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OUT-OF-PLANE STRESS AND DISPLACEMENT FOR THROUGH-THE-THICKNESS CRACKS IN PLATES OF FINITE THICKNESS

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The out-of-plane stress and displacement fields are investigated analytically for mode I through-the-thickness cracks in an infinite plate of finite thickness within the first-order plate theory. The developed method is based on the distributed dislocation approach and an earlier derived three-dimensional solution for an edge dislocation. Numerical results are obtained through application of Gauss–Chebyshev quadrature for both finite length and semiinfinite crack cases. The calculated stress and displacement fields are found to be in good agreement with already published experimental and finite element studies. Further results for the averaged through-the-thickness stress intensity factor are given and again found to be in good agreement with previous finite element values. The developed solutions can therefore be used in experimental techniques for the assessment of the stress intensity factor using the out-of-plane displacement measurements, for example by the interferometry method.

1. Introduction

The investigation of geometric singularities like edges or cracks in engineering structures requires an understanding of the complicated three-dimensional stress field surrounding the singularity. Over the past fifty years, analytical and numerical investigations in fracture mechanics have mainly focused on two-dimensional or axisymmetric geometries. This is due to the much needed simplifications that the classic two-dimensional theories of elasticity bring to the mathematical analysis. Three-dimensional effects are often acknowledged in these studies as the true crack tip stress field is always triaxial. However, the relationship between the actual three-dimensional distribution and the results obtained within the two-dimensional theories is still not completely understood. For that reason, three-dimensional crack problems have been identified as a critical area where further research is needed [Erdogan 2000].

The three-dimensional crack-front stress and displacement fields have been investigated by many researchers including Cruse [1970], Burton et al. [1984], Yang and Freund [1985], Nakamura and Parks [1988], Leung and Su [1995] and Nevalainen and Dodds [1995], to name only a few. In particular, Hartranft and Sih [1970] proposed an approximate three-dimensional theory and studied the effects of plate thickness on the stress intensity factor. A comprehensive literature review on the earlier investigations of three-dimensional crack problems is provided by Kwon and Sun [2000].

The triaxial stress state in the vicinity of a crack tip in a sufficiently brittle material has been found to have a significant influence on fracture behavior [Kong et al. 1995]. Yang and Freund [1985] and Yuan

Keywords: distributed dislocation technique, edge dislocation, out-of-plane constraint factor, out-of-plane displacement, plate thickness effect, through-the-thickness crack.

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and Brocks [1998] have shown that the specimen thickness significantly effects the crack tip stress and displacement fields, which play a crucial role in the initiation and propagation of cracks [Guo 2000].

A number of experimental investigations have been undertaken to determine the three-dimensional stress and displacement fields in the vicinity of the crack tip for a range of cracked geometries. In an experimental study by Rosakis and Ravi-Chandar [1986], the method of caustics by transmission and reflection was employed to determine the extent of the three-dimensional crack tip region. They found that plane stress conditions are recovered at a radial distance of around half the plate thickness, which confirms the analytical results of Yang and Freund [1985]. Similar conclusions were made by Pfaff et al. [1994] and Humbert et al. [2000] who utilized interferometry to determine the out-of-plane displacement field surrounding a mode I crack.

Theoretical investigations of three-dimensional crack tip stress and displacement fields have mainly utilized finite element (FE) techniques. Several researchers, including Nakamura and Parks [1988] and She and Guo [2007b], have provided detailed analyzes of the crack tip region for mode I and mixed mode (I–II) semiinfinite cracks, respectively. It was shown that the out-of-plane stress and displacement fields exhibit significant three-dimensional effects within a radial distance from the crack tip of about half the plate thickness and converge with the plane stress solutions at around 1.5 times the plate thickness. The FE results of Nakamura and Parks [1988] were found to be in reasonable agreement with the experimental study by Pfaff et al. [1994].

The purpose of this paper is to present an analytical method for calculating the stress and displacement fields at the tip of a through-the-thickness crack using the first order plate theory. This theory was previously utilized by Yang and Freund [1985] for investigating three-dimensional effects for a semiinfinite crack though only qualitative comparison with experimental results was provided in their work. The methods developed in the current paper are based on the distributed dislocation technique (DDT) and the solution for an edge dislocation in a plate of arbitrary thickness [Kotousov and Wang 2002]. Both semiinfinite and finite length cracks are investigated covering almost all geometries considered in the previous studies. The calculated results compare well with the previously published data. These solutions can therefore be used in experimental techniques for the assessment of the stress intensity factor using the out-of-plane displacement measurements, for example by the interferometry method.

In this paper, a brief review of the DDT for semiinfinite cracks is first given followed by the results for the out-of-plane constraint factor. In the next section, the formulation of the finite length crack problem is presented along with the results for the out-of-plane displacement.

2. Semiinfinite crack in a finite thickness plate

We will begin by providing an outline of the DDT as applied to a straight semiinfinite crack in a plate of thickness $2h$. A full description of the technique has already been given by Codrington and Kotousov [2007]; however, a brief review will be presented here for completeness. It is assumed that a through-the-thickness crack lies along the x axis ($-\infty < x < 1$) in an infinite plane and is subjected to a remotely applied mode I stress intensity factor K. If the crack is replaced with a continuous distribution of dislocations along the x axis then the y-stress field is given by the superposition principle as the singular integral equation [Hills et al. 1996]

$$\sigma_{yy}(x, y) = \frac{1}{\pi} \int_{-\infty}^{1} B_y(\xi)\, G_{yy}(x-\xi, y)\, d\xi. \tag{1}$$

In Equation (1), $B_y(\xi)$ is the unknown dislocation density function; it is related to the separation of the crack faces $g(\xi)$ by $B_y(\xi) = -dg(\xi)/d\xi$. The function $G_{yy}(x, y)$ is the dislocation influence function, which forms the singular kernel of the system. The influence functions depend on the geometry of the problem under investigation and various solutions are available in the literature. A comprehensive review is provided by Hills et al. [1996]. In the case of a plane stress or plane strain analysis, the y direction influence function for a dislocation in an infinite plane is given by Hills et al. [1996] as

$$G_{yy}(x, y) = \frac{2\mu}{(\kappa+1)} \frac{x}{\rho^4} (x^2 + 3y^2), \tag{2}$$

where μ is the shear modulus, κ is Kolosov's constant being either $(3-\nu)/(1+\nu)$ for plane stress or $3-4\nu$ for plane strain, with ν being Poisson's ratio, and $\rho^2 = x^2 + y^2$.

Three-dimensional geometry effects will be considered for the case of a finite thickness plate by applying the solution for an edge dislocation in an infinite plate of thickness $2h$ [Kotousov and Wang 2002]. The developed influence functions are based on first-order plate theory [Kane and Mindlin 1956] whereby it is assumed that the out-of-plane strain is uniform in the thickness direction. Namely, generalized plane strain conditions are assumed to exist. In addition, the simplification is made that the dislocation Burgers vector and each of the triaxial stress components are uniform across the thickness of the plate and are equal to the average through-the-thickness values. Results obtained from this theory have been shown to be in good agreement with the through-the-thickness averages from careful three-dimensional FE studies [Berto et al. 2004; Kaèianauskas et al. 2005; She and Guo 2007a]. The y direction influence function for the case of a finite thickness plate is determined by Kotousov and Wang [2002] as

$$G_{yy}(x, y) = -\frac{E}{4(1-\nu^2)} \frac{x}{\rho^2} \left[-(1-\nu^2) + \frac{4\nu^2}{(\lambda\rho)^2} - 2\nu^2 K_0(\lambda\rho) - \frac{2\nu^2\,(2+(\lambda\rho)^2)\,K_1(\lambda\rho)}{\lambda\rho} \right], \tag{3}$$

where E is Young's modulus, $K_0(\cdot)$ and $K_1(\cdot)$ are the modified Bessel functions of the second kind of order 0 and 1, respectively, and the parameter λ is given by

$$\lambda = \frac{1}{h}\sqrt{\frac{6}{1-\nu}}.$$

To solve the integral equation (1) via Gauss–Chebyshev quadrature, we need first to introduce the coordinate transformations

$$x = \frac{2t}{t+1}, \qquad \xi = \frac{2s}{s+1}, \tag{4}$$

which give rise to the transformed integral equation

$$\bar{\sigma}_{yy}(t, y) = \frac{2}{\pi} \int_{-1}^{1} \bar{B}_y(s) \bar{G}_{yy}(t-s, y) \frac{ds}{(s+1)^2}. \tag{5}$$

By applying Gauss–Chebyshev quadrature to (5), the integral is reduced to a linear series in n unknowns, $\bar{\phi}(s_i)$, such that

$$\bar{\sigma}_{yy}(t, y) = \frac{2}{n} \sum_{i=1}^{n} \frac{1}{s_i + 1} \bar{\phi}(s_i)\, \bar{G}_{yy}(t - s_i, y), \tag{6}$$

where n is the number of integration points and

$$s_i = \cos\left(\frac{2i - 1}{2n} \pi\right), \qquad i = 1 \cdots n.$$

Along the length of the crack, where $x < 1$ and $y = 0$, the summation (6) is only valid at the discrete collocation points, which are

$$t_k = \cos\left(\frac{k}{n} \pi\right), \quad k = 1 \ldots n - 1.$$

Outside of the crack interval, where $x > 1$ or $y \neq 0$, Equation (6) may be evaluated at any point.

The nonsingular function $\bar{\phi}(s)$ is related to the dislocation density by

$$\bar{B}_y(s) = \bar{\phi}(s)(1 + s)^{1/2}(1 - s)^{-1/2}. \tag{7}$$

It is assumed in Equation (7) that the stress field and, similarly, the gradient of the crack opening displacement are square root singular at the crack tip, where $x = s = 1$. Furthermore, the singularity is taken as being uniform across the entire thickness of the plate. This is due to the averaging nature of the first-order plate theory employed in the dislocation solution, which assumes a constant Burgers vector and uniform stresses across the plate thickness. Other singularities that are associated with three-dimensional geometry are, as a result, unable to be described in the analysis, for example the corner singularity found at the intersection of the crack front and the free surface of the plate [Benthem 1980]. As $x \to -\infty$ or $s \to -1$, the gradient of the crack opening displacement approaches zero and this has also been incorporated into Equation (7).

The through-the-thickness average crack tip stress intensity factor can be determined directly from an asymptotic analysis of the crack tip opening displacement or stress field near the crack tip. In the case of the plane stress or plane strain analysis, this gives

$$K_{\text{tip}} = \lim_{r \to 0} \sqrt{2\pi r}\, \frac{2\mu}{\kappa + 1} \frac{\partial \hat{g}(r)}{\partial r} = \sqrt{2\pi}\, \frac{2\mu}{\kappa + 1} \bar{\phi}(1), \quad (\theta = \pi) \tag{8}$$

and for the case of a finite thickness plate

$$K_{\text{tip}} = \lim_{r \to 0} \sqrt{2\pi r}\, \hat{\sigma}_{yy}(r, 0) = \frac{\sqrt{2\pi}\, E}{4(1 - \nu^2)} \bar{\phi}(1). \tag{9}$$

Here, $\hat{g}(r) = g(x)$ for $\theta = \pi$ or 0, $\hat{\sigma}_{yy}(r, \theta) = \sigma_{yy}(x, y)$ and the conversion between Cartesian and polar coordinates can be made via the transformations $x = r\cos(\theta) + 1$ and $y = r\sin(\theta)$. From inspection of Equation (9), it can be seen that the stress intensity factor for the finite thickness plate is simply the plane strain form of (8). However, it should be noted that the stress state is actually generalized plane strain, in accordance with the first-order plate theory, not plane strain as (9) suggests. The function $\bar{\phi}$ has only been defined at each of the integration points; therefore, $\bar{\phi}(1)$ may be found using the extrapolation formula obtained by Krenk [1975] and Hills et al. [1996]. In most cases it is generally sufficient to approximate

$\bar{\phi}(1)$ by $\bar{\phi}(s_1)$, since the computations for the exact value of $\bar{\phi}(1)$ are quite lengthy. In a similar manner, the side condition of a stress intensity factor, K_{far}, applied remotely from the crack tip can be written as

$$\bar{\phi}(S_n) = \frac{K_{\text{far}}}{\sqrt{2\pi}} \frac{\kappa+1}{2\mu} \tag{10}$$

for the cases of plane stress and strain. In the finite thickness plate analysis, it is assumed that plane stress conditions will prevail remotely from the crack tip [Yang and Freund 1985] and therefore the plane stress form of (10) is employed.

Use is now made of the requirement that the crack faces must remain traction free, which means that $\sigma_{yy}(x, 0) = 0$ along the crack length or simply $\bar{\sigma}_{yy}(t_k, 0) = 0$. This constraint, together with Equation (6), provides a system of $n-1$ linear equations in n unknowns $\bar{\phi}(s_i)$. Depending on whether a plane stress/strain analysis or a finite thickness plate analysis is undertaken, the choice of the kernel function $G_{yy}(x, y)$ as either (2) or (3) respectively is made. The nth equation which completes the set of linear equations is given by the condition of the remotely applied stress intensity factor (10). The set of n linear equations in n unknowns can now be solved via any standard method.

3. Results for the out-of-plane stress

In this section, results for the out-of-plane stress field are presented for a semiinfinite crack in a plate of finite thickness. The effect of the out-of-plane stresses are commonly described in the literature by the out-of-plane constraint factor

$$\widehat{T}_z(r, \theta) = T_z(x, y) = \frac{\sigma_{zz}(x, y)}{\nu[\sigma_{xx}(x, y) + \sigma_{yy}(x, y)]}$$

where the stress components $\sigma_{xx}(x, y)$ and $\sigma_{zz}(x, y)$ are found in a similar manner to (6) by replacing the kernel with

$$G_{xx}(x, y) = \frac{E}{4(1-\nu^2)} \frac{x}{\rho^2} \left[(1-\nu^2) + \frac{4\nu^2}{(\lambda\rho)^2} - 2\nu^2 K_0(\lambda\rho) - \frac{4\nu^2 K_1(\lambda\rho)}{\lambda\rho} \right],$$

$$G_{zz}(x, y) = \frac{E\nu}{2(1-\nu^2)} \frac{\lambda x}{\rho} K_1(\lambda\rho),$$

respectively [Kotousov and Wang 2002].

Results for the out-of-plane constraint factor crack are shown in Figure 1, for a semiinfinite, as a function of the radial distance from the crack tip to plate thickness ratio $r/2h$ with $\theta = 0$. FE results by Nakamura and Parks [1988] and She and Guo [2007b] for semiinfinite cracks are also provided as a comparison. Both the mid-thickness (MT) and through-the-thickness average (AV) FE values are given. It can be seen that the mid-thickness results are in better agreement with the present values than the average results are. This is due to the different modeling assumptions made in each of the studies. Namely, the current investigation makes the simplification of generalized plane strain conditions in the vicinity of the crack tip. Furthermore, FE techniques are limited by the finite mesh size in representing the singular stress field near the crack tip. Figure 1 shows that at the crack tip the conditions reach near plane strain while at approximately $r/2h = 1.5$ the plane stress solution is recovered. In these and all

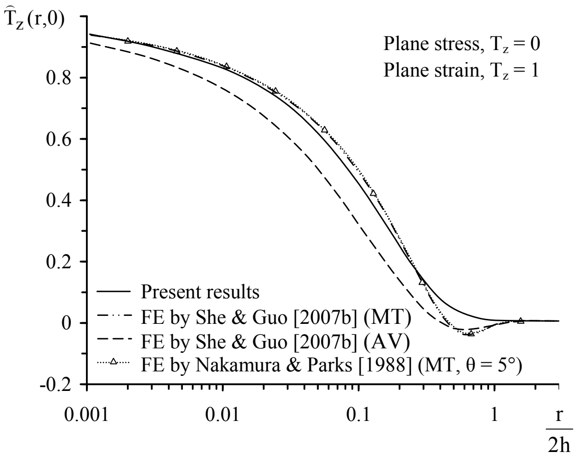

Figure 1. Out-of-plane constraint factor as a function of the ratio $r/2h$ for a semiinfinite crack ($\theta = 0°$).

subsequent calculations, Poisson's ratio is taken as $\nu = 0.3$ though any variation of ν has minimal effect on the constraint factor. Approximately 250 integration points are required to reach a convergence in the solution.

Figure 2 shows the results for the out-of-plane constraint factor as a function of θ for various $r/2h$. Again a semiinfinite crack model has been used. The mid-thickness FE results by She and Guo [2007b] are given and they show a good agreement with the present results. Results for the ratio of the average crack tip stress intensity factor to the far-field stress intensity factor are shown in Figure 3 as a function of Poisson's ratio. The crack tip stress intensity factor for the case of a finite thickness plate is determined by Equation (9). The present results are identical to the through-the-thickness average of the values presented by She and Guo [2007b] and Nakamura and Parks [1988] for semiinfinite cracks.

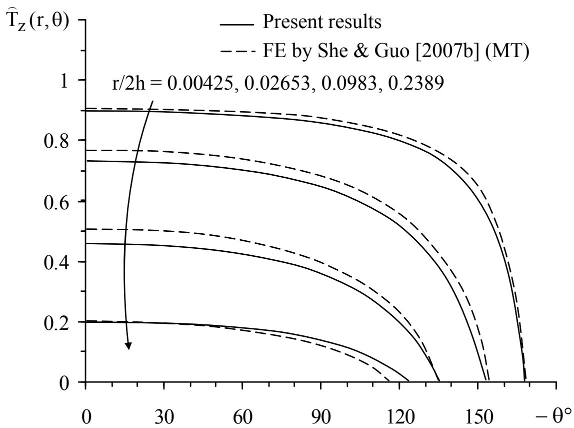

Figure 2. Out-of-plane constraint factor as a function of θ for various $r/2h$ for a semi-infinite crack.

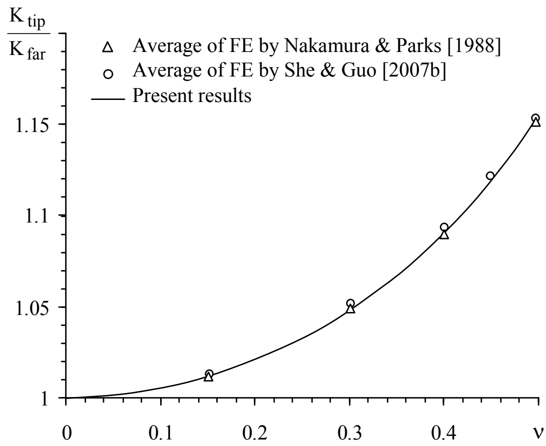

Figure 3. Ratio of the average crack tip stress intensity factor to the far-field stress intensity factor as a function of Poisson's ratio for a semiinfinite crack.

4. Results for the out-of-plane displacement

The out-of-plane displacement for the case of a semiinfinite crack can be determined at any point within the plate by the function

$$\widehat{u}_z(r, \theta, z) = u_z(x, y, z) = \frac{z}{\pi h} \int_{-\infty}^{1} B_y(\xi)\, G_{uz}(x-\xi, y)\, d\xi,$$

where the plate mid-thickness is at $z = 0$ and the plate surfaces are at $z = \pm h$. The displacement kernel for the finite thickness plate analysis is given by Kotousov and Wang [2002] as

$$G_{uz}(x, y) = -\frac{\nu h \lambda}{2} \frac{x}{\rho} \left[\frac{1}{\lambda \rho} - K_1(\lambda \rho) \right]$$

and in the case of plane stress is

$$G_{uz}(x, y) = -\frac{\nu h}{2} \frac{x}{\rho^2}.$$

The formulation of the finite length crack problem is very similar to that of the semiinfinite crack as outlined in Section 2 and thus most details are omitted. It is assumed that a through-the-thickness crack of length $2a$ lies within $-a < x < a$ on the x axis in an infinite plane and is subjected to remotely applied stress, $\sigma_{yy}^{\infty}(x)$. The governing singular integral equation therefore becomes [Hills et al. 1996]

$$\sigma_{yy}(x, y) = \frac{1}{\pi} \int_{-a}^{a} B_y(\xi)\, G_{yy}(x-\xi, y)\, d\xi + \sigma_{yy}^{\infty}(x). \tag{11}$$

Solution to the integral equation (11) follows via application of Gauss–Chebyshev quadrature in a similar manner as for the semiinfinite crack case. The transformations (4), however, are replaced with the new transformations:

$$x = at, \qquad \xi = as,$$

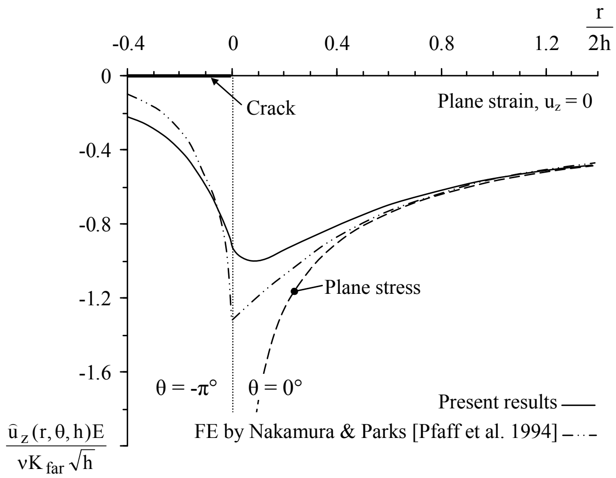

Figure 4. Normalized out-of-plane surface displacement as a function of $r/2h$ in the case of a semiinfinite crack.

and the nonsingular function $\bar{\phi}(s)$ is now related to the dislocation density by

$$\bar{B}_y(s) = \bar{\phi}(s)(1+s)^{-1/2}\,(1-s)^{-1/2}.$$

Here it is assumed that the dislocation density function is square root singular at both $s = -1$ and $s = 1$ since there is a singularity in the displacement gradient and stress fields at each of the crack tips.

The out-of-plane displacement for the finite length crack may be determined by

$$\widehat{u}_z(r,\theta,z) = u_z(x,y,z) = \frac{z}{\pi h}\int_{-a}^{a} B_y(\xi)\,G_{uz}(x-\xi,y)\,d\xi - \frac{\nu}{E}\,z\,\sigma_{yy}^{\infty}(x),$$

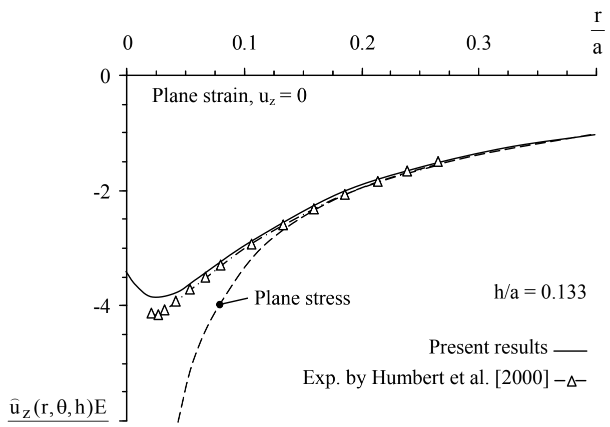

Figure 5. Normalized out-of-plane surface displacement as a function of r/a in the case of a finite length crack ($\theta = 0°$).

where the extra term is due to the uniform lateral contraction of the infinite plate loaded by the remote tensile stress $\sigma_{yy}^{\infty}(x)$. The conversion between Cartesian and polar coordinates is then made by $y = r \sin\theta$ and $x = r \cos\theta + a$.

Results for normalized out-of-plane surface displacement along the line of the crack are presented in Figure 4 in the case of a semiinfinite crack. The empirical fit by Pfaff et al. [1994] to the FE results by Nakamura and Parks [1988] is also given as a comparison. The difference between the finite element results and the present ones could be explained by the mesh refinement issues at the crack tip in the FE model. Figure 5 displays the results for the normalized out-of-plane displacement ahead of the crack tip for the case of a finite length crack. The present results are in good agreement with the experimental values of Humbert et al. [2000]. The difference is less than 10% and can be partially explained by the presence of the process zone at the crack tip where the material is subjected to inelastic deformations.

5. Conclusion

An analytical method is presented for calculating the out-of-plane stress and displacement fields in plates of finite thickness. The developed method is based on the DDT and the three-dimensional solution for an edge dislocation in plates of arbitrary thickness. Numerical results are obtained for both finite length and semiinfinite crack models through application of Gauss–Chebyshev quadrature. Results for the normalized out-of-plane constraint factor and the out-of-plane surface displacement are presented. The present values are compared with finite element and experimental results and found to be consistent. Further results for the ratio of the crack tip stress intensity factor to the applied stress intensity factor are given. A comparison with the through-the-thickness average values from previous finite element studies shows a very good agreement. These solutions can therefore be used in the experimental techniques for the assessment of the stress intensity factor using the out-of-plane displacement measurements.

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