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**BISTABLE STRUCTURES FOR ENERGY ABSORPTION  
I. METALLIC STRUCTURES UNDER TENSION**

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## **BISTABLE STRUCTURES FOR ENERGY ABSORPTION I. METALLIC STRUCTURES UNDER TENSION**

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This paper presents proof-of-concept experiments on metallic bistable structures, and is followed by a companion paper about experiments on composite bistable structures. A bistable structure is characterized by a stress/strain curve with stable branches separated by unstable branches. The authors were interested in a particular bistable structure, one that once activated, has a second stronger state which has the ability to sustain higher loads. This allows for a better distribution of damage. In addition, the structure keeps its integrity for a longer time, leading to a fail safe design. Results on metallic configurations under tensile loading are shown in this paper. In particular, chains with one, two, and three bistable elements of 5052-H32 aluminum were designed, manufactured, tested, and compared to their corresponding baselines. A strain energy increase from 11% to about 30% is shown with respect to the baselines. Moreover, a comparative study with A36 annealed mild steel and C10100 copper shows the effect of the different ductility and stiffness on energy absorption.

### **1. Introduction**

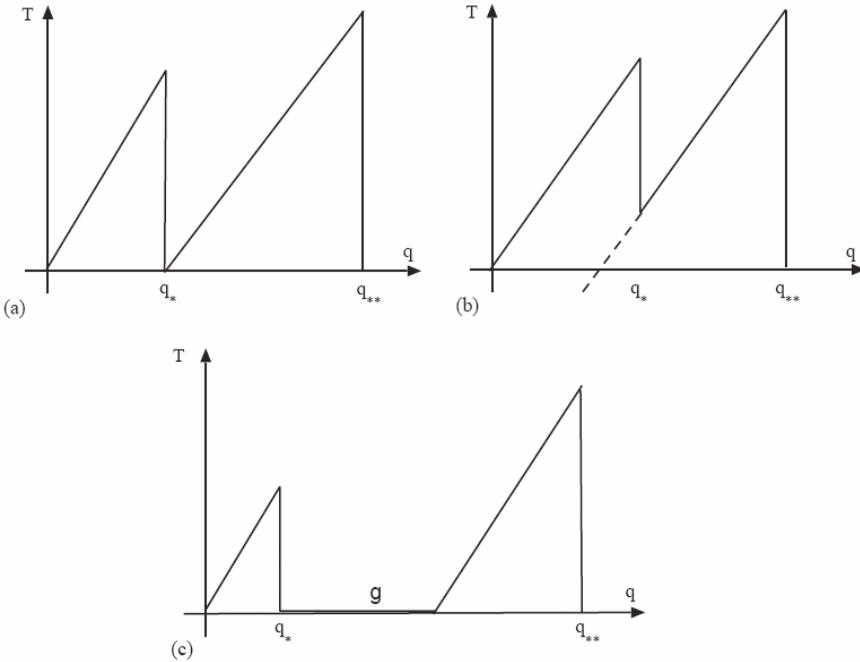
In traditional metallic structures under tensile loading, failure occurs when a weak location of the structure begins to yield, neck, and ultimately fracture. In such structures, only the energy absorption associated with yielding and fracture of a relatively small volume of the material is realized. Thus, the total energy absorbed is less than what would be available if the entire structure experienced necking and fracture. The metallic bistable structures discussed in this paper are designed to use necking and ductility properties of the material to absorb more energy, while maintaining the integrity of the structure. Results are shown for metallic chains under quasi-static tension. Chains with one to three bistable links of 5052-H32 aluminum alloy were examined, as well as the effect of ductility and strength of three different materials: A36 annealed mild steel, 5052-H32 aluminum alloy, and annealed C10100 copper.

In addition to ductility-driven metallic bistable structures, it is possible to design bistable structures made of composite materials, which are typically brittle. The design in this case is based on different mechanisms: the loading and unloading of the elastic structure, with tailoring of the strain capacity of the different subparts of the structure. Our companion paper discusses results for composite chains under quasi-static tension. The use of composites allows greater flexibility in tailoring the structure to the needed load carrying capability. The work in these two papers constitutes a first proof-of-concept of the ideas introduced by Cherkaev and Slepyan [1995]; see Figure 1.

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*Keywords:* tension, metal, energy absorption, bistable.

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**Figure 1.** Bistable force-elongation diagrams are characterized by stable branches connected by unstable branches, taken in this illustration as piecewise linear functions. Reproduced from [Cherkaev et al. 2005], courtesy of Andrej and Elena Cherkaev.

We have been collaborating with A. Cherkaev and E. Cherkaev since the summer of 2004, and preliminary work was presented at two conferences [Whitman et al. 2005a; 2005b]. A. Cherkaev and L. Slepyan introduced the concepts of *main links* and *waiting links*. Main links are designed to break first, by having, for example, a smaller cross-sectional area, or by being made of a weaker material. Waiting links become active as the attached main link breaks. They provide a redundant load path and continue deforming under loading while preventing overall fracture. Moreover, the partially damaged state (the load/displacement of the overall structure after the first main link has broken and the attached waiting link becomes active) has a higher load-carrying capability than the undamaged state (the load/displacement of the overall structure before the first main link breaks). In these papers, chains made of main links and waiting links are defined as *bistable chains*. Further research and numerical simulations by Cherkaev, Slepyan and their collaborators showed that bistable chains are able to sustain a higher dynamic loading (in particular, impact and explosion) without breaking, compared to conventional chain structures, since they spread damage across a larger part of the volume [Cherkaev and Zhornitskaya 2004; Slepyan and Ayzenberg 1997; Cherkaev et al. 2005; Slepyan et al. 2005].

A literature survey showed that, while the idea of bistable structures is not new, its design, driven on material-based mechanisms and with the objective of higher energy absorption, is indeed novel. Bistable structures have been used for robotics, semiconductor research, smart morphing structures [Schultz 2005], deployable structures, and shells [Kebadze et al. 2004; Santer and Pellegrino 2004]. Dancila

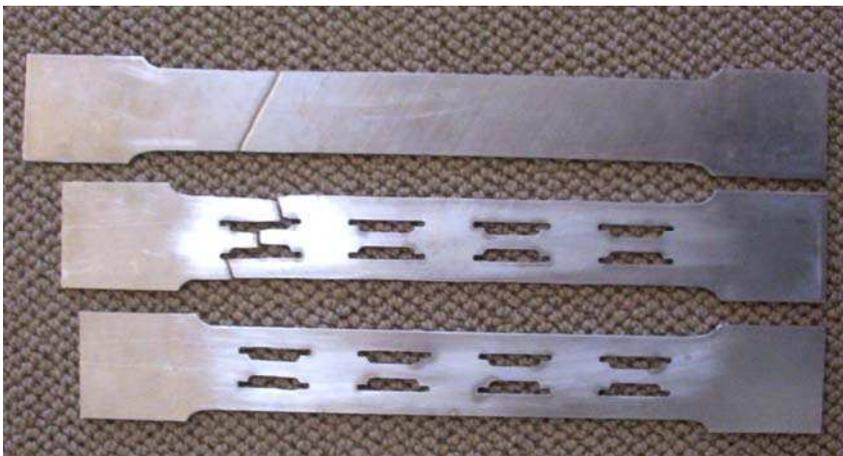
and Armanios [2001] did not use the word ‘bistable’, but showed a one-dimensional composite member with a progressive failure and yield-type response similar to the stress/strain curve of a bistable structure. Dancila [1998], in a Ph.D. thesis, discussed a flexible one-dimensional structure made of prepreg, Scotch 898 High Performance Filament Tape made by the 3M Corporation, embedded with E-glass fibers. No progress of the idea past these 1D specimens seems to have appeared in the literature.

Further work is currently being carried out on these concepts by the authors of this paper, in collaboration with A. Cherkaev, E. Cherkaev and L. Slepyan. Long-term applications are expected for design of crashworthy structures in mechanical, aerospace and civil engineering. Examples are the design of structural components for aircraft, floors, ceilings, crumple zones in cars, guard-rails, ship hulls, as well as design of redundant, ductile, and impact-resistant parts of civil infrastructures in earthquake-prone or hazard-prone areas.

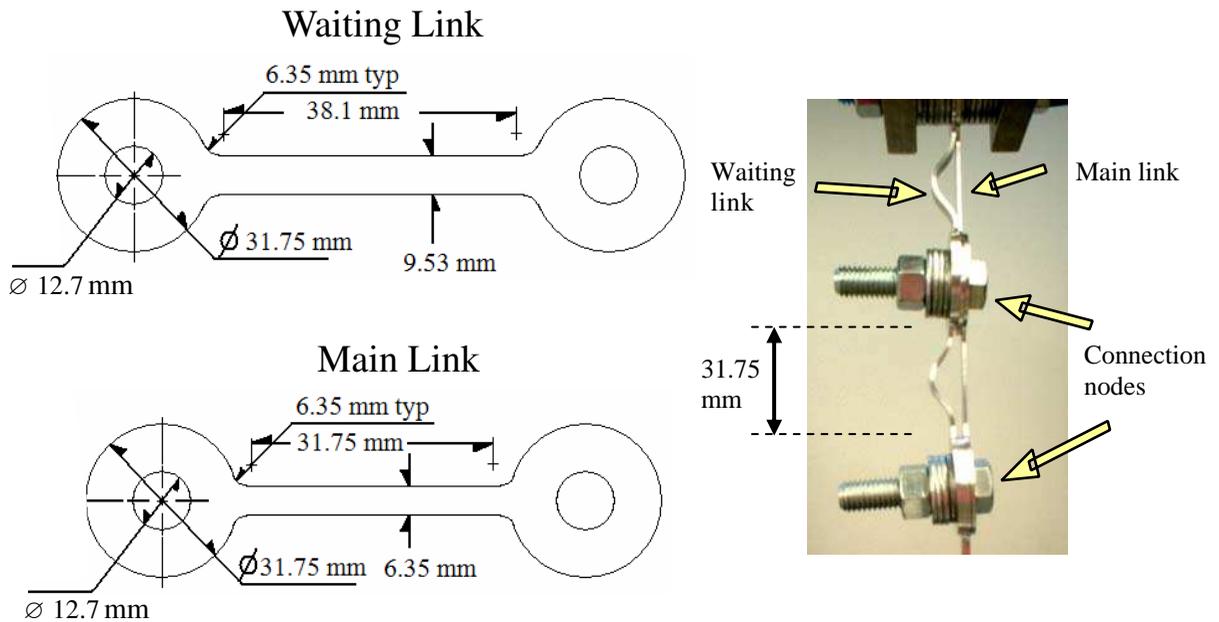
## 2. Design of a single bistable link

The ductility and strength of main links and waiting links need to be tailored appropriately to have the desired bistable behavior and synergistic mechanism with a controlled, high energy failure. The first studies were carried out on a mildly strain-hardened 5000 series aluminum, namely 5052-H32 aluminum. While the property of thermal hardening of 2000-, 6000- and 7000-series aluminum alloys would prove more useful for engineering applications, tests using a 5000-series configuration proved adequate for the scope of this study. The chosen alloy has a published elongation to failure of 12% [Oberg et al. 2000]. Figure 2 shows one of the first iterations, which failed.

By trial and error, a design with dog-bone shaped specimens was found to give the desired response. A dog-bone shape is also consistent with standard tensile testing (per ASTM standard E8). The material used was 1.59 mm thick. Dimensions and shapes of main links and waiting links are shown in Figure 3, left. The waiting link was curved to shape, and is 20% longer and 50% wider than the main link, as shown in Figure 3, right. The connection nodes, that is, the fasteners connecting the main link to a waiting link, had rather significant weight and stiffness with respect to the thin specimens.



**Figure 2.** Some first, unsuccessful designs. Specimens are made of 5052-H32 aluminum.



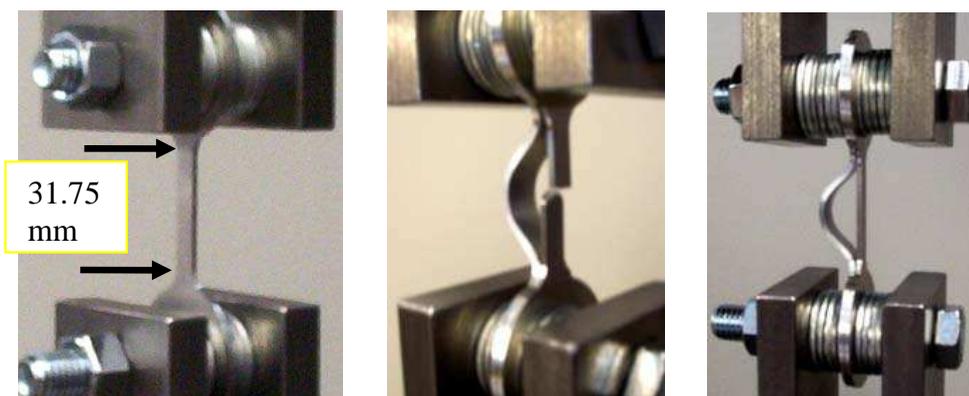
**Figure 3.** Left: Dimension of the main link and waiting link (sketch not to scale). The waiting link is 20% longer than the main link, that is, the percent wait is 20%. Right: Detail of main links, waiting links and connection nodes in an intact three-link chain of 5052-H32 aluminum, whose behavior under tension is shown in Figure 6.

Extensometers were not available which could span the gauge lengths of the specimens, so the cross-head displacement had to be used to assess the strain of the specimens. Any deformation outside the gauge lengths was to be avoided, which required the use of heavy and stiff connection nodes. The focus of this research was the behavior of the bistable links and not node or connection optimization. It was necessary to maximize the deformation of main and waiting links with respect to the connections.

Tests were conducted using either a 222 kN screw-driven machine or a 44.5 kN screw-driven machine (based on availability), at a rate of 1.5 mm/minute. The first machine was preferred, due to its simpler load train.

In this paper we introduce the notion of *percent wait*, the percent difference of the waiting link length relative to the attached main link length. The main link of a given material will fail at a particular elongation. Cherkaev and Slepian predicted that the waiting link would be disabled until the elongation to failure of the attached main link is reached. Using our definition of percent wait, this would mean that a percent wait equivalent to the elongation to failure would be needed for the mechanism to work.

A number of different materials, thicknesses and shapes were tested, which showed that the straightening of the curved waiting link could not be ignored in the process. It was observed through testing (discussed below) that an aluminum main link with a nominal elongation to failure equal to 15% needed a waiting link with percent wait equal to 20%, that is, an increase of 5 percent wait over that expected by the theory. This difference is due to the nonlinear response of the waiting link as it straightens. The waiting link not only had to be able to sustain the maximum load of the attached main link, but also a



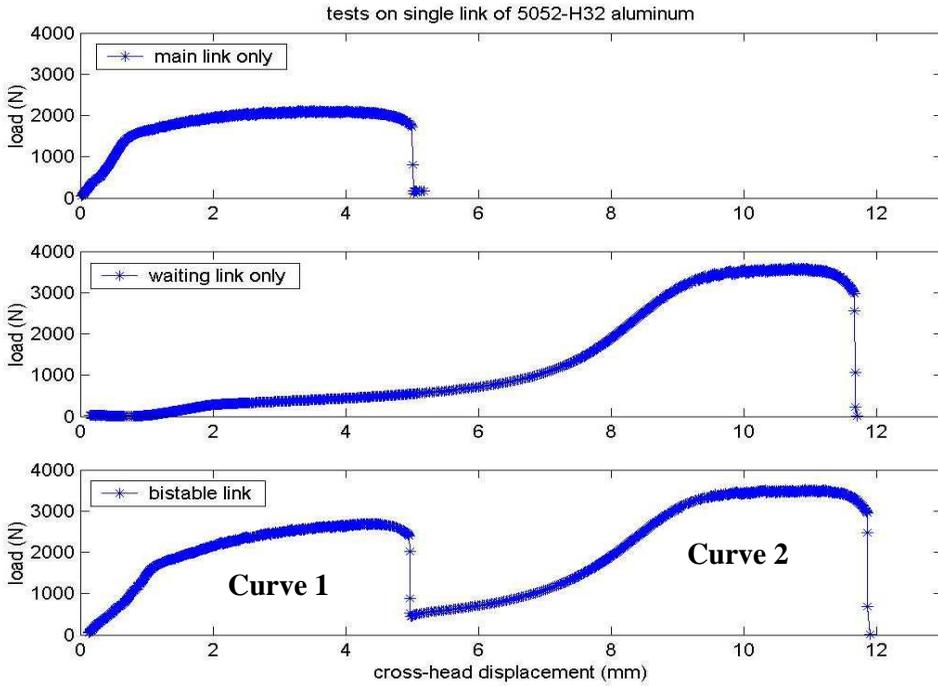
**Figure 4.** Tests on 5052-H32 aluminum. Left: a main link (Test A). Middle: a waiting link, where the main link has been cut (Test B). Right: full link (Test C). The load/displacement curves are shown in Figure 5.

sufficient additional load to allow the fracture of another main link in the chain (for chains having more than one main link and one waiting link). This is why the waiting link in Figure 3 is 20% longer than the main link.

The three tests that led us to the conclusions above were: Test A, a tensile test on a main link by itself, Test B, a tensile test to identify the behavior of a waiting link, and Test C, a tensile test on a main link connected to a waiting link. Figure 4 shows photographs of these tests. Test A (left pane) determined the overall elongation and maximum load of one single main link of 5052-H32 aluminum, following the ASTM standard E8. Test B evaluated the waiting link's response (middle pane). This required that the waiting link had the same node-to-node length as the main link. During the initial part of the test, a waiting link by itself would elongate past this node-to-node distance as the load increased to the 44.5 N preload. To avoid this, a main link was added and connected to the waiting link, forming the full link. After the preload was established — that is, when there was no slack in the load train — the main link was cut across its section, ensuring further testing of a waiting link of the correct dimensions. A nonlinear load/cross-head displacement curve was obtained as a result of the waiting link's straightening and strain hardening effects caused by the initial curved shape of the link. The acceptable waiting link dimensions and response were thus determined experimentally. Finally, Test C was carried out on the full bistable link, that is, the main link plus waiting link (Figure 4, right). Figure 5 shows the load/displacement curves corresponding to these tests.

**Test A.** The main link had length of 31.75 mm. The maximum cross-head displacement recorded was 5.15 mm, thus resulting in a 16.2% elongation to failure ( $5.15/31.75 \cdot 100$ ). The maximum load was reached at an elongation of 10.3% ( $3.27/31.75 \cdot 100$ ). Thus, the final 5.9% elongation was due to necking.

Had the main link been twice as long, the elongation to failure would have been different: the main link would presumably have reached the same maximum load at the same elongation, 10.3%, but the strain energy involved in the link's further extension and necking (the additional 1.09 mm) would have been associated with twice as much length, with a resulting elongation of  $1.09/(31.75 \cdot 2) \cdot 100 = 1.72\%$ .



**Figure 5.** Load/displacement curves for tests A, B, C of the 5052-H32 aluminum specimens shown in Figure 4. Responses of main link (top), waiting link (center), and full main link plus waiting link (bottom).

This, added to the elongation at maximum load, 10.3%, would result in a 12.0% elongation to failure. These considerations show that testing on the main link is needed, because the elongation to failure changes with the length of the specimen.

**Test B.** As discussed above, the elongation to failure of the main link was 16.2%. The waiting link's elongation to failure was a smaller 14%. Some of the strain energy involved in the process was used to straighten the waiting link, which had been initially bent to shape, and was 6.36 mm longer than the main link. Hence, the elongation to failure was calculated by subtracting 6.36 mm from the overall 11.7 mm. Also, the maximum load for the waiting link was 3602 N, a 70% increase with respect to the maximum load of the main link. Recall that the waiting link is 50% wider than the corresponding main link, hence a 50% increase should have been obtained on account of this width difference. The additional 20% increase of maximum load should be attributed to the waiting link's strain hardening process, since it was curved to shape and then straightened during testing. This also illustrates the difficulty of predicting response of the structure without conducting experiments.

**Test C.** The full link was tested next. As mentioned earlier, the waiting link associated with a failed main link had to carry not only the maximum load of another intact main link but also additional load due to the straightening of the latter's waiting link. This allowed for greater energy absorption of the chain. Curve 1 in Figure 5 corresponds to the bistable link, which is undamaged until the main link

breaks, indicated by the drop in Curve 1. Peak load is 27.2% higher than the maximum load for Test A (the tensile behavior of the main link by itself). Curve 2 corresponds to the now partially damaged link (following Curve 1, the main link is now broken, and the load is carried by the waiting link still attached to the main link). The peak load of Curve 2 in is 2.23% lower than the maximum load for Test B (the tensile behavior of the waiting link by itself), but it is also 30.6% higher than the peak of Curve 1 (the tensile behavior of the bistable link before the main link's failure). Therefore, the curve not only exhibits bistable behavior, but has a partially damaged state with higher load carrying capability than its corresponding undamaged state.

The strain energies associated with these tests were calculated by numerical integration using the trapezoid rule (through the MATLAB routine 'trapz'), and were found to be 8.89J for Test A (the main link), 16.0J for Test B (the waiting link), and 25.1J for Test C (the full link), which is equivalent to the sum of the first two tests.

The following section will show that this bistable structure can absorb more energy than its baseline, depending on the number of links in the chain.

### 3. Proof of increased energy absorption of bistable links

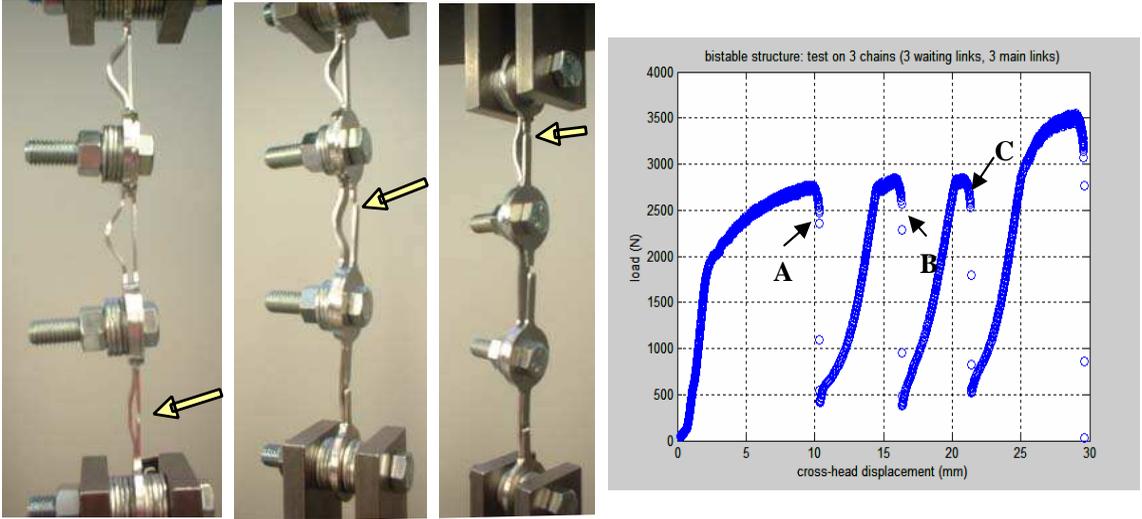
Three different chains of bistable links, manufactured with 5052-H32 aluminum, were tested to investigate the effect of additional necking on the overall energy absorption of the specimens, and to identify the actual increase of energy absorption of these configurations. Also, three corresponding baseline specimens were tested, where the formation of a single necking region caused the failure of the entire specimen. These latter specimens had the same cross-section as each full bistable link. Recall that the main link and the waiting link were made of 1.59 mm thick sheet material, and were 6.35 mm and 9.52 mm wide, respectively. The baseline had the same thickness and a width of 15.9 mm, the sum of the two widths. The baseline length was one, two and three times the length of the single bistable link plus an additional 12%. This 12% accounts for the additional material present in the waiting link, which is 20% longer than its corresponding main link. Normalizing with respect to the width, one obtains  $0.20 \cdot 9.52/15.9 = 12\%$ . The material used in the nodes was not accounted for in the baseline.

Figure 6 shows a tensile experiment on a three-link chain with its corresponding load/displacement curve. Figure 7 shows the load/displacement curves for all bistable and baseline specimens. The absorbed energy was calculated, and is reported in Table 1. These data indicate that at least three bistable links would be needed for a consistent increase of energy. For three bistable elements, the energy increase is about 30% with respect to the baseline specimens.

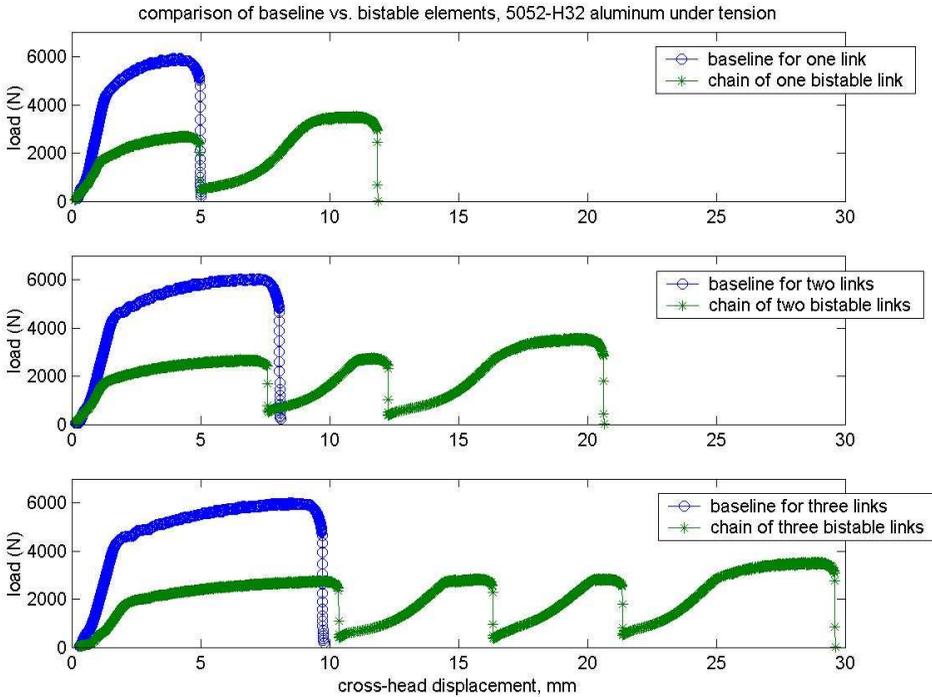
Future work will include uncertainty analysis. The goal of the current study was to identify a metallic bistable structure with the desired behavior. The authors plan to optimize test conditions, manufacturing, or any other elements of the process that would reduce data scatter for metallic bistable configurations.

### 4. Comparison of materials with different ductility

An investigation was carried out using two ductile materials in addition to the 5052-H32 aluminum: A36 annealed mild steel, and C10100 annealed copper. These materials were selected because of their published high ductility. This study aimed to assess how materials with different ductility and strength compared to each other. The dimensions of the main links and waiting links were the same for all



**Figure 6.** A chain of 5052-H32 aluminum with three bistable links. Left: Experiment. Arrows highlight the fracture and straightening of the waiting links in time. Right: Load/displacement curve. Points A, B, and C correspond to fracture of the first, second, and third main links.



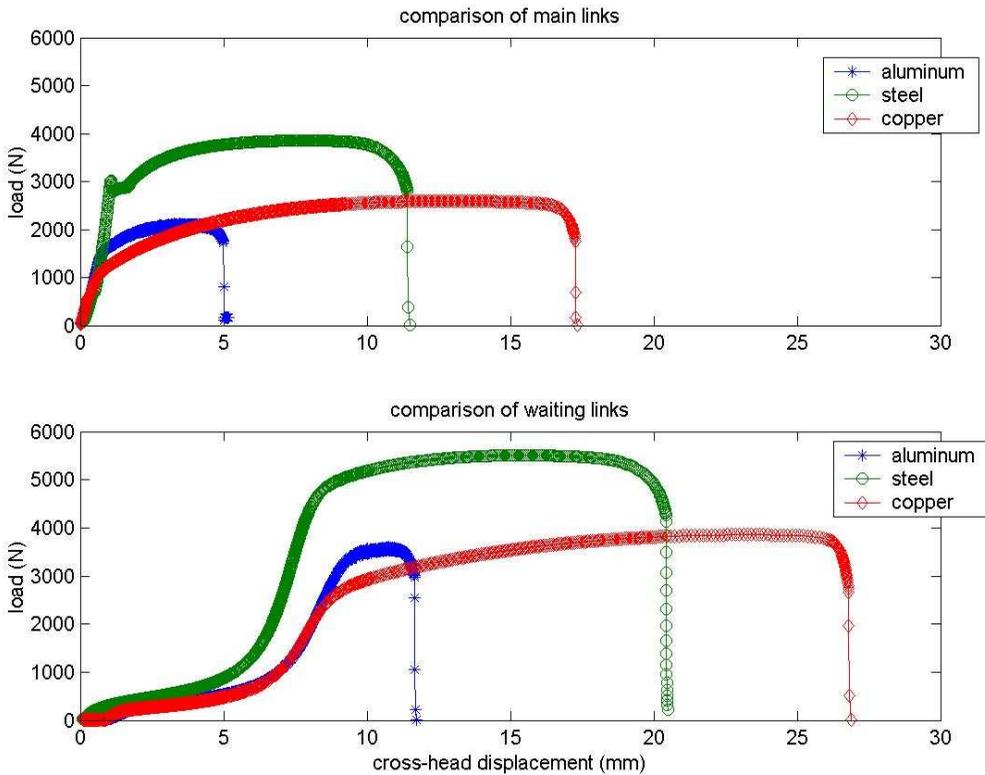
**Figure 7.** Comparison of bistable links and baseline links. From top to bottom, displacement/load curves for one, two, and three links of 5052-H32 aluminum.

<i>Number of links</i>	<i>Energy of bistable specimens (J)</i>	<i>Energy of baseline specimens (J)</i>	<i>Bistable/baseline energy ratio (%)</i>
1	25.1	22.3	112.4
2	43.1	38.6	111.6
3	61.3	46.7	131.3

**Table 1.** Energy absorbed in the yielding and fracture process of one to three links of bistable elements (5052-H32 aluminum), compared to corresponding baselines.

materials, that is, those shown in Figure 3. Tensile experiments were run on the main links alone and on the waiting links alone. Tensile experiments on the bistable link were omitted since the percent wait, 20%, was less than the elongation to failure for steel and copper, which prevented the desired behavior in the resulting main link + waiting link (that is, a second phase stronger than the first).

Figure 8 shows the load/displacement curves for the three materials. Steel was considerably stronger than the other materials, while copper had the highest elongation.



**Figure 8.** Comparison of main links and waiting links for three materials: 5052-H32 aluminum, A36 mild steel, and C10100 annealed copper.

Table 2 reports data on elongation to failure, elongation at maximum load, maximum load, energy absorbed, and its normalized value with respect to the density. Note the differences in the behavior of main links and waiting links regarding elongation, maximum load and absorbed energy.

Assume that necking starts at the maximum load. The aluminum main link, for example, will spend  $(16.2 - 10.3)\% = 5.9\%$  of its elongation in necking and fracturing, while its associated waiting link will spend  $(14 - 11.5)\% = 2.5\%$  in the same physical operation.

Inspecting maximum load, one can expect the maximum load of waiting links to be 1.5 times higher than for the corresponding main links, because the waiting link has a 50% wider section. This turned out to be a relatively good estimate (within 5%) for steel and copper, but not for aluminum, where the waiting link had a maximum load 13% higher than the predicted value. Therefore, it seems that the strain hardening in aluminum due to the bending to shape and straightening of the waiting link indeed does increase the maximum load. Steel and copper, however, had a small decrease of maximum load (within 5%).

Considering energies absorbed, it was expected that waiting links, which are 20% longer and 50% wider than the main link, would absorb  $(1 + 0.50) \cdot (1 + 0.20) = 1.8$  times more energy than the corresponding main link. This turned to be a correct estimate for aluminum and copper (within 1%). However, steel absorbed 7.6% more energy than expected.

Finally, one can calculate the absorbed energy per density (using  $2680 \text{ kg/m}^3$  for 5052-H32 aluminum,  $7850 \text{ kg/m}^3$  for A36 steel, and  $8940 \text{ kg/m}^3$  for C10100 copper). In this latter case, the steel main link and waiting link are found to have superior energy absorption/density with respect to the other materials investigated.

We see from our comparative study of the three materials that an estimate for maximum load and absorbed energy may be a reasonable approximation for some materials, but not others. It may not capture different behavior as the waiting links are first bent to shape and then straightened under loading. It is important to test waiting links in order to correctly design a bistable structure.

	<i>Elongation to failure (%)</i>	<i>Elongation at maximum load (%)</i>	<i>Maximum load (N)</i>	<i>Absorbed energy (J)</i>	<i>Absorbed energy / density (J m<sup>3</sup>/kg)</i>
<b>Main link</b>					
<i>Aluminum</i>	16.2	10.3	2122	8.9	0.0033
<i>Steel</i>	36.0	24.8	3854	38.6	0.0049
<i>Copper</i>	54.4	40.8	2594	38.6	0.0043
<b>Waiting link</b>					
<i>Aluminum</i>	14.0	11.5	3603	16.0	0.0060
<i>Steel</i>	37.0	23.9	5506	74.8	0.0095
<i>Copper</i>	53.7	43.3	3855	69.9	0.0078

**Table 2.** Data for main links and waiting links made with three materials: 5052-H32 aluminum, A36 annealed mild steel, and C10100 annealed copper.

Further studies on metallic bistable structures will adopt steel for both main links and waiting links, because of the higher performance per density of this material compared to aluminum and copper.

## 5. Summary and conclusions

This paper presents an experimental investigation of metallic bistable structures loaded in quasi-static tension. The concepts of bistable structures for energy absorption were introduced by Cherkaev, Slepian and their collaborators starting in 1995. However, this paper is the first actual experimental study which backs their theories. Bistable structures are material-based mechanisms that may absorb more energy than their corresponding baselines because of a more efficient way of using the material's ductility and multiple necking regions. Moreover, the two main components of these structures, *main links* and *waiting links*, are designed to ensure a redundant load path and a delayed and controlled failure. Results were shown and discussed as follows for the different investigations:

***Design of a main link and waiting link such that the corresponding bistable link has a partially damaged state which can absorb a higher load than the undamaged state.*** Trial-and-error experimental testing on 5052-H32 aluminum (Tests A, B, and C in this paper) allowed us to design appropriate dimensions of main links and waiting links to reach this goal. It is observed that the response of the waiting link cannot be predicted by conventional tensile testing, and that the so-called *percent wait* (the percent difference in waiting link length with respect to the attached main link length) may need to be higher than the elongation to failure predicted by Cherkaev, Slepian, and their collaborators.

***Study of the energy absorption with respect to the number of bistable links.*** The behavior of chains with one, two and three bistable links of 5052-H32 aluminum was compared to the corresponding baselines of the same material. Results show that a three-link bistable chain absorbs 30% more energy than its baseline.

***Study of main links and waiting links of three different materials: 5052-H32 aluminum, A36 mild, annealed steel and C10100 annealed copper.*** Main links and waiting links had dimensions as in Figure 3. This work shows that predictions of maximum loads and absorbed energy may not capture correctly the behavior of waiting links for all materials, thus testing is recommended. Moreover, main links and waiting links made of steel turned out to have a better performance (in terms of maximum loads and energy absorbed per unit density) than for the other materials.

In conclusion, this paper discusses initial experimental work on metallic bistable structures for energy absorption. Further investigation will concentrate on identifying the role of the connection nodes, collecting more data for statistical significance, optimizing the manufacturing and testing process to reduce data scatter, using steel for main links and waiting links, and extending these concepts to different types of loading (such as bending, shear and impact). At this stage, it is not possible to give guidelines for designing such structures, but future work will include optimization and numerical simulations, and will allow us eventually to reach this goal. The authors strongly believe that bistable structures have the potential to offer revolutionary energy absorption and crash worthiness compared to traditional structures, and that they would be very useful in applications where fail-safe behavior is critical.

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