

Appendix A Exact solution

In this section, the realization of the exact solution is investigated. The obtained twelve constants: c_{01} , a_{21} , b_{21} , b_{02} , c_{02} , a_{22} , b_{22} , c_{22} , d_{22} , b_{03} , c_{23} , and d_{23} of are given by

$$c_{01} = (E_1 E_2 R_2^2 s) / (E_2 (-1 + \nu_1) (-E_3 (-R_2^2 (-1 + \nu_2)) + R_1^2 (1 + \nu_2))) + E_2 (R_1 - R_2) (R_1 + R_2) (1 + \nu_3) + E_1 (E_3 (R_1 - R_2) (R_1 + R_2) (-1 + \nu_2^2) - E_2 (R_1^2 (-1 + \nu_2) - R_2^2 (1 + \nu_2)) (1 + \nu_3)), \quad (A 1)$$

$$a_{21} = (-4E_1 E_2 R_2^2 s (E_2 (E_3 (R_2^6 (-3 + \nu_1) (-3 + \nu_2) - 3R_1^4 R_2^2 (1 + \nu_1) (1 + \nu_2) + 2R_1^6 (3 + \nu_1) (1 + \nu_2)) - E_2 (R_1 - R_2) (R_1 + R_2) (-R_1^2 R_2^2 (-3 + \nu_1) - R_2^4 (-3 + \nu_1) + 2R_1^4 (3 + \nu_1) (1 + \nu_3)) + E_1 (E_3 (1 + \nu_2) (-R_2^6 (-3 + \nu_2)) + 3R_1^4 R_2^2 (1 + \nu_2) - 2R_1^6 (3 + \nu_2)) + E_2 (-3R_1^4 R_2^2 (1 + \nu_2) + R_2^6 (1 + \nu_2) + 2R_1^6 (3 + \nu_2) (1 + \nu_3)))) / (-2E_1 E_2 (E_3^2 (R_1 - R_2) (R_1 + R_2) (1 + \nu_2) (R_1^6 (-3 + \nu_1 (-1 + \nu_2) - \nu_2) (-3 + \nu_2) - R_2^6 (5 + \nu_1 (-1 + \nu_2) - \nu_2) (-3 + \nu_2) + R_1^2 R_2^4 (3 + 9\nu_1 + 4(-2 + \nu_1)\nu_2 + 3(-1 + \nu_1)\nu_2^2) + R_1^4 R_2^2 (9 - \nu_1(3 + \nu_2) (-1 + 3\nu_2) + \nu_2(4 + 3\nu_2))) + E_2^2 (R_1 - R_2) (R_1 + R_2) (R_1^6 (-3 - \nu_1 + (-1 + \nu_1)\nu_2) - R_1^4 R_2^2 (3 + \nu_1 + 3(-1 + \nu_1)\nu_2) + R_1^2 R_2^4 (-9 + 5\nu_1 + 3(-1 + \nu_1)\nu_2) + R_2^6 (-5 + \nu_1 + \nu_2 - \nu_1\nu_2)) (-3 + \nu_3) (1 + \nu_3) - 2E_2 E_3 (-4R_1^6 R_2^2 (-1 + \nu_1)\nu_2 (1 + \nu_2) (-1 + \nu_3) - 4R_1^2 R_2^6 (-1 + \nu_1)\nu_2 (1 + \nu_2) (-1 + \nu_3) + 6R_1^4 R_2^4 (-1 + \nu_1) (1 + \nu_2)^2 (-1 + \nu_3) + R_1^8 (-3 - \nu_1 + (-1 + \nu_1)\nu_2) (-3 - \nu_2 + (-1 + \nu_2)\nu_3) + R_2^8 (5 - \nu_1 + (-1 + \nu_1)\nu_2) (5 - \nu_2 + (-1 + \nu_2)\nu_3))) + E_2^2 (-3 + \nu_1) (1 + \nu_1) (E_3^2 (R_1^8 (-3 + \nu_2) (1 + \nu_2) + R_2^8 (-3 + \nu_2) (1 + \nu_2) - 4R_1^6 R_2^2 (1 + \nu_2)^2 + 6R_1^4 R_2^4 (1 + \nu_2)^2 - 4R_1^2 R_2^6 (3 + \nu_2^2)) + E_2^2 (R_1 - R_2)^4 (R_1 + R_2)^4 (-3 + \nu_3) (1 + \nu_3) + 2E_2 E_3 (R_1 - R_2) (R_1 + R_2) (-R_1^2 R_2^4 (-5 + 3\nu_2 (-1 + \nu_3) + \nu_3)) + R_1^4 R_2^2 (-1 + 3\nu_2 (-1 + \nu_3) + 5\nu_3) + R_2^6 (5 - \nu_2 + (-1 + \nu_2)\nu_3) + R_1^6 (3 + \nu_2 + \nu_3 - \nu_2\nu_3))) + E_1^2 (E_3^2 (R_1 - R_2)^2 (R_1 + R_2)^2 (1 + \nu_2)^2 (R_1^4 (-3 + \nu_2)^2 + R_2^4 (-3 + \nu_2)^2 - 2R_1^2 R_2^2 (-3 + \nu_2 (6 + \nu_2))) + E_2^2 (R_1^8 (-3 + \nu_2) (1 + \nu_2) + R_2^8 (-3 + \nu_2) (1 + \nu_2) + 6R_1^4 R_2^4 (1 + \nu_2)^2 - 4R_1^2 R_2^6 (1 + \nu_2)^2 - 4R_1^6 R_2^2 (3 + \nu_2^2)) (-3 + \nu_3) (1 + \nu_3) - 2E_2 E_3 (R_1 - R_2) (R_1 + R_2) (1 + \nu_2) (R_1^6 (-3 + \nu_2) (-3 - \nu_2 + (-1 + \nu_2)\nu_3) - R_2^6 (-3 + \nu_2) (5 - \nu_2 + (-1 + \nu_2)\nu_3) + R_1^2 R_2^4 (-3(-1 + \nu_2)(5 + \nu_2) + (3 + \nu_2)(-1 + 3\nu_2)\nu_3) + R_1^4 R_2^2 (3(7 + \nu_2^2) - (9 + \nu_2(4 + 3\nu_2))\nu_3))))), \quad (A 2)$$

$$\begin{aligned}
b_{21} = & (-8E_1E_2R_1^2(R_1-R_2)R_2^2(R_1+R_2)s(E_2(1+\nu_1)-E_1(1+\nu_2))(-E_3(1+\nu_2))+ \\
& E_2(1+\nu_3)))/(-2E_1E_2(E_3^2(R_1-R_2)(R_1+R_2)(1+\nu_2)(R_1^6(-3+\nu_1(-1+\nu_2)-\nu_2) \\
& (-3+\nu_2)-R_2^6(5+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)+R_1^2R_2^4(3+9\nu_1+4(-2+\nu_1)\nu_2+ \\
& 3(-1+\nu_1)\nu_2^2)+R_1^4R_2^2(9-\nu_1(3+\nu_2)(-1+3\nu_2)+\nu_2(4+3\nu_2)))+E_2^2(R_1-R_2) \\
& (R_1+R_2)(R_1^6(-3-\nu_1+(-1+\nu_1)\nu_2)-R_1^4R_2^2(3+\nu_1+3(-1+\nu_1)\nu_2)+R_1^2R_2^4 \\
& (-9+5\nu_1+3(-1+\nu_1)\nu_2)+R_2^6(-5+\nu_1+\nu_2-\nu_1\nu_2))(-3+\nu_3)(1+\nu_3)-2E_2E_3 \\
& (-4R_1^6R_2^2(-1+\nu_1)\nu_2(1+\nu_2)(-1+\nu_3)-4R_1^2R_2^6(-1+\nu_1)\nu_2(1+\nu_2)(-1+\nu_3)+ \\
& 6R_1^4R_2^4(-1+\nu_1)(1+\nu_2)^2(-1+\nu_3)+R_1^8(-3-\nu_1+(-1+\nu_1)\nu_2)(-3-\nu_2+(-1+\nu_2)\nu_3)+ \\
& R_2^8(5-\nu_1+(-1+\nu_1)\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)))+E_2^2(-3+\nu_1)(1+\nu_1)(E_3^2(R_1^8(-3+\nu_2) \\
& (1+\nu_2)+R_2^8(-3+\nu_2)(1+\nu_2)-4R_1^6R_2^2(1+\nu_2)^2+6R_1^4R_2^4(1+\nu_2)^2-4R_1^2R_2^6 \\
& (3+\nu_2^2))+E_2^2(R_1-R_2)^4(R_1+R_2)^4(-3+\nu_3)(1+\nu_3)+2E_2E_3(R_1-R_2)(R_1+R_2) \\
& (-R_1^2R_2^4(-5+3\nu_2(-1+\nu_3)+\nu_3))+R_1^4R_2^2(-1+3\nu_2(-1+\nu_3)+5\nu_3)+R_2^6(5-\nu_2+ \\
& (-1+\nu_2)\nu_3)+R_1^6(3+\nu_2+\nu_3-\nu_2\nu_3)))+E_1^2(E_3^2(R_1-R_2)^2(R_1+R_2)^2(1+\nu_2)^2 \\
& (R_1^4(-3+\nu_2)^2+R_2^4(-3+\nu_2)^2-2R_1^2R_2^2(-3+\nu_2(6+\nu_2)))+E_2^2(R_1^8(-3+\nu_2)(1+\nu_2) \\
& +R_2^8(-3+\nu_2)(1+\nu_2)+6R_1^4R_2^4(1+\nu_2)^2-4R_1^2R_2^6(1+\nu_2)^2-4R_1^6R_2^2(3+\nu_2^2))(-3+\nu_3) \\
& (1+\nu_3)-2E_2E_3(R_1-R_2)(R_1+R_2)(1+\nu_2)(R_1^6(-3+\nu_2)(-3-\nu_2+(-1+\nu_2)\nu_3)- \\
& R_2^6(-3+\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)+R_1^2R_2^4(-3(-1+\nu_2)(5+\nu_2)+(3+\nu_2)(-1+3\nu_2)\nu_3)+ \\
& R_1^4R_2^2(3(7+\nu_2^2)-(9+\nu_2(4+3\nu_2))\nu_3))))) , \tag{A 3}
\end{aligned}$$

$$\begin{aligned}
b_{02} = & (E_2R_1^2R_2^2s(E_2-E_2\nu_1+E_1(-1+\nu_2)))/(E_2(-1+\nu_1)(E_3(-R_2^2(-1+\nu_2))+ \\
& R_1^2(1+\nu_2))-E_2(R_1-R_2)(R_1+R_2)(1+\nu_3))+E_1(-E_3(R_1-R_2)(R_1+R_2)(-1+\nu_2) \\
& (1+\nu_2))+E_2(R_1^2(-1+\nu_2)-R_2^2(1+\nu_2))(1+\nu_3)) , \tag{A 4}
\end{aligned}$$

$$\begin{aligned}
c_{02} = & -(E_2R_2^2s(E_1+E_2-E_2\nu_1+E_1\nu_2))/(2(E_2(-1+\nu_1)(E_3(-R_2^2(-1+\nu_2))+ \\
& R_1^2(1+\nu_2))-E_2(R_1-R_2)(R_1+R_2)(1+\nu_3))+E_1(-E_3(R_1-R_2)(R_1+R_2)(-1+\nu_2) \\
& (1+\nu_2))+E_2(R_1^2(-1+\nu_2)-R_2^2(1+\nu_2))(1+\nu_3)) , \tag{A 5}
\end{aligned}$$

$$\begin{aligned}
a_{22} = & (E_2 R_2^2 s(-E_2^2(-3+\nu_1)(1+\nu_1)(E_3(R_2^6(-3+\nu_2)-4R_1^6(1+\nu_2)+3R_1^4 R_2^2 \\
& (1+\nu_2))+E_2(4R_1^6-3R_1^4 R_2^2-R_2^6)(1+\nu_3)))+2E_1 E_2(E_3(-4R_1^6(-1+\nu_1)\nu_2(1+\nu_2)+ \\
& 3R_1^4 R_2^2(-1+\nu_1)(1+\nu_2)^2+R_2^6(-3+\nu_2)(5-\nu_1+(-1+\nu_1)\nu_2))+E_2(4R_1^6(-1+\nu_1)\nu_2- \\
& 3R_1^4 R_2^2(-1+\nu_1)(1+\nu_2)+R_2^6(-5+\nu_1+\nu_2-\nu_1\nu_2))(1+\nu_3))+E_1^2(E_3(1+\nu_2) \\
& (-R_2^6(-3+\nu_2)^2)-3R_1^4 R_2^2(1+\nu_2)^2+4R_1^6(3+\nu_2^2))-E_2(-R_2^6(-3+\nu_2)(1+\nu_2))- \\
& 3R_1^4 R_2^2(1+\nu_2)^2+4R_1^6(3+\nu_2^2))(1+\nu_3)))/(-2E_1 E_2(E_3^2(R_1-R_2)(R_1+R_2) \\
& (1+\nu_2)(R_1^6(-3+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)-R_2^6(5+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)+ \\
& R_1^2 R_2^4(3+9\nu_1+4(-2+\nu_1)\nu_2+3(-1+\nu_1)\nu_2^2)+R_1^4 R_2^2(9-\nu_1(3+\nu_2)(-1+3\nu_2)+ \\
& \nu_2(4+3\nu_2)))+E_2^2(R_1-R_2)(R_1+R_2)(R_1^6(-3-\nu_1+(-1+\nu_1)\nu_2)-R_1^4 R_2^2 \\
& (3+\nu_1+3(-1+\nu_1)\nu_2)+R_1^2 R_2^4(-9+5\nu_1+3(-1+\nu_1)\nu_2)+R_2^6(-5+\nu_1+\nu_2-\nu_1\nu_2)) \\
& (-3+\nu_3)(1+\nu_3)-2E_2 E_3(-4R_1^6 R_2^2(-1+\nu_1)\nu_2(1+\nu_2)(-1+\nu_3)-4R_1^2 R_2^6(-1+\nu_1) \\
& \nu_2(1+\nu_2)(-1+\nu_3)+6R_1^4 R_2^4(-1+\nu_1)(1+\nu_2)^2(-1+\nu_3)+R_1^8(-3-\nu_1+(-1+\nu_1)\nu_2) \\
& (-3-\nu_2+(-1+\nu_2)\nu_3)+R_2^8(5-\nu_1+(-1+\nu_1)\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)))+E_2^2(-3+\nu_1) \\
& (1+\nu_1)(E_3^2(R_1^8(-3+\nu_2)(1+\nu_2)+R_2^8(-3+\nu_2)(1+\nu_2)-4R_1^6 R_2^2(1+\nu_2)^2+ \\
& 6R_1^4 R_2^4(1+\nu_2)^2-4R_1^2 R_2^6(3+\nu_2^2))+E_2^2(R_1-R_2)^4(R_1+R_2)^4(-3+\nu_3)(1+\nu_3)+ \\
& 2E_2 E_3(R_1-R_2)(R_1+R_2)(-R_1^2 R_2^4(-5+3\nu_2(-1+\nu_3)+\nu_3))+R_1^4 R_2^2 \\
& (-1+3\nu_2(-1+\nu_3)+5\nu_3)+R_2^6(5-\nu_2+(-1+\nu_2)\nu_3)+R_1^6(3+\nu_2+\nu_3-\nu_2\nu_3)))+ \\
& E_1^2(E_3^2(R_1-R_2)^2(R_1+R_2)^2(1+\nu_2)^2(R_1^4(-3+\nu_2)^2+R_2^4(-3+\nu_2)^2-2R_1^2 R_2^2(-3+ \\
& \nu_2(6+\nu_2)))+E_2^2(R_1^8(-3+\nu_2)(1+\nu_2)+R_2^8(-3+\nu_2)(1+\nu_2)+6R_1^4 R_2^4(1+\nu_2)^2- \\
& 4R_1^2 R_2^6(1+\nu_2)^2-4R_1^6 R_2^2(3+\nu_2^2))(-3+\nu_3)(1+\nu_3)-2E_2 E_3(R_1-R_2)(R_1+R_2) \\
& (1+\nu_2)(R_1^6(-3+\nu_2)(-3-\nu_2+(-1+\nu_2)\nu_3)-R_2^6(-3+\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)+ \\
& R_1^2 R_2^4(-3(-1+\nu_2)(5+\nu_2)+(3+\nu_2)(-1+3\nu_2)\nu_3)+R_1^4 R_2^2(3(7+\nu_2^2)- \\
& (9+\nu_2(4+3\nu_2))\nu_3)))).
\end{aligned} \tag{A 6}$$

$$\begin{aligned}
b_{22} = & (2E_2R_1^2(R_1-R_2)R_2^2(R_1+R_2)s(E_1+3E_2-E_2\nu_1+E_1\nu_2)(-(E_2(1+\nu_1))+ \\
& E_1(1+\nu_2))(-E_3(1+\nu_2))+E_2(1+\nu_3)))/(-2E_1E_2(E_3^2(R_1-R_2)(R_1+R_2)(1+\nu_2) \\
& (R_1^6(-3+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)-R_2^6(5+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)+R_1^2R_2^4 \\
& (3+9\nu_1+4(-2+\nu_1)\nu_2+3(-1+\nu_1)\nu_2^2)+R_1^4R_2^2(9-\nu_1(3+\nu_2)(-1+3\nu_2)+ \\
& \nu_2(4+3\nu_2)))+E_2^2(R_1-R_2)(R_1+R_2)(R_1^6(-3-\nu_1+(-1+\nu_1)\nu_2)-R_1^4R_2^2 \\
& (3+\nu_1+3(-1+\nu_1)\nu_2)+R_1^2R_2^4(-9+5\nu_1+3(-1+\nu_1)\nu_2)+R_2^6(-5+\nu_1+\nu_2-\nu_1\nu_2)) \\
& (-3+\nu_3)(1+\nu_3)-2E_2E_3(-4R_1^6R_2^2(-1+\nu_1)\nu_2(1+\nu_2)(-1+\nu_3)-4R_1^2R_2^6(-1+\nu_1) \\
& \nu_2(1+\nu_2)(-1+\nu_3)+6R_1^4R_2^4(-1+\nu_1)(1+\nu_2)^2(-1+\nu_3)+R_1^8(-3-\nu_1+(-1+\nu_1)\nu_2) \\
& (-3-\nu_2+(-1+\nu_2)\nu_3)+R_2^8(5-\nu_1+(-1+\nu_1)\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)))+E_2^2(-3+\nu_1) \\
& (1+\nu_1)(E_3^2(R_1^8(-3+\nu_2)(1+\nu_2)+R_2^8(-3+\nu_2)(1+\nu_2)-4R_1^6R_2^2(1+\nu_2)^2+ \\
& 6R_1^4R_2^4(1+\nu_2)^2-4R_1^2R_2^6(3+\nu_2^2))+E_2^2(R_1-R_2)^4(R_1+R_2)^4(-3+\nu_3)(1+\nu_3)+ \\
& 2E_2E_3(R_1-R_2)(R_1+R_2)(-(R_1^2R_2^4(-5+3\nu_2(-1+\nu_3)+\nu_3))+R_1^4R_2^2 \\
& (-1+3\nu_2(-1+\nu_3)+5\nu_3)+R_2^6(5-\nu_2+(-1+\nu_2)\nu_3)+R_1^6(3+\nu_2+\nu_3-\nu_2\nu_3)))+ \\
& E_1^2(E_3^2(R_1-R_2)^2(R_1+R_2)^2(1+\nu_2)^2(R_1^4(-3+\nu_2)^2+R_2^4(-3+\nu_2)^2-2R_1^2R_2^2(-3+ \\
& \nu_2(6+\nu_2)))+E_2^2(R_1^8(-3+\nu_2)(1+\nu_2)+R_2^8(-3+\nu_2)(1+\nu_2)+6R_1^4R_2^4(1+\nu_2)^2- \\
& 4R_1^2R_2^6(1+\nu_2)^2-4R_1^6R_2^2(3+\nu_2^2))(-3+\nu_3)(1+\nu_3)-2E_2E_3(R_1-R_2)(R_1+R_2) \\
& (1+\nu_2)(R_1^6(-3+\nu_2)(-3-\nu_2+(-1+\nu_2)\nu_3)-R_2^6(-3+\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)+ \\
& R_1^2R_2^4(-3(-1+\nu_2)(5+\nu_2)+(3+\nu_2)(-1+3\nu_2)\nu_3)+R_1^4R_2^2(3(7+\nu_2^2)- \\
& (9+\nu_2(4+3\nu_2))\nu_3)))).
\end{aligned} \tag{A 7}$$

$$\begin{aligned}
c_{22} = & -((E_2 R_1^4 R_2^4 s(-E_2(1+\nu_1))+E_1(1+\nu_2))(-E_2(-3+\nu_1))(-E_3(-R_2^4 \\
& (-3+\nu_2))+R_1^4(1+\nu_2))+E_2(R_1^4-R_2^4)(1+\nu_3))+E_1(-E_3(R_1^4-R_2^4)(-3+\nu_2) \\
& (1+\nu_2))+E_2(R_1^4(-3+\nu_2)-R_2^4(1+\nu_2))(1+\nu_3)))/(-2E_1E_2(E_3^2(R_1-R_2)(R_1+R_2) \\
& (1+\nu_2)(R_1^6(-3+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)-R_2^6(5+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)+ \\
& R_1^2R_2^4(3+9\nu_1+4(-2+\nu_1)\nu_2+3(-1+\nu_1)\nu_2^2)+R_1^4R_2^2(9-\nu_1(3+\nu_2)(-1+3\nu_2)+ \\
& \nu_2(4+3\nu_2)))+E_2^2(R_1-R_2)(R_1+R_2)(R_1^6(-3-\nu_1+(-1+\nu_1)\nu_2)-R_1^4R_2^2 \\
& (3+\nu_1+3(-1+\nu_1)\nu_2)+R_1^2R_2^4(-9+5\nu_1+3(-1+\nu_1)\nu_2)+R_2^6(-5+\nu_1+\nu_2-\nu_1\nu_2)) \\
& (-3+\nu_3)(1+\nu_3)-2E_2E_3(-4R_1^6R_2^2(-1+\nu_1)\nu_2(1+\nu_2)(-1+\nu_3)-4R_1^2R_2^6(-1+\nu_1) \\
& \nu_2(1+\nu_2)(-1+\nu_3)+6R_1^4R_2^4(-1+\nu_1)(1+\nu_2)^2(-1+\nu_3)+R_1^8(-3-\nu_1+(-1+\nu_1)\nu_2) \\
& (-3-\nu_2+(-1+\nu_2)\nu_3)+R_2^8(5-\nu_1+(-1+\nu_1)\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)))+E_2^2(-3+\nu_1) \\
& (1+\nu_1)(E_3^2(R_1^8(-3+\nu_2)(1+\nu_2)+R_2^8(-3+\nu_2)(1+\nu_2)-4R_1^6R_2^2(1+\nu_2)^2+ \\
& 6R_1^4R_2^4(1+\nu_2)^2-4R_1^2R_2^6(3+\nu_2^2))+E_2^2(R_1-R_2)^4(R_1+R_2)^4(-3+\nu_3)(1+\nu_3)+ \\
& 2E_2E_3(R_1-R_2)(R_1+R_2)(-R_1^2R_2^4(-5+3\nu_2(-1+\nu_3)+\nu_3))+R_1^4R_2^2 \\
& (-1+3\nu_2(-1+\nu_3)+5\nu_3)+R_2^6(5-\nu_2+(-1+\nu_2)\nu_3)+R_1^6(3+\nu_2+\nu_3-\nu_2\nu_3)))+ \\
& E_1^2(E_3^2(R_1-R_2)^2(R_1+R_2)^2(1+\nu_2)^2(R_1^4(-3+\nu_2)^2+R_2^4(-3+\nu_2)^2-2R_1^2R_2^2(-3+ \\
& \nu_2(6+\nu_2)))+E_2^2(R_1^8(-3+\nu_2)(1+\nu_2)+R_2^8(-3+\nu_2)(1+\nu_2)+6R_1^4R_2^4(1+\nu_2)^2- \\
& 4R_1^2R_2^6(1+\nu_2)^2-4R_1^6R_2^2(3+\nu_2^2))(-3+\nu_3)(1+\nu_3)-2E_2E_3(R_1-R_2)(R_1+R_2) \\
& (1+\nu_2)(R_1^6(-3+\nu_2)(-3-\nu_2+(-1+\nu_2)\nu_3)-R_2^6(-3+\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)+ \\
& R_1^2R_2^4(-3(-1+\nu_2)(5+\nu_2)+(3+\nu_2)(-1+3\nu_2)\nu_3)+R_1^4R_2^2(3(7+\nu_2)- \\
& (9+\nu_2(4+3\nu_2))\nu_3)))).
\end{aligned} \tag{A 8}$$

$$\begin{aligned}
d_{22} = & (2E_2R_1^2R_2^2s(-E_2(1+\nu_1))+E_1(1+\nu_2))(-E_2(-3+\nu_1)(-E_3(-R_2^6 \\
& (-3+\nu_2))+R_1^6(1+\nu_2)))+E_2(R_1^6-R_2^6)(1+\nu_3))+E_1(-E_3(R_1^6-R_2^6)(-3+\nu_2) \\
& (1+\nu_2))+E_2(R_1^6(-3+\nu_2)-R_2^6(1+\nu_2))(1+\nu_3)))/(-2E_1E_2(E_3^2(R_1-R_2)(R_1+R_2) \\
& (1+\nu_2)(R_1^6(-3+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)-R_2^6(5+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)+ \\
& R_1^2R_2^4(3+9\nu_1+4(-2+\nu_1)\nu_2+3(-1+\nu_1)\nu_2^2))+R_1^4R_2^2(9-\nu_1(3+\nu_2)(-1+3\nu_2)+ \\
& \nu_2(4+3\nu_2)))+E_2^2(R_1-R_2)(R_1+R_2)(R_1^6(-3-\nu_1+(-1+\nu_1)\nu_2)-R_1^4R_2^2 \\
& (3+\nu_1+3(-1+\nu_1)\nu_2)+R_1^2R_2^4(-9+5\nu_1+3(-1+\nu_1)\nu_2)+R_2^6(-5+\nu_1+\nu_2-\nu_1\nu_2)) \\
& (-3+\nu_3)(1+\nu_3)-2E_2E_3(-4R_1^6R_2^2(-1+\nu_1)\nu_2(1+\nu_2)(-1+\nu_3)-4R_1^2R_2^6(-1+\nu_1) \\
& \nu_2(1+\nu_2)(-1+\nu_3)+6R_1^4R_2^4(-1+\nu_1)(1+\nu_2)^2(-1+\nu_3)+R_1^8(-3-\nu_1+(-1+\nu_1)\nu_2) \\
& (-3-\nu_2+(-1+\nu_2)\nu_3)+R_2^8(5-\nu_1+(-1+\nu_1)\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)))+E_2^2(-3+\nu_1) \\
& (1+\nu_1)(E_3^2(R_1^8(-3+\nu_2)(1+\nu_2)+R_2^8(-3+\nu_2)(1+\nu_2)-4R_1^6R_2^2(1+\nu_2)^2+ \\
& 6R_1^4R_2^4(1+\nu_2)^2-4R_1^2R_2^6(3+\nu_2^2))+E_2^2(R_1-R_2)^4(R_1+R_2)^4(-3+\nu_3)(1+\nu_3)+ \\
& 2E_2E_3(R_1-R_2)(R_1+R_2)(-R_1^2R_2^4(-5+3\nu_2(-1+\nu_3)+\nu_3))+R_1^4R_2^2 \\
& (-1+3\nu_2(-1+\nu_3)+5\nu_3)+R_2^6(5-\nu_2+(-1+\nu_2)\nu_3)+R_1^6(3+\nu_2+\nu_3-\nu_2\nu_3)))+ \\
& E_1^2(E_3^2(R_1-R_2)^2(R_1+R_2)^2(1+\nu_2)^2(R_1^4(-3+\nu_2)^2+R_2^4(-3+\nu_2)^2-2R_1^2R_2^2(-3+ \\
& \nu_2(6+\nu_2)))+E_2^2(R_1^8(-3+\nu_2)(1+\nu_2)+R_2^8(-3+\nu_2)(1+\nu_2)+6R_1^4R_2^4(1+\nu_2)^2- \\
& 4R_1^2R_2^6(1+\nu_2)^2-4R_1^6R_2^2(3+\nu_2^2))(-3+\nu_3)(1+\nu_3)-2E_2E_3(R_1-R_2)(R_1+R_2) \\
& (1+\nu_2)(R_1^6(-3+\nu_2)(-3-\nu_2+(-1+\nu_2)\nu_3)-R_2^6(-3+\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)+ \\
& R_1^2R_2^4(-3(-1+\nu_2)(5+\nu_2)+(3+\nu_2)(-1+3\nu_2)\nu_3)+R_1^4R_2^2(3(7+\nu_2^2)- \\
& (9+\nu_2(4+3\nu_2))\nu_3)))).
\end{aligned} \tag{A 9}$$

$$\begin{aligned}
b_{03} = & (R_2^2s(E_2(-1+\nu_1)(-E_3(-R_2^2(-1+\nu_2))+R_1^2(1+\nu_2)))+E_2(R_1-R_2) \\
& (R_1+R_2)(-1+\nu_3))+E_1(E_3(R_1-R_2)(R_1+R_2)(-1+\nu_2^2)-E_2(R_1^2(-1+\nu_2)- \\
& R_2^2(1+\nu_2))(-1+\nu_3)))/(2(E_2(-1+\nu_1)(E_3(-R_2^2(-1+\nu_2))+R_1^2(1+\nu_2))- \\
& E_2(R_1-R_2)(R_1+R_2)(1+\nu_3))+E_1(-E_3(R_1-R_2)(R_1+R_2)(-1+\nu_2)(1+\nu_2))+ \\
& E_2(R_1^2(-1+\nu_2)-R_2^2(1+\nu_2))(1+\nu_3))),
\end{aligned} \tag{A 10}$$

$$\begin{aligned}
c_{23} = & -(\mathbf{R}_2^4 s(-2\mathbf{E}_1 \mathbf{E}_2 (\mathbf{E}_3^2 (\mathbf{R}_1 - \mathbf{R}_2) (\mathbf{R}_1 + \mathbf{R}_2) (1 + \nu_2) (\mathbf{R}_1^6 (-3 + \nu_1 (-1 + \nu_2) - \nu_2) \\
& (-3 + \nu_2) - \mathbf{R}_2^6 (5 + \nu_1 (-1 + \nu_2) - \nu_2) (-3 + \nu_2) + \mathbf{R}_1^2 \mathbf{R}_2^4 (3 + 9\nu_1 + 4(-2 + \nu_1)\nu_2 + \\
& 3(-1 + \nu_1)\nu_2^2) + \mathbf{R}_1^4 \mathbf{R}_2^2 (9 - \nu_1 (3 + \nu_2) (-1 + 3\nu_2) + \nu_2 (4 + 3\nu_2))) + \mathbf{E}_2^2 (\mathbf{R}_1 - \mathbf{R}_2) \\
& (\mathbf{R}_1 + \mathbf{R}_2) (\mathbf{R}_1^6 (-3 - \nu_1 + (-1 + \nu_1)\nu_2) - \mathbf{R}_1^4 \mathbf{R}_2^2 (3 + \nu_1 + 3(-1 + \nu_1)\nu_2) + \mathbf{R}_1^2 \mathbf{R}_2^4 \\
& (-9 + 5\nu_1 + 3(-1 + \nu_1)\nu_2) + \mathbf{R}_2^6 (-5 + \nu_1 + \nu_2 - \nu_1 \nu_2)) (1 + \nu_3)^2 - 2\mathbf{E}_2 \mathbf{E}_3 (-4\mathbf{R}_1^6 \mathbf{R}_2^2 \\
& (-1 + \nu_1)\nu_2 (1 + \nu_2) (1 + \nu_3) + \mathbf{R}_1^8 (-1 + \nu_2) (-3 - \nu_1 + (-1 + \nu_1)\nu_2) (1 + \nu_3) + \mathbf{R}_2^8 \\
& (-1 + \nu_2) (5 - \nu_1 + (-1 + \nu_1)\nu_2) (1 + \nu_3) - 4\mathbf{R}_1^2 \mathbf{R}_2^6 (-1 + \nu_1) (1 + \nu_2) (2 + \nu_2 + \nu_2 \nu_3) + \\
& 2\mathbf{R}_1^4 \mathbf{R}_2^4 (-1 + \nu_1) (1 + \nu_2) (7 + 3\nu_2 + 3(1 + \nu_2)\nu_3))) + \mathbf{E}_2^2 (-3 + \nu_1) (1 + \nu_1) (\mathbf{E}_3^2 (\mathbf{R}_1^8 \\
& (-3 + \nu_2) (1 + \nu_2) + \mathbf{R}_2^8 (-3 + \nu_2) (1 + \nu_2) - 4\mathbf{R}_1^6 \mathbf{R}_2^2 (1 + \nu_2)^2 + 6\mathbf{R}_1^4 \mathbf{R}_2^4 (1 + \nu_2)^2 - \\
& 4\mathbf{R}_1^2 \mathbf{R}_2^6 (3 + \nu_2^2)) + \mathbf{E}_2^2 (\mathbf{R}_1 - \mathbf{R}_2)^4 (\mathbf{R}_1 + \mathbf{R}_2)^4 (1 + \nu_3)^2 + 2\mathbf{E}_2 \mathbf{E}_3 (\mathbf{R}_1 - \mathbf{R}_2) (\mathbf{R}_1 + \mathbf{R}_2) \\
& (-\mathbf{R}_1^6 (-1 + \nu_2) (1 + \nu_3)) + \mathbf{R}_2^6 (-1 + \nu_2) (1 + \nu_3) + \mathbf{R}_1^4 \mathbf{R}_2^2 (5 + 3\nu_2) (1 + \nu_3) - \mathbf{R}_1^2 \mathbf{R}_2^4 \\
& (9 + \nu_3 + 3\nu_2 (1 + \nu_3))) + \mathbf{E}_1^2 (\mathbf{E}_3^2 (\mathbf{R}_1 - \mathbf{R}_2)^2 (\mathbf{R}_1 + \mathbf{R}_2)^2 (1 + \nu_2)^2 (\mathbf{R}_1^4 (-3 + \nu_2)^2 + \\
& \mathbf{R}_2^4 (-3 + \nu_2)^2 - 2\mathbf{R}_1^2 \mathbf{R}_2^2 (-3 + \nu_2 (6 + \nu_2))) + \mathbf{E}_2^2 (\mathbf{R}_1^8 (-3 + \nu_2) (1 + \nu_2) + \mathbf{R}_2^8 (-3 + \nu_2) \\
& (1 + \nu_2) + 6\mathbf{R}_1^4 \mathbf{R}_2^4 (1 + \nu_2)^2 - 4\mathbf{R}_1^2 \mathbf{R}_2^6 (1 + \nu_2)^2 - 4\mathbf{R}_1^6 \mathbf{R}_2^2 (3 + \nu_2^2)) (1 + \nu_3)^2 - 2\mathbf{E}_2 \mathbf{E}_3 \\
& (\mathbf{R}_1 - \mathbf{R}_2) (\mathbf{R}_1 + \mathbf{R}_2) (1 + \nu_2) (\mathbf{R}_1^6 (-3 + \nu_2) (-1 + \nu_2) (1 + \nu_3) - \mathbf{R}_2^6 (-3 + \nu_2) (-1 + \nu_2) \\
& (1 + \nu_3) - \mathbf{R}_1^4 \mathbf{R}_2^2 (9 + \nu_2 (4 + 3\nu_2)) (1 + \nu_3) + \mathbf{R}_1^2 \mathbf{R}_2^4 (5 - 3\nu_3 + \nu_2 (3\nu_2 (1 + \nu_3) + \\
& 8(2 + \nu_3)))))) / (4(-2\mathbf{E}_1 \mathbf{E}_2 (\mathbf{E}_3^2 (\mathbf{R}_1 - \mathbf{R}_2) (\mathbf{R}_1 + \mathbf{R}_2) (1 + \nu_2) (\mathbf{R}_1^6 (-3 + \nu_1 (-1 + \nu_2) - \nu_2) \\
& (-3 + \nu_2) - \mathbf{R}_2^6 (5 + \nu_1 (-1 + \nu_2) - \nu_2) (-3 + \nu_2) + \mathbf{R}_1^2 \mathbf{R}_2^4 (3 + 9\nu_1 + 4(-2 + \nu_1)\nu_2 + \\
& 3(-1 + \nu_1)\nu_2^2) + \mathbf{R}_1^4 \mathbf{R}_2^2 (9 - \nu_1 (3 + \nu_2) (-1 + 3\nu_2) + \nu_2 (4 + 3\nu_2))) + \mathbf{E}_2^2 (\mathbf{R}_1 - \mathbf{R}_2) \\
& (\mathbf{R}_1 + \mathbf{R}_2) (\mathbf{R}_1^6 (-3 - \nu_1 + (-1 + \nu_1)\nu_2) - \mathbf{R}_1^4 \mathbf{R}_2^2 (3 + \nu_1 + 3(-1 + \nu_1)\nu_2) + \mathbf{R}_1^2 \mathbf{R}_2^4 \\
& (-9 + 5\nu_1 + 3(-1 + \nu_1)\nu_2) + \mathbf{R}_2^6 (-5 + \nu_1 + \nu_2 - \nu_1 \nu_2)) (-3 + \nu_3) (1 + \nu_3) - 2\mathbf{E}_2 \mathbf{E}_3 \\
& (-4\mathbf{R}_1^6 \mathbf{R}_2^2 (-1 + \nu_1)\nu_2 (1 + \nu_2) (-1 + \nu_3) - 4\mathbf{R}_1^2 \mathbf{R}_2^6 (-1 + \nu_1)\nu_2 (1 + \nu_2) (-1 + \nu_3) + \\
& 6\mathbf{R}_1^4 \mathbf{R}_2^4 (-1 + \nu_1) (1 + \nu_2)^2 (-1 + \nu_3) + \mathbf{R}_1^8 (-3 - \nu_1 + (-1 + \nu_1)\nu_2) (-3 - \nu_2 + (-1 + \nu_2) \\
& \nu_3) + \mathbf{R}_2^8 (5 - \nu_1 + (-1 + \nu_1)\nu_2) (5 - \nu_2 + (-1 + \nu_2)\nu_3))) + \mathbf{E}_2^2 (-3 + \nu_1) (1 + \nu_1) (\mathbf{E}_3^2 \\
& (\mathbf{R}_1^8 (-3 + \nu_2) (1 + \nu_2) + \mathbf{R}_2^8 (-3 + \nu_2) (1 + \nu_2) - 4\mathbf{R}_1^6 \mathbf{R}_2^2 (1 + \nu_2)^2 + 6\mathbf{R}_1^4 \mathbf{R}_2^4 (1 + \nu_2)^2 - \\
& 4\mathbf{R}_1^2 \mathbf{R}_2^6 (3 + \nu_2^2)) + \mathbf{E}_2^2 (\mathbf{R}_1 - \mathbf{R}_2)^4 (\mathbf{R}_1 + \mathbf{R}_2)^4 (-3 + \nu_3) (1 + \nu_3) + 2\mathbf{E}_2 \mathbf{E}_3 (\mathbf{R}_1 - \mathbf{R}_2) \\
& (\mathbf{R}_1 + \mathbf{R}_2) (-\mathbf{R}_1^2 \mathbf{R}_2^4 (-5 + 3\nu_2 (-1 + \nu_3) + \nu_3)) + \mathbf{R}_1^4 \mathbf{R}_2^2 (-1 + 3\nu_2 (-1 + \nu_3) + 5\nu_3) + \\
& \mathbf{R}_2^6 (5 - \nu_2 + (-1 + \nu_2)\nu_3) + \mathbf{R}_1^6 (3 + \nu_2 + \nu_3 - \nu_2 \nu_3))) + \mathbf{E}_1^2 (\mathbf{E}_3^2 (\mathbf{R}_1 - \mathbf{R}_2)^2 (\mathbf{R}_1 + \mathbf{R}_2)^2 \\
& (1 + \nu_2)^2 (\mathbf{R}_1^4 (-3 + \nu_2)^2 + \mathbf{R}_2^4 (-3 + \nu_2)^2 - 2\mathbf{R}_1^2 \mathbf{R}_2^2 (-3 + \nu_2 (6 + \nu_2))) + \mathbf{E}_2^2 (\mathbf{R}_1^8 (-3 + \nu_2) \\
& (1 + \nu_2) + \mathbf{R}_2^8 (-3 + \nu_2) (1 + \nu_2) + 6\mathbf{R}_1^4 \mathbf{R}_2^4 (1 + \nu_2)^2 - 4\mathbf{R}_1^2 \mathbf{R}_2^6 (1 + \nu_2)^2 - 4\mathbf{R}_1^6 \mathbf{R}_2^2 \\
& (3 + \nu_2^2)) (-3 + \nu_3) (1 + \nu_3) - 2\mathbf{E}_2 \mathbf{E}_3 (\mathbf{R}_1 - \mathbf{R}_2) (\mathbf{R}_1 + \mathbf{R}_2) (1 + \nu_2) (\mathbf{R}_1^6 (-3 + \nu_2) \\
& (-3 - \nu_2 + (-1 + \nu_2)\nu_3) - \mathbf{R}_2^6 (-3 + \nu_2) (5 - \nu_2 + (-1 + \nu_2)\nu_3) + \mathbf{R}_1^2 \mathbf{R}_2^4 (-3(-1 + \nu_2) \\
& (5 + \nu_2) + (3 + \nu_2) (-1 + 3\nu_2)\nu_3) + \mathbf{R}_1^4 \mathbf{R}_2^2 (3(7 + \nu_2^2) - (9 + \nu_2 (4 + 3\nu_2))\nu_3))))),
\end{aligned} \tag{A 11}$$

$$\begin{aligned}
d_{23} = & (\mathbf{R}_2^2 s (\mathbf{E}_2^2 (-3 + \nu_1) (1 + \nu_1) (\mathbf{E}_3^2 (\mathbf{R}_1^8 (-3 + \nu_2) (1 + \nu_2) + \mathbf{R}_2^8 (-3 + \nu_2) (1 + \nu_2) - \\
& 4\mathbf{R}_1^6 \mathbf{R}_2^2 (1 + \nu_2)^2 + 6\mathbf{R}_1^4 \mathbf{R}_2^4 (1 + \nu_2)^2 - 4\mathbf{R}_1^2 \mathbf{R}_2^6 (3 + \nu_2^2)) - 2\mathbf{E}_2 \mathbf{E}_3 (\mathbf{R}_1 - \mathbf{R}_2) (\mathbf{R}_1 + \mathbf{R}_2) \\
& (-\mathbf{R}_1^6 - 5\mathbf{R}_1^4 \mathbf{R}_2^2 + \mathbf{R}_1^2 \mathbf{R}_2^4 + \mathbf{R}_2^6 + (\mathbf{R}_1^2 - \mathbf{R}_2^2)^3 \nu_2) (1 + \nu_3) + \mathbf{E}_2^2 (\mathbf{R}_1^2 - \mathbf{R}_2^2)^4 (1 + \nu_3)^2) - 2\mathbf{E}_1 \mathbf{E}_2 \\
& (\mathbf{E}_3^2 (\mathbf{R}_1 - \mathbf{R}_2) (\mathbf{R}_1 + \mathbf{R}_2) (1 + \nu_2) (\mathbf{R}_1^6 (-3 + \nu_1 (-1 + \nu_2) - \nu_2) (-3 + \nu_2) - \mathbf{R}_2^6 (5 + \nu_1 (-1 + \nu_2) - \\
& \nu_2) (-3 + \nu_2) + \mathbf{R}_1^2 \mathbf{R}_2^4 (3 + 9\nu_1 + 4(-2 + \nu_1)\nu_2 + 3(-1 + \nu_1)\nu_2^2) + \mathbf{R}_1^4 \mathbf{R}_2^2 (9 - \nu_1 (3 + \nu_2) \\
& (-1 + 3\nu_2) + \nu_2 (4 + 3\nu_2))) - 2\mathbf{E}_2 \mathbf{E}_3 (\mathbf{R}_1^8 (-3 + \nu_1 (-1 + \nu_2) - \nu_2) (-1 + \nu_2) + \mathbf{R}_2^8 (5 + \\
& \nu_1 (-1 + \nu_2) - \nu_2) (-1 + \nu_2) - 4\mathbf{R}_1^6 \mathbf{R}_2^2 (-1 + \nu_1)\nu_2 (1 + \nu_2) - 4\mathbf{R}_1^2 \mathbf{R}_2^6 (-1 + \nu_1)\nu_2 (1 + \nu_2) + \\
& 6\mathbf{R}_1^4 \mathbf{R}_2^4 (-1 + \nu_1) (1 + \nu_2)^2) (1 + \nu_3) + \mathbf{E}_2^2 (\mathbf{R}_1 - \mathbf{R}_2) (\mathbf{R}_1 + \mathbf{R}_2) (\mathbf{R}_1^6 (-3 + \nu_1 (-1 + \nu_2) - \nu_2) - \\
& \mathbf{R}_1^4 \mathbf{R}_2^2 (3 + \nu_1 + 3(-1 + \nu_1)\nu_2) + \mathbf{R}_1^2 \mathbf{R}_2^4 (-9 + 5\nu_1 + 3(-1 + \nu_1)\nu_2) + \mathbf{R}_2^6 (-5 + \nu_1 + \nu_2 - \nu_1 \nu_2)) \\
& (1 + \nu_3)^2) + \mathbf{E}_1^2 (\mathbf{E}_3^2 (\mathbf{R}_1 - \mathbf{R}_2)^2 (\mathbf{R}_1 + \mathbf{R}_2)^2 (1 + \nu_2)^2 (\mathbf{R}_1^4 (-3 + \nu_2)^2 + \mathbf{R}_2^4 (-3 + \nu_2)^2 - \\
& 2\mathbf{R}_1^2 \mathbf{R}_2^2 (-3 + \nu_2 (6 + \nu_2))) - 2\mathbf{E}_2 \mathbf{E}_3 (\mathbf{R}_1 - \mathbf{R}_2) (\mathbf{R}_1 + \mathbf{R}_2) (1 + \nu_2) (\mathbf{R}_1^6 (-3 + \nu_2) (-1 + \nu_2) - \\
& \mathbf{R}_2^6 (-3 + \nu_2) (-1 + \nu_2) + \mathbf{R}_1^2 \mathbf{R}_2^4 (3 + \nu_2) (-1 + 3\nu_2) - \mathbf{R}_1^4 \mathbf{R}_2^2 (9 + \nu_2 (4 + 3\nu_2))) (1 + \nu_3) + \\
& \mathbf{E}_2^2 (\mathbf{R}_1^8 (-3 + \nu_2) (1 + \nu_2) + \mathbf{R}_2^8 (-3 + \nu_2) (1 + \nu_2) + 6\mathbf{R}_1^4 \mathbf{R}_2^4 (1 + \nu_2)^2 - 4\mathbf{R}_1^2 \mathbf{R}_2^6 (1 + \nu_2)^2 - \\
& 4\mathbf{R}_1^6 \mathbf{R}_2^2 (3 + \nu_2^2)) (1 + \nu_3)^2)) / (2(-2\mathbf{E}_1 \mathbf{E}_2 (\mathbf{E}_3^2 (\mathbf{R}_1 - \mathbf{R}_2) (\mathbf{R}_1 + \mathbf{R}_2) (1 + \nu_2) \\
& (\mathbf{R}_1^6 (-3 + \nu_1 (-1 + \nu_2) - \nu_2) (-3 + \nu_2) - \mathbf{R}_2^6 (5 + \nu_1 (-1 + \nu_2) - \nu_2) (-3 + \nu_2) + \mathbf{R}_1^2 \mathbf{R}_2^4 \\
& (3 + 9\nu_1 + 4(-2 + \nu_1)\nu_2 + 3(-1 + \nu_1)\nu_2^2) + \mathbf{R}_1^4 \mathbf{R}_2^2 (9 - \nu_1 (3 + \nu_2) (-1 + 3\nu_2) + \\
& \nu_2 (4 + 3\nu_2))) + \mathbf{E}_2^2 (\mathbf{R}_1 - \mathbf{R}_2) (\mathbf{R}_1 + \mathbf{R}_2) (\mathbf{R}_1^6 (-3 - \nu_1 + (-1 + \nu_1)\nu_2) - \mathbf{R}_1^4 \mathbf{R}_2^2 \\
& (3 + \nu_1 + 3(-1 + \nu_1)\nu_2) + \mathbf{R}_1^2 \mathbf{R}_2^4 (-9 + 5\nu_1 + 3(-1 + \nu_1)\nu_2) + \mathbf{R}_2^6 (-5 + \nu_1 + \nu_2 - \nu_1 \nu_2)) \\
& (-3 + \nu_3) (1 + \nu_3) - 2\mathbf{E}_2 \mathbf{E}_3 (-4\mathbf{R}_1^6 \mathbf{R}_2^2 (-1 + \nu_1)\nu_2 (1 + \nu_2) (-1 + \nu_3) - 4\mathbf{R}_1^2 \mathbf{R}_2^6 (-1 + \nu_1) \\
& \nu_2 (1 + \nu_2) (-1 + \nu_3) + 6\mathbf{R}_1^4 \mathbf{R}_2^4 (-1 + \nu_1) (1 + \nu_2)^2 (-1 + \nu_3) + \mathbf{R}_1^8 (-3 - \nu_1 + (-1 + \nu_1)\nu_2) \\
& (-3 - \nu_2 + (-1 + \nu_2)\nu_3) + \mathbf{R}_2^8 (5 - \nu_1 + (-1 + \nu_1)\nu_2) (5 - \nu_2 + (-1 + \nu_2)\nu_3))) + \mathbf{E}_2^2 (-3 + \nu_1) \\
& (1 + \nu_1) (\mathbf{E}_3^2 (\mathbf{R}_1^8 (-3 + \nu_2) (1 + \nu_2) + \mathbf{R}_2^8 (-3 + \nu_2) (1 + \nu_2) - 4\mathbf{R}_1^6 \mathbf{R}_2^2 (1 + \nu_2)^2 + \\
& 6\mathbf{R}_1^4 \mathbf{R}_2^4 (1 + \nu_2)^2 - 4\mathbf{R}_1^2 \mathbf{R}_2^6 (3 + \nu_2^2)) + \mathbf{E}_2^2 (\mathbf{R}_1 - \mathbf{R}_2)^4 (\mathbf{R}_1 + \mathbf{R}_2)^4 (-3 + \nu_3) (1 + \nu_3) + \\
& 2\mathbf{E}_2 \mathbf{E}_3 (\mathbf{R}_1 - \mathbf{R}_2) (\mathbf{R}_1 + \mathbf{R}_2) (-\mathbf{R}_1^2 \mathbf{R}_2^4 (-5 + 3\nu_2 (-1 + \nu_3) + \nu_3)) + \mathbf{R}_1^4 \mathbf{R}_2^2 \\
& (-1 + 3\nu_2 (-1 + \nu_3) + 5\nu_3) + \mathbf{R}_2^6 (5 - \nu_2 + (-1 + \nu_2)\nu_3) + \mathbf{R}_1^6 (3 + \nu_2 + \nu_3 - \nu_2 \nu_3))) + \\
& \mathbf{E}_1^2 (\mathbf{E}_3^2 (\mathbf{R}_1 - \mathbf{R}_2)^2 (\mathbf{R}_1 + \mathbf{R}_2)^2 (1 + \nu_2)^2 (\mathbf{R}_1^4 (-3 + \nu_2)^2 + \mathbf{R}_2^4 (-3 + \nu_2)^2 - 2\mathbf{R}_1^2 \mathbf{R}_2^2 (-3 + \\
& \nu_2 (6 + \nu_2))) + \mathbf{E}_2^2 (\mathbf{R}_1^8 (-3 + \nu_2) (1 + \nu_2) + \mathbf{R}_2^8 (-3 + \nu_2) (1 + \nu_2) + 6\mathbf{R}_1^4 \mathbf{R}_2^4 (1 + \nu_2)^2 - \\
& 4\mathbf{R}_1^2 \mathbf{R}_2^6 (1 + \nu_2)^2 - 4\mathbf{R}_1^6 \mathbf{R}_2^2 (3 + \nu_2^2)) (-3 + \nu_3) (1 + \nu_3) - 2\mathbf{E}_2 \mathbf{E}_3 (\mathbf{R}_1 - \mathbf{R}_2) (\mathbf{R}_1 + \mathbf{R}_2) \\
& (1 + \nu_2) (\mathbf{R}_1^6 (-3 + \nu_2) (-3 - \nu_2 + (-1 + \nu_2)\nu_3) - \mathbf{R}_2^6 (-3 + \nu_2) (5 - \nu_2 + (-1 + \nu_2)\nu_3) + \\
& \mathbf{R}_1^2 \mathbf{R}_2^4 (-3(-1 + \nu_2)(5 + \nu_2) + (3 + \nu_2)(-1 + 3\nu_2)\nu_3) + \mathbf{R}_1^4 \mathbf{R}_2^2 (3(7 + \nu_2^2) - \\
& (9 + \nu_2 (4 + 3\nu_2))\nu_3))))). \tag{A 12}
\end{aligned}$$

Thus, the exact solutions of stress, strain, and displacement can be calculated by

substituting these constants into Eq. (9) to obtain the stress function of Φ_1 , Φ_2 , and Φ_3 .

Furthermore, the circumferential stressed of disk, ring, and plate are also determined from Eq. (10) as follows;

$$\begin{aligned}
\sigma_{\theta\theta 1} = & 2(E_1 E_2 R_2^2 s) / (E_2 (-1 + \nu_1) (- (E_3 (- (R_2^2 (-1 + \nu_2))) + R_1^2 (1 + \nu_2))) + E_2 (R_1 - R_2) \\
& (R_1 + R_2) (1 + \nu_3)) + E_1 (E_3 (R_1 - R_2) (R_1 + R_2) (-1 + \nu_2^2) - E_2 (R_1^2 (-1 + \nu_2) - R_2^2 (1 + \nu_2)) \\
& (1 + \nu_3))) + (2(-4E_1 E_2 R_2^2 s (E_2 (E_3 (R_2^6 (-3 + \nu_1) (-3 + \nu_2) - 3R_1^4 R_2^2 (1 + \nu_1) (1 + \nu_2) + \\
& 2R_1^6 (3 + \nu_1) (1 + \nu_2)) - E_2 (R_1 - R_2) (R_1 + R_2) (- (R_1^2 R_2^2 (-3 + \nu_1)) - R_2^4 (-3 + \nu_1) + \\
& 2R_1^4 (3 + \nu_1) (1 + \nu_3)) + E_1 (E_3 (1 + \nu_2) (- (R_2^6 (-3 + \nu_2)) + 3R_1^4 R_2^2 (1 + \nu_2) - 2R_1^6 \\
& (3 + \nu_2)) + E_2 (-3R_1^4 R_2^2 (1 + \nu_2) + R_2^6 (1 + \nu_2) + 2R_1^6 (3 + \nu_2)) (1 + \nu_3)))) / (-2E_1 E_2 \\
& (E_3^2 (R_1 - R_2) (R_1 + R_2) (1 + \nu_2) (R_1^6 (-3 + \nu_1 (-1 + \nu_2) - \nu_2) (-3 + \nu_2) - R_2^6 (5 + \nu_1 \\
& (-1 + \nu_2) - \nu_2) (-3 + \nu_2) + R_1^2 R_2^4 (3 + 9\nu_1 + 4(-2 + \nu_1)\nu_2 + 3(-1 + \nu_1)\nu_2^2) + R_1^4 R_2^2 \\
& (9 - \nu_1 (3 + \nu_2) (-1 + 3\nu_2) + \nu_2 (4 + 3\nu_2))) + E_2^2 (R_1 - R_2) (R_1 + R_2) (R_1^6 (-3 - \nu_1 + \\
& (-1 + \nu_1)\nu_2) - R_1^4 R_2^2 (3 + \nu_1 + 3(-1 + \nu_1)\nu_2) + R_1^2 R_2^4 (-9 + 5\nu_1 + 3(-1 + \nu_1)\nu_2) + \\
& R_2^6 (-5 + \nu_1 + \nu_2 - \nu_1\nu_2)) (-3 + \nu_3) (1 + \nu_3) - 2E_2 E_3 (-4R_1^6 R_2^2 (-1 + \nu_1)\nu_2 (1 + \nu_2) \\
& (-1 + \nu_3) - 4R_1^2 R_2^6 (-1 + \nu_1)\nu_2 (1 + \nu_2) (-1 + \nu_3) + 6R_1^4 R_2^4 (-1 + \nu_1) (1 + \nu_2)^2 (-1 + \nu_3) + \\
& R_1^8 (-3 - \nu_1 + (-1 + \nu_1)\nu_2) (-3 - \nu_2 + (-1 + \nu_2)\nu_3) + R_2^8 (5 - \nu_1 + (-1 + \nu_1)\nu_2) (5 - \nu_2 + \\
& (-1 + \nu_2)\nu_3))) + E_2^2 (-3 + \nu_1) (1 + \nu_1) (E_3 (R_1^8 (-3 + \nu_2) (1 + \nu_2) + R_2^8 (-3 + \nu_2) (1 + \nu_2) - \\
& 4R_1^6 R_2^2 (1 + \nu_2)^2 + 6R_1^4 R_2^4 (1 + \nu_2)^2 - 4R_1^2 R_2^6 (3 + \nu_2^2)) + E_2^2 (R_1 - R_2)^4 (R_1 + R_2)^4 \\
& (-3 + \nu_3) (1 + \nu_3) + 2E_2 E_3 (R_1 - R_2) (R_1 + R_2) (- (R_1^2 R_2^4 (-5 + 3\nu_2 (-1 + \nu_3) + \nu_3)) + \\
& R_1^4 R_2^2 (-1 + 3\nu_2 (-1 + \nu_3) + 5\nu_3) + R_2^6 (5 - \nu_2 + (-1 + \nu_2)\nu_3) + R_1^6 (3 + \nu_2 + \nu_3 - \nu_2\nu_3))) + \\
& E_1^2 (E_3^2 (R_1 - R_2)^2 (R_1 + R_2)^2 (1 + \nu_2)^2 (R_1^4 (-3 + \nu_2)^2 + R_2^4 (-3 + \nu_2)^2 - 2R_1^2 R_2^2 (-3 + \\
& \nu_2 (6 + \nu_2))) + E_2^2 (R_1^8 (-3 + \nu_2) (1 + \nu_2) + R_2^8 (-3 + \nu_2) (1 + \nu_2) + 6R_1^4 R_2^4 (1 + \nu_2)^2 - \\
& 4R_1^2 R_2^6 (1 + \nu_2)^2 - 4R_1^6 R_2^2 (3 + \nu_2^2)) (-3 + \nu_3) (1 + \nu_3) - 2E_2 E_3 (R_1 - R_2) (R_1 + R_2) \\
& (1 + \nu_2) (R_1^6 (-3 + \nu_2) (-3 - \nu_2 + (-1 + \nu_2)\nu_3) - R_2^6 (-3 + \nu_2) (5 - \nu_2 + (-1 + \nu_2)\nu_3) + \\
& R_1^2 R_2^4 (-3(-1 + \nu_2)(5 + \nu_2) + (3 + \nu_2)(-1 + 3\nu_2)\nu_3) + R_1^4 R_2^2 (3(7 + \nu_2^2) - \\
& (9 + \nu_2 (4 + 3\nu_2))\nu_3))) + 12(-8E_1 E_2 R_1^2 (R_1 - R_2) R_2^2 (R_1 + R_2) s (E_2 (1 + \nu_1) - \\
& E_1 (1 + \nu_2)) (- (E_3 (1 + \nu_2)) + E_2 (1 + \nu_3))) / (-2E_1 E_2 (E_3^2 (R_1 - R_2) (R_1 + R_2) (1 + \nu_2) \\
& (R_1^6 (-3 + \nu_1 (-1 + \nu_2) - \nu_2) (-3 + \nu_2) - R_2^6 (5 + \nu_1 (-1 + \nu_2) - \nu_2) (-3 + \nu_2) + R_1^2 R_2^4 \\
& (3 + 9\nu_1 + 4(-2 + \nu_1)\nu_2 + 3(-1 + \nu_1)\nu_2^2) + R_1^4 R_2^2 (9 - \nu_1 (3 + \nu_2) (-1 + 3\nu_2) + \\
& \nu_2 (4 + 3\nu_2))) + E_2^2 (R_1 - R_2) (R_1 + R_2) (R_1^6 (-3 - \nu_1 + (-1 + \nu_1)\nu_2) - R_1^4 R_2^2 \\
& (3 + \nu_1 + 3(-1 + \nu_1)\nu_2) + R_1^2 R_2^4 (-9 + 5\nu_1 + 3(-1 + \nu_1)\nu_2) + R_2^6 (-5 + \nu_1 + \nu_2 - \nu_1\nu_2)) \\
& (-3 + \nu_3) (1 + \nu_3) - 2E_2 E_3 (-4R_1^6 R_2^2 (-1 + \nu_1)\nu_2 (1 + \nu_2) (-1 + \nu_3) - 4R_1^2 R_2^6 (-1 + \nu_1)
\end{aligned}$$

$$\begin{aligned}
& \nu_2(1+\nu_2)(-1+\nu_3)+6R_1^4R_2^4(-1+\nu_1)(1+\nu_2)^2(-1+\nu_3)+R_1^8(-3-\nu_1+(-1+\nu_1)\nu_2) \\
& (-3-\nu_2+(-1+\nu_2)\nu_3)+R_2^8(5-\nu_1+(-1+\nu_1)\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3))+E_2^2(-3+\nu_1) \\
& (1+\nu_1)(E_3^2(R_1^8(-3+\nu_2)(1+\nu_2)+R_2^8(-3+\nu_2)(1+\nu_2)-4R_1^6R_2^2(1+\nu_2)^2+6R_1^4R_2^4 \\
& (1+\nu_2)^2-4R_1^2R_2^6(3+\nu_2^2))+E_2^2(R_1-R_2)^4(R_1+R_2)^4(-3+\nu_3)(1+\nu_3)+2E_2E_3 \\
& (R_1-R_2)(R_1+R_2)(-R_1^2R_2^4(-5+3\nu_2(-1+\nu_3)+\nu_3))+R_1^4R_2^2(-1+3\nu_2(-1+\nu_3)+ \\
& 5\nu_3)+R_2^6(5-\nu_2+(-1+\nu_2)\nu_3)+R_1^6(3+\nu_2+\nu_3-\nu_2\nu_3))+E_1^2(E_3^2(R_1-R_2)^2 \\
& (R_1+R_2)^2(1+\nu_2)^2(R_1^4(-3+\nu_2)^2+R_2^4(-3+\nu_2)^2-2R_1^2R_2^2(-3+\nu_2(6+\nu_2))))+ \\
& E_2^2(R_1^8(-3+\nu_2)(1+\nu_2)+R_2^8(-3+\nu_2)(1+\nu_2)+6R_1^4R_2^4(1+\nu_2)^2-4R_1^2R_2^6(1+\nu_2)^2- \\
& 4R_1^6R_2^2(3+\nu_2^2))(-3+\nu_3)(1+\nu_3)-2E_2E_3(R_1-R_2)(R_1+R_2)(1+\nu_2)(R_1^6(-3+\nu_2) \\
& (-3-\nu_2+(-1+\nu_2)\nu_3)-R_2^6(-3+\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)+R_1^2R_2^4(-3(-1+\nu_2) \\
& (5+\nu_2)+(3+\nu_2)(-1+3\nu_2)\nu_3)+R_1^4R_2^2(3(7+\nu_2^2)-(9+\nu_2(4+3\nu_2))\nu_3)))r^2) \cos(2\theta), \tag{A 13}
\end{aligned}$$

$$\begin{aligned}
\sigma_{\theta\theta 2} = & 2-(E_2R_2^2s(E_1+E_2-E_2\nu_1+E_1\nu_2))/(2(E_2(-1+\nu_1)(E_3(-R_2^2(-1+\nu_2))+ \\
& R_1^2(1+\nu_2))-E_2(R_1-R_2)(R_1+R_2)(1+\nu_3))+E_1(-E_3(R_1-R_2)(R_1+R_2)(-1+\nu_2) \\
& (1+\nu_2))+E_2(R_1^2(-1+\nu_2)-R_2^2(1+\nu_2))(1+\nu_3)))-E_2R_1^2R_2^2s(E_2-E_2\nu_1+ \\
& E_1(-1+\nu_2)))/(E_2(-1+\nu_1)(E_3(-R_2^2(-1+\nu_2))+R_1^2(1+\nu_2))-E_2(R_1-R_2)(R_1+R_2) \\
& (1+\nu_3))+E_1(-E_3(R_1-R_2)(R_1+R_2)(-1+\nu_2)(1+\nu_2))+E_2(R_1^2(-1+\nu_2)- \\
& R_2^2(1+\nu_2))(1+\nu_3)))r^{-2} + (6-((E_2R_1^4R_2^4s(-E_2(1+\nu_1))+E_1(1+\nu_2))(-E_2 \\
& (-3+\nu_1)(-E_3(-R_2^4(-3+\nu_2))+R_1^4(1+\nu_2))+E_2(R_1^4-R_2^4)(1+\nu_3))+E_1(-E_3 \\
& (R_1^4-R_2^4)(-3+\nu_2)(1+\nu_2))+E_2(R_1^4(-3+\nu_2)-R_2^4(1+\nu_2))(1+\nu_3))))/ \\
& (-2E_1E_2(E_3^2(R_1-R_2)(R_1+R_2)(1+\nu_2)(R_1^6(-3+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)- \\
& R_2^6(5+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)+R_1^2R_2^4(3+9\nu_1+4(-2+\nu_1)\nu_2+3(-1+\nu_1)\nu_2^2)+ \\
& R_1^4R_2^2(9-\nu_1(3+\nu_2)(-1+3\nu_2)+\nu_2(4+3\nu_2)))+E_2^2(R_1-R_2)(R_1+R_2)(R_1^6 \\
& (-3-\nu_1+(-1+\nu_1)\nu_2)-R_1^4R_2^2(3+\nu_1+3(-1+\nu_1)\nu_2)+R_1^2R_2^4(-9+5\nu_1+ \\
& 3(-1+\nu_1)\nu_2)+R_2^6(-5+\nu_1+\nu_2-\nu_1\nu_2))(-3+\nu_3)(1+\nu_3)-2E_2E_3(-4R_1^6R_2^2(-1+\nu_1) \\
& \nu_2(1+\nu_2)(-1+\nu_3)-4R_1^2R_2^6(-1+\nu_1)\nu_2(1+\nu_2)(-1+\nu_3)+6R_1^4R_2^4(-1+\nu_1)(1+\nu_2)^2 \\
& (-1+\nu_3)+R_1^8(-3-\nu_1+(-1+\nu_1)\nu_2)(-3-\nu_2+(-1+\nu_2)\nu_3)+R_2^8(5-\nu_1+(-1+\nu_1)\nu_2) \\
& (5-\nu_2+(-1+\nu_2)\nu_3)))+E_2^2(-3+\nu_1)(1+\nu_1)(E_3^2(R_1^8(-3+\nu_2)(1+\nu_2)+R_2^8(-3+\nu_2) \\
& (1+\nu_2)-4R_1^6R_2^2(1+\nu_2)^2+6R_1^4R_2^4(1+\nu_2)^2-4R_1^2R_2^6(3+\nu_2^2))+E_2^2(R_1-R_2)^4 \\
& (R_1+R_2)^4(-3+\nu_3)(1+\nu_3)+2E_2E_3(R_1-R_2)(R_1+R_2)(-R_1^2R_2^4(-5+3\nu_2 \\
& (-1+\nu_3)+\nu_3))+R_1^4R_2^2(-1+3\nu_2(-1+\nu_3)+5\nu_3)+R_2^6(5-\nu_2+(-1+\nu_2)\nu_3)+ \\
& R_1^6(3+\nu_2+\nu_3-\nu_2\nu_3))+E_1^2(E_3^2(R_1-R_2)^2(R_1+R_2)^2(1+\nu_2)^2(R_1^4(-3+\nu_2)^2+
\end{aligned}$$

$$\begin{aligned}
& \mathbf{R}_2^4(-3+\nu_2)^2-2\mathbf{R}_1^2\mathbf{R}_2^2(-3+\nu_2(6+\nu_2))+\mathbf{E}_2^2(\mathbf{R}_1^8(-3+\nu_2)(1+\nu_2)+\mathbf{R}_2^8(-3+\nu_2) \\
& (1+\nu_2)+6\mathbf{R}_1^4\mathbf{R}_2^4(1+\nu_2)^2-4\mathbf{R}_1^2\mathbf{R}_2^6(1+\nu_2)^2-4\mathbf{R}_1^6\mathbf{R}_2^2(3+\nu_2^2))(-3+\nu_3)(1+\nu_3)- \\
& 2\mathbf{E}_2\mathbf{E}_3(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(1+\nu_2)(\mathbf{R}_1^6(-3+\nu_2)(-3-\nu_2+(-1+\nu_2)\nu_3)-\mathbf{R}_2^6(-3+\nu_2) \\
& (5-\nu_2+(-1+\nu_2)\nu_3)+\mathbf{R}_1^2\mathbf{R}_2^4(-3(-1+\nu_2)(5+\nu_2)+(3+\nu_2)(-1+3\nu_2)\nu_3)+\mathbf{R}_1^4\mathbf{R}_2^2 \\
& (3(7+\nu_2^2)-(9+\nu_2(4+3\nu_2))\nu_3))))r^{-4}+2(\mathbf{E}_2\mathbf{R}_2^2s(-(\mathbf{E}_2^2(-3+\nu_1)(1+\nu_1)(\mathbf{E}_3(\mathbf{R}_2^6 \\
& (-3+\nu_2)-4\mathbf{R}_1^6(1+\nu_2)+3\mathbf{R}_1^4\mathbf{R}_2^2(1+\nu_2))+\mathbf{E}_2(4\mathbf{R}_1^6-3\mathbf{R}_1^4\mathbf{R}_2^2-\mathbf{R}_2^6)(1+\nu_3)))+2\mathbf{E}_1\mathbf{E}_2 \\
& (\mathbf{E}_3(-4\mathbf{R}_1^6(-1+\nu_1)\nu_2(1+\nu_2)+3\mathbf{R}_1^4\mathbf{R}_2^2(-1+\nu_1)(1+\nu_2)^2+\mathbf{R}_2^6(-3+\nu_2)(5-\nu_1+ \\
& (-1+\nu_1)\nu_2))+\mathbf{E}_2(4\mathbf{R}_1^6(-1+\nu_1)\nu_2-3\mathbf{R}_1^4\mathbf{R}_2^2(-1+\nu_1)(1+\nu_2)+\mathbf{R}_2^6(-5+\nu_1+\nu_2-\nu_1\nu_2)) \\
& (1+\nu_3))+\mathbf{E}_1^2(\mathbf{E}_3(1+\nu_2)(-\mathbf{R}_2^6(-3+\nu_2)^2)-3\mathbf{R}_1^4\mathbf{R}_2^2(1+\nu_2)^2+4\mathbf{R}_1^6(3+\nu_2^2))-\mathbf{E}_2 \\
& (-\mathbf{R}_2^6(-3+\nu_2)(1+\nu_2))-3\mathbf{R}_1^4\mathbf{R}_2^2(1+\nu_2)^2+4\mathbf{R}_1^6(3+\nu_2^2))(1+\nu_3)))/(-2\mathbf{E}_1\mathbf{E}_2 \\
& (\mathbf{E}_3^2(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(1+\nu_2)(\mathbf{R}_1^6(-3+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)-\mathbf{R}_2^6(5+ \\
& \nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)+\mathbf{R}_1^2\mathbf{R}_2^4(3+9\nu_1+4(-2+\nu_1)\nu_2+3(-1+\nu_1)\nu_2^2)+\mathbf{R}_1^4\mathbf{R}_2^2 \\
& (9-\nu_1(3+\nu_2)(-1+3\nu_2)+\nu_2(4+3\nu_2)))+\mathbf{E}_2^2(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(\mathbf{R}_1^6(-3-\nu_1+ \\
& (-1+\nu_1)\nu_2)-\mathbf{R}_1^4\mathbf{R}_2^2(3+\nu_1+3(-1+\nu_1)\nu_2)+\mathbf{R}_1^2\mathbf{R}_2^4(-9+5\nu_1+3(-1+\nu_1)\nu_2)+ \\
& \mathbf{R}_2^6(-5+\nu_1+\nu_2-\nu_1\nu_2))(-3+\nu_3)(1+\nu_3)-2\mathbf{E}_2\mathbf{E}_3(-4\mathbf{R}_1^6\mathbf{R}_2^2(-1+\nu_1)\nu_2(1+\nu_2) \\
& (-1+\nu_3)-4\mathbf{R}_1^2\mathbf{R}_2^6(-1+\nu_1)\nu_2(1+\nu_2)(-1+\nu_3)+6\mathbf{R}_1^4\mathbf{R}_2^4(-1+\nu_1)(1+\nu_2)^2(-1+\nu_3)+ \\
& \mathbf{R}_1^8(-3-\nu_1+(-1+\nu_1)\nu_2)(-3-\nu_2+(-1+\nu_2)\nu_3)+\mathbf{R}_2^8(5-\nu_1+(-1+\nu_1)\nu_2)(5-\nu_2+ \\
& (-1+\nu_2)\nu_3)))+\mathbf{E}_2^2(-3+\nu_1)(1+\nu_1)(\mathbf{E}_3^2(\mathbf{R}_1^8(-3+\nu_2)(1+\nu_2)+\mathbf{R}_2^8(-3+\nu_2)(1+\nu_2)- \\
& 4\mathbf{R}_1^6\mathbf{R}_2^2(1+\nu_2)^2+6\mathbf{R}_1^4\mathbf{R}_2^4(1+\nu_2)^2-4\mathbf{R}_1^2\mathbf{R}_2^6(3+\nu_2^2))+\mathbf{E}_2^2(\mathbf{R}_1-\mathbf{R}_2)^4(\mathbf{R}_1+\mathbf{R}_2)^4 \\
& (-3+\nu_3)(1+\nu_3)+2\mathbf{E}_2\mathbf{E}_3(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(-\mathbf{R}_1^2\mathbf{R}_2^4(-5+3\nu_2(-1+\nu_3)+\nu_3))+ \\
& \mathbf{R}_1^4\mathbf{R}_2^2(-1+3\nu_2(-1+\nu_3)+5\nu_3)+\mathbf{R}_2^6(5-\nu_2+(-1+\nu_2)\nu_3)+\mathbf{R}_1^6(3+\nu_2+\nu_3-\nu_2\nu_3)))+ \\
& \mathbf{E}_1^2(\mathbf{E}_3^2(\mathbf{R}_1-\mathbf{R}_2)^2(\mathbf{R}_1+\mathbf{R}_2)^2(1+\nu_2)^2(\mathbf{R}_1^4(-3+\nu_2)^2+\mathbf{R}_2^4(-3+\nu_2)^2-2\mathbf{R}_1^2\mathbf{R}_2^2(-3+ \\
& \nu_2(6+\nu_2)))+\mathbf{E}_2^2(\mathbf{R}_1^8(-3+\nu_2)(1+\nu_2)+\mathbf{R}_2^8(-3+\nu_2)(1+\nu_2)+6\mathbf{R}_1^4\mathbf{R}_2^4(1+\nu_2)^2- \\
& 4\mathbf{R}_1^2\mathbf{R}_2^6(1+\nu_2)^2-4\mathbf{R}_1^6\mathbf{R}_2^2(3+\nu_2^2))(-3+\nu_3)(1+\nu_3)-2\mathbf{E}_2\mathbf{E}_3(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2) \\
& (1+\nu_2)(\mathbf{R}_1^6(-3+\nu_2)(-3-\nu_2+(-1+\nu_2)\nu_3)-\mathbf{R}_2^6(-3+\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)+ \\
& \mathbf{R}_1^2\mathbf{R}_2^4(-3(-1+\nu_2)(5+\nu_2)+(3+\nu_2)(-1+3\nu_2)\nu_3)+\mathbf{R}_1^4\mathbf{R}_2^2(3(7+\nu_2^2)-(9+\nu_2 \\
& (4+3\nu_2))\nu_3))))+12(2\mathbf{E}_2\mathbf{R}_1^2(\mathbf{R}_1-\mathbf{R}_2)\mathbf{R}_2^2(\mathbf{R}_1+\mathbf{R}_2)s(\mathbf{E}_1+3\mathbf{E}_2-\mathbf{E}_2\nu_1+\mathbf{E}_1\nu_2) \\
& (-\mathbf{E}_2(1+\nu_1))+\mathbf{E}_1(1+\nu_2))(-\mathbf{E}_3(1+\nu_2))+\mathbf{E}_2(1+\nu_3)))/(-2\mathbf{E}_1\mathbf{E}_2(\mathbf{E}_3^2(\mathbf{R}_1-\mathbf{R}_2) \\
& (\mathbf{R}_1+\mathbf{R}_2)(1+\nu_2)(\mathbf{R}_1^6(-3+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)-\mathbf{R}_2^6(5+\nu_1(-1+\nu_2)-\nu_2) \\
& (-3+\nu_2)+\mathbf{R}_1^2\mathbf{R}_2^4(3+9\nu_1+4(-2+\nu_1)\nu_2+3(-1+\nu_1)\nu_2^2)+\mathbf{R}_1^4\mathbf{R}_2^2(9-\nu_1(3+\nu_2) \\
& (-1+3\nu_2)+\nu_2(4+3\nu_2)))+\mathbf{E}_2^2(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(\mathbf{R}_1^6(-3-\nu_1+(-1+\nu_1)\nu_2)-\mathbf{R}_1^4\mathbf{R}_2^2 \\
& (3+\nu_1+3(-1+\nu_1)\nu_2)+\mathbf{R}_1^2\mathbf{R}_2^4(-9+5\nu_1+3(-1+\nu_1)\nu_2)+\mathbf{R}_2^6(-5+\nu_1+\nu_2-\nu_1\nu_2)) \\
& (-3+\nu_3)(1+\nu_3)-2\mathbf{E}_2\mathbf{E}_3(-4\mathbf{R}_1^6\mathbf{R}_2^2(-1+\nu_1)\nu_2(1+\nu_2)(-1+\nu_3)-4\mathbf{R}_1^2\mathbf{R}_2^6(-1+\nu_1) \\
& \nu_2(1+\nu_2)(-1+\nu_3)+6\mathbf{R}_1^4\mathbf{R}_2^4(-1+\nu_1)(1+\nu_2)^2(-1+\nu_3)+\mathbf{R}_1^8(-3-\nu_1+(-1+\nu_1)\nu_2)
\end{aligned}$$

$$\begin{aligned}
& (-3-\nu_2+(-1+\nu_2)\nu_3)+\mathbf{R}_2^8(5-\nu_1+(-1+\nu_1)\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)))+\mathbf{E}_2^2(-3+\nu_1) \\
& (1+\nu_1)(\mathbf{E}_3^2(\mathbf{R}_1^8(-3+\nu_2)(1+\nu_2)+\mathbf{R}_2^8(-3+\nu_2)(1+\nu_2)-4\mathbf{R}_1^6\mathbf{R}_2^2(1+\nu_2)^2+ \\
& 6\mathbf{R}_1^4\mathbf{R}_2^4(1+\nu_2)^2-4\mathbf{R}_1^2\mathbf{R}_2^6(3+\nu_2^2))+\mathbf{E}_2^2(\mathbf{R}_1-\mathbf{R}_2)^4(\mathbf{R}_1+\mathbf{R}_2)^4(-3+\nu_3)(1+\nu_3)+ \\
& 2\mathbf{E}_2\mathbf{E}_3(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(-\mathbf{R}_1^2\mathbf{R}_2^4(-5+3\nu_2(-1+\nu_3)+\nu_3))+\mathbf{R}_1^4\mathbf{R}_2^2(-1+3\nu_2 \\
& (-1+\nu_3)+5\nu_3)+\mathbf{R}_2^6(5-\nu_2+(-1+\nu_2)\nu_3)+\mathbf{R}_1^6(3+\nu_2+\nu_3-\nu_2\nu_3)))+\mathbf{E}_1^2(\mathbf{E}_3^2(\mathbf{R}_1-\mathbf{R}_2)^2 \\
& (\mathbf{R}_1+\mathbf{R}_2)^2(1+\nu_2)^2(\mathbf{R}_1^4(-3+\nu_2)^2+\mathbf{R}_2^4(-3+\nu_2)^2-2\mathbf{R}_1^2\mathbf{R}_2^2(-3+\nu_2(6+\nu_2))))+\mathbf{E}_2^2(\mathbf{R}_1^8 \\
& (-3+\nu_2)(1+\nu_2)+\mathbf{R}_2^8(-3+\nu_2)(1+\nu_2)+6\mathbf{R}_1^4\mathbf{R}_2^4(1+\nu_2)^2-4\mathbf{R}_1^2\mathbf{R}_2^6(1+\nu_2)^2-4\mathbf{R}_1^6\mathbf{R}_2^2 \\
& (3+\nu_2^2))(-3+\nu_3)(1+\nu_3)-2\mathbf{E}_2\mathbf{E}_3(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(1+\nu_2)(\mathbf{R}_1^6(-3+\nu_2)(-3-\nu_2+ \\
& (-1+\nu_2)\nu_3)-\mathbf{R}_2^6(-3+\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)+\mathbf{R}_1^2\mathbf{R}_2^4(-3(-1+\nu_2)(5+\nu_2)+(3+\nu_2) \\
& (-1+3\nu_2)\nu_3)+\mathbf{R}_1^4\mathbf{R}_2^2(3(7+\nu_2^2)-(9+\nu_2(4+3\nu_2))\nu_3))))r^2)\cos(2\theta),
\end{aligned} \tag{A 14}$$

$$\begin{aligned}
\sigma_{\theta\theta 3} = & s/2-(\mathbf{R}_2^2s(\mathbf{E}_2(-1+\nu_1)(-\mathbf{E}_3(-\mathbf{R}_2^2(-1+\nu_2))+\mathbf{R}_1^2(1+\nu_2)))+\mathbf{E}_2(\mathbf{R}_1-\mathbf{R}_2) \\
& (\mathbf{R}_1+\mathbf{R}_2)(-1+\nu_3))+\mathbf{E}_1(\mathbf{E}_3(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(-1+\nu_2^2)-\mathbf{E}_2(\mathbf{R}_1^2(-1+\nu_2)- \\
& \mathbf{R}_2^2(1+\nu_2))(-1+\nu_3)))/(2(\mathbf{E}_2(-1+\nu_1)(\mathbf{E}_3(-\mathbf{R}_2^2(-1+\nu_2))+\mathbf{R}_1^2(1+\nu_2))- \\
& \mathbf{E}_2(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(1+\nu_3))+\mathbf{E}_1(-\mathbf{E}_3(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(-1+\nu_2)(1+\nu_2))+ \\
& \mathbf{E}_2(\mathbf{R}_1^2(-1+\nu_2)-\mathbf{R}_2^2(1+\nu_2))(1+\nu_3))))r^{-2}+(6-(\mathbf{R}_2^4s(-2\mathbf{E}_1\mathbf{E}_2(\mathbf{E}_3^2(\mathbf{R}_1-\mathbf{R}_2) \\
& (\mathbf{R}_1+\mathbf{R}_2)(1+\nu_2)(\mathbf{R}_1^6(-3+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)-\mathbf{R}_2^6(5+\nu_1(-1+\nu_2)-\nu_2) \\
& (-3+\nu_2)+\mathbf{R}_1^2\mathbf{R}_2^4(3+9\nu_1+4(-2+\nu_1)\nu_2+3(-1+\nu_1)\nu_2^2)+\mathbf{R}_1^4\mathbf{R}_2^2(9-\nu_1(3+\nu_2) \\
& (-1+3\nu_2)+\nu_2(4+3\nu_2)))+\mathbf{E}_2^2(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(\mathbf{R}_1^6(-3-\nu_1+(-1+\nu_1)\nu_2)- \\
& \mathbf{R}_1^4\mathbf{R}_2^2(3+\nu_1+3(-1+\nu_1)\nu_2)+\mathbf{R}_1^2\mathbf{R}_2^4(-9+5\nu_1+3(-1+\nu_1)\nu_2)+\mathbf{R}_2^6(-5+\nu_1+ \\
& \nu_2-\nu_1\nu_2))(1+\nu_3)^2-2\mathbf{E}_2\mathbf{E}_3(-4\mathbf{R}_1^6\mathbf{R}_2^2(-1+\nu_1)\nu_2(1+\nu_2)(1+\nu_3)+\mathbf{R}_1^8(-1+\nu_2) \\
& (-3-\nu_1+(-1+\nu_1)\nu_2)(1+\nu_3)+\mathbf{R}_2^8(-1+\nu_2)(5-\nu_1+(-1+\nu_1)\nu_2)(1+\nu_3)-4\mathbf{R}_1^2\mathbf{R}_2^6 \\
& (-1+\nu_1)(1+\nu_2)(2+\nu_2+\nu_2\nu_3)+2\mathbf{R}_1^4\mathbf{R}_2^4(-1+\nu_1)(1+\nu_2)(7+3\nu_2+3(1+\nu_2)\nu_3)))+ \\
& \mathbf{E}_2^2(-3+\nu_1)(1+\nu_1)(\mathbf{E}_3^2(\mathbf{R}_1^8(-3+\nu_2)(1+\nu_2)+\mathbf{R}_2^8(-3+\nu_2)(1+\nu_2)-4\mathbf{R}_1^6\mathbf{R}_2^2 \\
& (1+\nu_2)^2+6\mathbf{R}_1^4\mathbf{R}_2^4(1+\nu_2)^2-4\mathbf{R}_1^2\mathbf{R}_2^6(3+\nu_2^2))+\mathbf{E}_2^2(\mathbf{R}_1-\mathbf{R}_2)^4(\mathbf{R}_1+\mathbf{R}_2)^4(1+\nu_3)^2+ \\
& 2\mathbf{E}_2\mathbf{E}_3(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(-\mathbf{R}_1^6(-1+\nu_2)(1+\nu_3))+\mathbf{R}_2^6(-1+\nu_2)(1+\nu_3)+\mathbf{R}_1^4\mathbf{R}_2^2 \\
& (5+3\nu_2)(1+\nu_3)-\mathbf{R}_1^2\mathbf{R}_2^4(9+\nu_3+3\nu_2(1+\nu_3)))+\mathbf{E}_1^2(\mathbf{E}_3^2(\mathbf{R}_1-\mathbf{R}_2)^2(\mathbf{R}_1+\mathbf{R}_2)^2 \\
& (1+\nu_2)^2(\mathbf{R}_1^4(-3+\nu_2)^2+\mathbf{R}_2^4(-3+\nu_2)^2-2\mathbf{R}_1^2\mathbf{R}_2^2(-3+\nu_2(6+\nu_2))))+\mathbf{E}_2^2(\mathbf{R}_1^8(-3+\nu_2) \\
& (1+\nu_2)+\mathbf{R}_2^8(-3+\nu_2)(1+\nu_2)+6\mathbf{R}_1^4\mathbf{R}_2^4(1+\nu_2)^2-4\mathbf{R}_1^2\mathbf{R}_2^6(1+\nu_2)^2-4\mathbf{R}_1^6\mathbf{R}_2^2(3+\nu_2^2)) \\
& (1+\nu_3)^2-2\mathbf{E}_2\mathbf{E}_3(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(1+\nu_2)(\mathbf{R}_1^6(-3+\nu_2)(-1+\nu_2)(1+\nu_3)-\mathbf{R}_2^6 \\
& (-3+\nu_2)(-1+\nu_2)(1+\nu_3)-\mathbf{R}_1^4\mathbf{R}_2^2(9+\nu_2(4+3\nu_2))(1+\nu_3)+\mathbf{R}_1^2\mathbf{R}_2^4(5-3\nu_3+ \\
& \nu_2(3\nu_2(1+\nu_3)+8(2+\nu_3)))))))/(4(-2\mathbf{E}_1\mathbf{E}_2(\mathbf{E}_3^2(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(1+\nu_2)(\mathbf{R}_1^6
\end{aligned}$$

$$\begin{aligned}
& (-3+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)-\mathbf{R}_2^6(5+\nu_1(-1+\nu_2)-\nu_2)(-3+\nu_2)+\mathbf{R}_1^2\mathbf{R}_2^4 \\
& (3+9\nu_1+4(-2+\nu_1)\nu_2+3(-1+\nu_1)\nu_2^2)+\mathbf{R}_1^4\mathbf{R}_2^2(9-\nu_1(3+\nu_2)(-1+3\nu_2)+ \\
& \nu_2(4+3\nu_2)))+\mathbf{E}_2^2(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(\mathbf{R}_1^6(-3-\nu_1+(-1+\nu_1)\nu_2)-\mathbf{R}_1^4\mathbf{R}_2^2(3+\nu_1+ \\
& 3(-1+\nu_1)\nu_2)+\mathbf{R}_1^2\mathbf{R}_2^4(-9+5\nu_1+3(-1+\nu_1)\nu_2)+\mathbf{R}_2^6(-5+\nu_1+\nu_2-\nu_1\nu_2))(-3+\nu_3) \\
& (1+\nu_3)-2\mathbf{E}_2\mathbf{E}_3(-4\mathbf{R}_1^6\mathbf{R}_2^2(-1+\nu_1)\nu_2(1+\nu_2)(-1+\nu_3)-4\mathbf{R}_1^2\mathbf{R}_2^6(-1+\nu_1)\nu_2(1+\nu_2) \\
& (-1+\nu_3))+6\mathbf{R}_1^4\mathbf{R}_2^4(-1+\nu_1)(1+\nu_2)^2(-1+\nu_3)+\mathbf{R}_1^8(-3-\nu_1+(-1+\nu_1)\nu_2)(-3-\nu_2+ \\
& (-1+\nu_2)\nu_3)+\mathbf{R}_2^8(5-\nu_1+(-1+\nu_1)\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)))+\mathbf{E}_2^2(-3+\nu_1)(1+\nu_1) \\
& (\mathbf{E}_3^2(\mathbf{R}_1^8(-3+\nu_2)(1+\nu_2)+\mathbf{R}_2^8(-3+\nu_2)(1+\nu_2)-4\mathbf{R}_1^6\mathbf{R}_2^2(1+\nu_2)^2+6\mathbf{R}_1^4\mathbf{R}_2^4(1+\nu_2)^2- \\
& 4\mathbf{R}_1^2\mathbf{R}_2^6(3+\nu_2^2))+\mathbf{E}_2^2(\mathbf{R}_1-\mathbf{R}_2)^4(\mathbf{R}_1+\mathbf{R}_2)^4(-3+\nu_3)(1+\nu_3)+2\mathbf{E}_2\mathbf{E}_3(\mathbf{R}_1-\mathbf{R}_2) \\
& (\mathbf{R}_1+\mathbf{R}_2)(-\mathbf{R}_1^2\mathbf{R}_2^4(-5+3\nu_2(-1+\nu_3)+\nu_3))+\mathbf{R}_1^4\mathbf{R}_2^2(-1+3\nu_2(-1+\nu_3)+5\nu_3)+ \\
& \mathbf{R}_2^6(5-\nu_2+(-1+\nu_2)\nu_3)+\mathbf{R}_1^6(3+\nu_2+\nu_3-\nu_2\nu_3)))+\mathbf{E}_1^2(\mathbf{E}_3^2(\mathbf{R}_1-\mathbf{R}_2)^2(\mathbf{R}_1+\mathbf{R}_2)^2 \\
& (1+\nu_2)^2(\mathbf{R}_1^4(-3+\nu_2)^2+\mathbf{R}_2^4(-3+\nu_2)^2-2\mathbf{R}_1^2\mathbf{R}_2^2(-3+\nu_2(6+\nu_2)))+\mathbf{E}_2^2(\mathbf{R}_1^8(-3+\nu_2) \\
& (1+\nu_2)+\mathbf{R}_2^8(-3+\nu_2)(1+\nu_2)+6\mathbf{R}_1^4\mathbf{R}_2^4(1+\nu_2)^2-4\mathbf{R}_1^2\mathbf{R}_2^6(1+\nu_2)^2-4\mathbf{R}_1^6\mathbf{R}_2^2 \\
& (3+\nu_2^2))(-3+\nu_3)(1+\nu_3)-2\mathbf{E}_2\mathbf{E}_3(\mathbf{R}_1-\mathbf{R}_2)(\mathbf{R}_1+\mathbf{R}_2)(1+\nu_2)(\mathbf{R}_1^6(-3+\nu_2) \\
& (-3-\nu_2+(-1+\nu_2)\nu_3)-\mathbf{R}_2^6(-3+\nu_2)(5-\nu_2+(-1+\nu_2)\nu_3)+\mathbf{R}_1^2\mathbf{R}_2^4(-3(-1+\nu_2) \\
& (5+\nu_2)+(3+\nu_2)(-1+3\nu_2)\nu_3)+\mathbf{R}_1^4\mathbf{R}_2^2(3(7+\nu_2^2)-(9+\nu_2(4+3\nu_2))\nu_3))))r^{-4}- \\
& s/2)\cos(2\theta).
\end{aligned} \tag{A 15}$$

For other stress, strain, and displacement components, they can be calculated by the same procedure from Eqs. (6) and (12) which are not represent here due to the lengthy expressions.