le \Omega < 10$, and when $\Omega \ge 10$ it is constant for both the referred materials and for different dissipations. The phase velocity decreases as the dissipation b_1 increases, and it is less for material I than for material II. The same figure also shows that when $b_1 = 1$, the phase velocity is same for both materials. In Figure 2 (middle), the group velocity as a function of frequency is presented for first mode. It is clear that for $b_1 = 0.1$ and 1, both materials have the same group velocity, while when

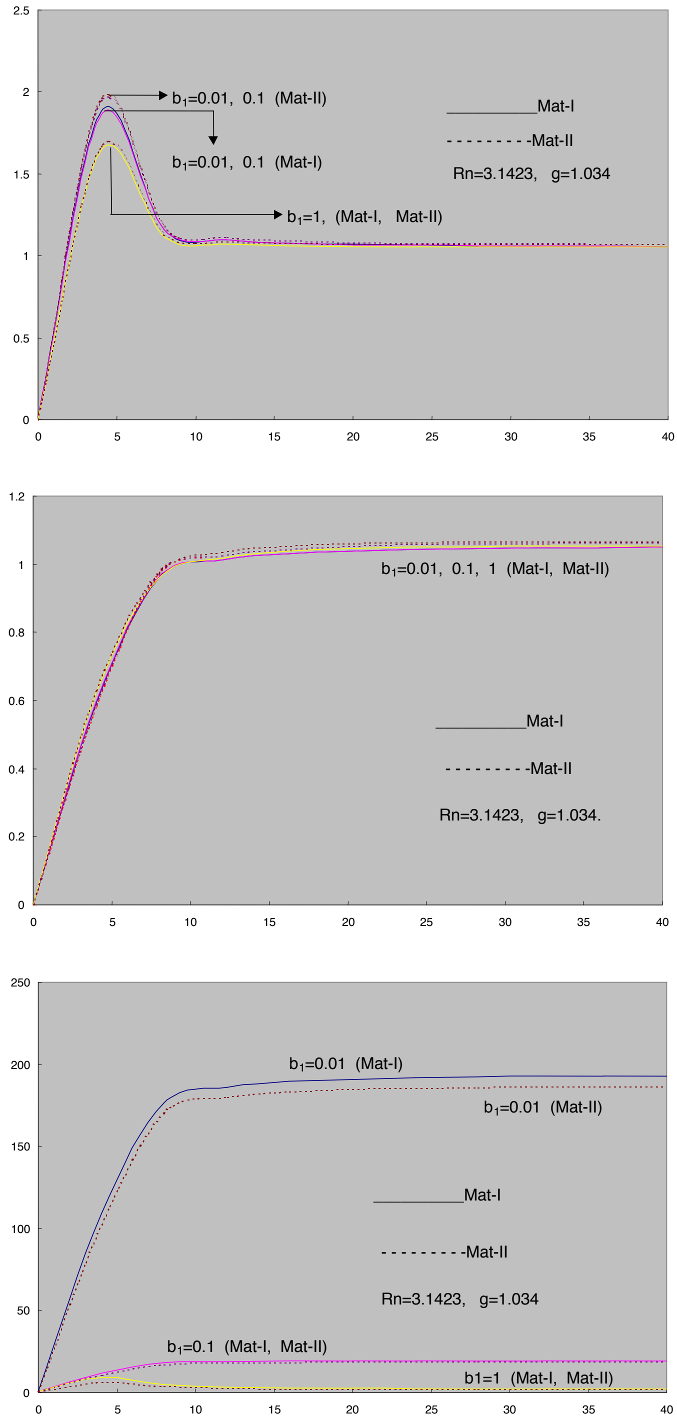


Figure 1. Torsional vibrations of hollow poroelastic cylinder, thin shell. The graphs show the phase velocity (top), group velocity (middle) and attenuation (bottom) as functions of frequency for the first mode, using reduced (nondimensional) variables.

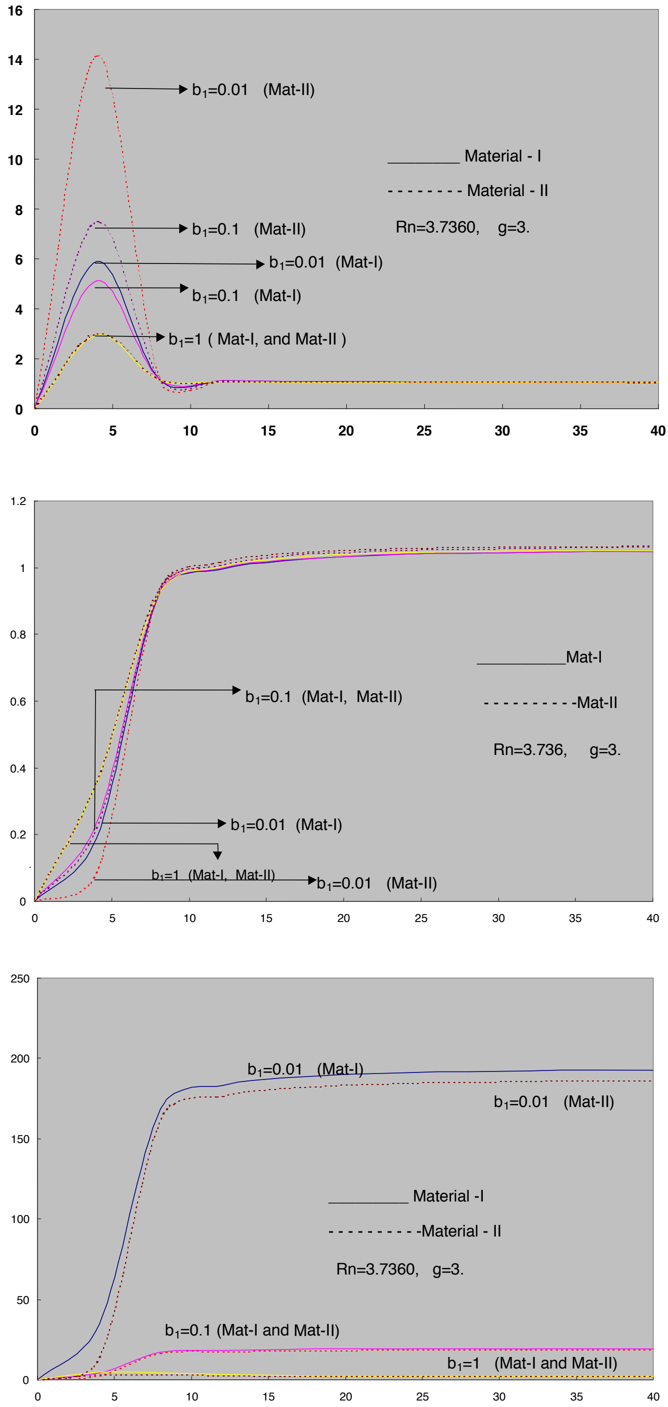
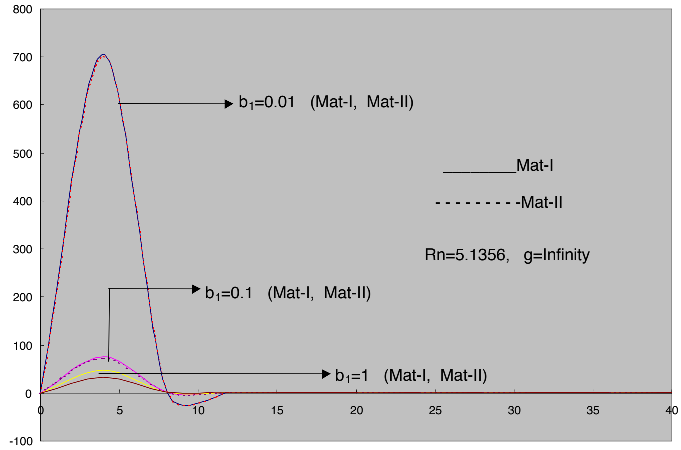
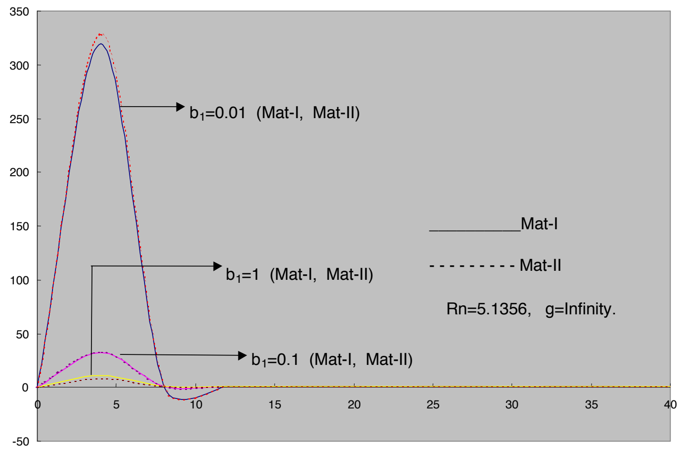


Figure 2. Torsional vibrations of hollow poroelastic cylinder, thick shell. The graphs show the phase velocity (top), group velocity (middle) and attenuation (bottom) as functions of frequency for the first mode, using reduced (nondimensional) variables.





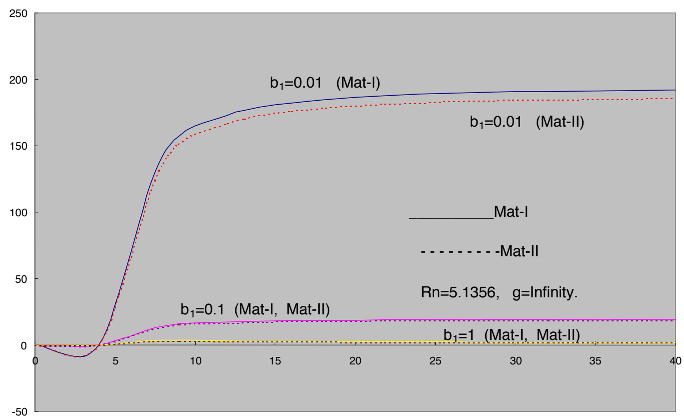


Figure 3. Torsional vibrations of solid poroelastic cylinder. The graphs show the phase velocity (top), group velocity (middle) and attenuation (bottom) as functions of frequency for the first mode, using reduced (nondimensional) variables.

$b_1 = 0.01$ the group velocity in material II is less than that of material I for $0 < \Omega < 5$. When $\Omega \geq 5$ the group velocity in materials I and II is almost the same for all dissipations. The attenuation is presented in Figure 2 (bottom) for a thick poroelastic shell in the case of the first mode. Its variation is similar to that of a thin shell.

The phase velocity of a poroelastic solid cylinder for the first mode is shown in Figure 3 (top). The phase velocity takes the same path for both materials when $b_1 = 0.01, 0.1$ and 1, but it decreases as dissipation increases. The group velocity for a poroelastic solid cylinder for the first mode is shown in Figure 3 (middle). Its variation is similar to that of the phase velocity (top figure). The group velocity of a poroelastic solid cylinder is seen to be less than the phase velocity for both materials. The attenuation of a poroelastic solid cylinder for the first mode is presented in Figure 3 (bottom). The attenuation in both materials is the same when $b_1 = 0.01, 0.1$ and 1; the figure also shows that the attenuation is higher for $b_1 = 0.01$ than for $b_1 = 0.1$ and 1.

6. Concluding remarks

The investigation of torsional vibrations of hollow poroelastic cylinders for different dissipations in the cases of a thin poroelastic shell, a thick poroelastic shell and a poroelastic solid cylinder has lead to the following conclusion:

(i) The phase velocity increases as we progress from a hollow poroelastic cylinder through thin and thick poroelastic shells to a poroelastic solid cylinder.

(ii) In general, the group velocity is less than the phase velocity.

(iii) The presence of a coupling parameter reduces the phase and group velocities.

(iv) It is observed that the increasing of the mass of a solid reduces both phase and group velocities.

(v) An increase in dissipation reduces the phase and group velocities as well as the attenuation for both materials.

(vi) There is no significant variation in attenuation between a thin poroelastic shell, a thick shell and a poroelastic solid cylinder.

(vii) The phase and group velocities for the second mode are in general higher than the corresponding values for the first mode, in all cases.

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References

[Abramowitz 1964] A. Abramowitz and I. A. Stegun (editors), *Handbook of mathematical functions with formulas, graphs, and mathematical tables*, edited by A. Abramowitz and I. A. Stegun, National Bureau of Standards Applied Mathematics Series **55**, U.S. Government Printing Office, Washington, DC, 1964. MR 29 #4914 Zbl 0171.38503

[Armenàkas 1965] A. E. Armenàkas, "Torsional waves in composite rods", *J. Acoust. Soc. Am.* **38**:3 (1965), 439–446.

[Biot 1956] M. A. Biot, "Theory of propagation of elastic waves in a fluid-saturated porous solid, I: low-frequency range", *J. Acoust. Soc. Am.* **28**:2 (1956), 168–178.

[Chao et al. 2004] G. Chao, D. M. J. Smeulders, and M. E. H. van Dongen, "Shock-induced borehole waves in porous formations: theory and experiments", *J. Acoust. Soc. Am.* **116**:2 (2004), 693–702.

[Coussy et al. 1998] O. Coussy, L. Dormieux, and E. Detournay, "From mixture theory to Biot's approach for porous media", *Int. J. Solids Struct.* **35**:34–35 (1998), 4619–4635.

[Degrande et al. 1998] G. Degrande, G. De Roeck, P. Van Den Broeck, and D. Smeulders, "Wave propagation in layered dry, saturated and unsaturated poroelastic media", *Int. J. Solids Struct.* **35**:34–35 (1998), 4753–4778.

[Fatt 1959] I. Fatt, "The Biot–Willis elastic coefficients for a sandstone", *J. Appl. Mech.* (*ASME*) **26** (1959), 296–297.

[Gazis 1959] D. C. Gazis, "Three-dimensional investigation of the propagation of waves in hollow circular cylinders, I: Analytical foundation", *J. Acoust. Soc. Am.* **31**:5 (1959), 568–573.

[Gelinsky et al. 1998] S. Gelinsky, S. A. Shapiro, T. Muller, and B. Gurevich, "Dynamic poroelasticity of thinly layered structures", *Int. J. Solids Struct.* **35**:34–35 (1998), 4739–4751.

[Kolsky 1963] H. Kolsky, *Stress waves in solids*, Dover, New York, 1963.

[Love 1944] A. E. H. Love, *A treatise on the mathematical theory of elasticity*, Dover, New York, 1944.

[Malla Reddy and Tajuddin 2000] P. Malla Reddy and M. Tajuddin, "Exact analysis of the plane-strain vibrations of thick-walled hollow poroelastic cylinders", *Int. J. Solids Struct.* **37**:25 (2000), 3439–3456.

[Malla Reddy and Tajuddin 2003] P. Malla Reddy and M. Tajuddin, "Edge waves in poroelastic plate under plane-stress conditions", *J. Acoust. Soc. Am.* **114**:1 (2003), 185–193.

[Stanisic and Osburn 1967] M. M. Stanisic and C. M. Osburn, "On torsional vibrations of inhomogeneous anisotropic hollow cylinder", *Z. Angew. Math. Mech.* **47** (1967), 465–466.

[Tajuddin and Ahmed Shah 2006] M. Tajuddin and S. Ahmed Shah, "Circumferential waves of infinite hollow poroelastic cylinders", *J. Appl. Mech.* (*ASME*) **73**:4 (2006), 705–708.

[Tajuddin and Sarma 1980] M. Tajuddin and K. S. Sarma, "Torsional vibrations of poroelastic cylinders", *J. Appl. Mech.* (*ASME*) **47** (1980), 214–216.

[Wisse et al. 2002] C. J. Wisse, D. M. J. Smeulders, M. E. H. van Dongen, and G. Chao, "Guided wave modes in porous cylinders: experimental results", *J. Acoust. Soc. Am.* **112**:3 (2002), 890–895.

[Yew and Jogi 1976] C. H. Yew and P. N. Jogi, "Study of wave motions in fluid-saturated porous rocks", *J. Acoust. Soc. Am.* **60**:1 (1976), 2–8.

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