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AN INTRODUCTION OF THE LOCAL DISPLACEMENTS OF MASS AND ELECTRIC CHARGE PHENOMENA INTO THE MODEL OF THE MECHANICS OF POLARIZED ELECTROMAGNETIC SOLIDS

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Using the fundamental principles of thermodynamics of irreversible processes and continuum mechanics and electrodynamics, a complete set of equations of the thermomechanics of an electroconducting polarized medium has been obtained by taking into account the local displacements of mass and electric charge. To determine the thermodynamic state, two additional state parameters, namely the induced mass and the gradient of the energy measure of mass displacement, have been introduced. Two other parameters, the energy measure of mass displacement and the mass displacement vector, have been coupled to the aforementioned parameters. Such an extension of the state parameter space allows one to describe the near-surface inhomogeneity of the stress-strained state and the electric polarization as well as the surface charges and the electromagnetic signals induced by the surface formation.

1. Introduction

The theory of coupled electro-magneto-thermo-mechanical processes in polarized media has been the subject of many investigations, as have the applied problems of electrodynamics and mechanics of polarized structure [de Groot and Mazur 1962; Karnaukhov and Kirichok 1988; Maugin 1988; Nowacki 1983; Sedov 1997; Khoroshun 2006; Burak 1967]. Studying the process of electric polarization (the local displacement of electric charge) authors usually do not take into account the accompanying local displacement of mass, for example, the relative displacement of nuclei and electrons or of hydrogen and oxygen atoms in a water molecule, etc. Note also that the displacement of mass can arise without electric polarization [Hrytsyna and Kondrat 2006], for example, in the case of accelerated motion of a body with mass asymmetric molecules. The process of the mass displacement in thermomechanical systems was reported for the first time in [Burak 1987]. Later the studies in this direction have been concerned mostly with interfacial phenomena including the strength of the surface layers [Burak et al. 1991; Hrytsyna et al. 2006].

The purpose of this paper is the formulation and analysis of a mathematical model for the description of electro-magneto-thermo-mechanical processes in electroconducting polarized solids while taking into account the displacement of electric charges (polarization) and local displacement of mass.

Keywords: coupled electro-magneto-thermo-mechanical processes, electroconducting polarized nonferromagnetic solids, local displacements of mass and electric charges, interfacial phenomena.

2. The model

2A. ***Investigation object.*** We consider an isotropic thermoelastic polarized nonferromagnetic solid under the synergistic influence of external stresses, temperature gradients, and electromagnetic fields which induce mechanical, thermal, and electromagnetic processes in the body (domain (V)) enclosed by surface (Σ). The electromagnetic field causes the ordering of bound electric charges (polarizations) that is described by densities of electric flux $\mathbf{J}_{es}$ and mass flux $\mathbf{J}_{ms}$. The mass flux is caused by the difference in mass of bounded positive and negative charges.

All fields, which characterize the processes in solids, must satisfy the fundamental physical laws such as the conservation laws of mass, momentum, angular momentum, entropy, and energy.

The initial relations of the proposed model are based on the Euler approach.

2B. ***Electrodynamics Equations.*** Maxwell's equations can be written in the local form [Bredov et al. 1985; Landau and Lifshitz 1984; Tamm 1979]

$$\nabla\cdot\mathbf{B}=0,\quad \nabla\cdot\mathbf{D}=\rho_e,\quad \nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t},\quad \nabla\times\mathbf{H}=\mathbf{J}_{ef},\tag{2-1}$$

where $\mathbf{E}$, $\mathbf{H}$ are the electric and magnetic fields; $\mathbf{D}$, $\mathbf{B}$ are the vectors of electric and magnetic inductions; for nonferromagnetic mediums $\mathbf{B}=\mu_0\mathbf{H}$; $\mathbf{D}=\varepsilon_0\mathbf{E}+\mathbf{\Pi}_e$, where $\mathbf{\Pi}_e\equiv\mathbf{P}$ denotes the local displacement of electric charge (polarization); ε_0, μ_0 are the electric permittivity and the magnetic permeability of vacuum (electric and magnetic constants); ρ_e is the density of free electric charge; $\mathbf{J}_{ef}=\mathbf{J}_e+\mathbf{J}_{ed}+\mathbf{J}_{es}$ is the density of the total electric current; $\mathbf{J}_e$ is the density of electric current (convection and conduction currents); $\mathbf{J}_{ed}=\varepsilon_0\,(\partial\mathbf{E}/\partial t)$; $\mathbf{J}_{es}=\partial\mathbf{\Pi}_e/\partial t$ is the density of current, caused by ordering of a charged system (polarization current), and ∇ is the Hamilton operator.

We introduce the density of induced charge $\rho_{e\pi}$ [Bredov et al. 1985], and require that for an arbitrary solid of finite size (domain (V)) the vector $\mathbf{\Pi}_e$ of the local displacement of the electric charge and the density $\rho_{e\pi}$ satisfy (see also Bredov et al. [1985])

$$\int_{(V)}\mathbf{\Pi}_e\,dV=\int_{(V)}\rho_{e\pi}\mathbf{r}\,dV,\tag{2-2}$$

where $\mathbf{r}$ is the position vector. Taking into account the arbitrariness of the domain (V), the independence of Equation (2-2) from the choice of frame and the identity $\mathbf{a}\cdot\mathbf{\Pi_e}=(\mathbf{\Pi_e}\cdot\nabla)\,(\mathbf{a}\cdot\mathbf{r})$, where $\mathbf{a}$ is an arbitrary constant vector, from (2-2) we deduce

$$\int_{(V)}\rho_{e\pi}\,dV=0,\qquad \rho_{e\pi}=-\nabla\cdot\mathbf{\Pi}_e.\tag{2-3}$$

After differentiation by time, the second relation of the set (2-3) and using $\mathbf{J}_{es}=\partial\mathbf{\Pi}_e/\partial t$ we obtain the following equation

$$\frac{\partial\rho_{e\pi}}{\partial t}+\nabla\cdot\mathbf{J}_{es}=0,$$

which has the form of the conservation law of induced electric charges [Bredov et al. 1985]. From now on instead of $\mathbf{\Pi}_e$ we shall use the standard notation for the polarization vector $\mathbf{P}$.

2C. ***The mass balance equation.*** The mass balance equation in the integral form is given by

$$\frac{d}{dt}\int_{(V)} \rho\, dV = -\oint_{(\Sigma)} \mathbf{J}_* \cdot \mathbf{n}\, d\Sigma, \tag{2-4}$$

where ρ is the mass density, $\mathbf{J}_*$ denotes the density of the mass flux, and $\mathbf{n}$ is the outward unit normal to the surface (Σ). We assume that the density of mass flux $\mathbf{J}_*$ is the sum of the convective component $\mathbf{J}_{mc} = \rho\mathbf{v}_*$, where $\mathbf{v}_*$ is the average velocity of the displaced particles of the body, and the component $\mathbf{J}_{ms}$ related to the ordering of structure of a physically small element of the body. Thus the equation of mass balance (2-4) can be written as follows

$$\frac{d}{dt}\int_{(V)} \rho\, dV = -\oint_{(\Sigma)} (\rho\mathbf{v}_* + \mathbf{J}_{ms}) \cdot \mathbf{n}\, d\Sigma.$$

We introduce the local mass displacement vector as $\mathbf{\Pi}_m(t) = \int_0^t \mathbf{J}_{ms}(t')\, dt'$. Then, for the flux $\mathbf{J}_{ms}$ we obtain

$$\mathbf{J}_{ms} = \partial\mathbf{\Pi}_m/\partial t. \tag{2-5}$$

Thus the velocity $\mathbf{v}$ of the center of mass is $\mathbf{v} = \dfrac{1}{\rho}\Big(\rho\mathbf{v}_* + \dfrac{\partial\mathbf{\Pi}_m}{\partial t}\Big)$, and the equation of mass balance can now be written in the standard form

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0. \tag{2-6}$$

By analogy with the induced charge, we introduce the density of induced mass $\rho_{m\pi}$, which has the dimension of mass density, so that $\rho_{m\pi}$, $\mathbf{\Pi}_m$, $\mathbf{r}$ satisfy (see Equation (2-2))

$$\int_{(V)} \mathbf{\Pi}_m\, dV = \int_{(V)} \rho_{m\pi}\mathbf{r}\, dV. \tag{2-7}$$

From Equation (2-7) one deduces the following relations [Bredov et al. 1985]

$$\int_{(V)} \rho_{m\pi}\, dV = 0, \qquad \rho_{m\pi} = -\nabla\cdot\mathbf{\Pi}_m. \tag{2-8}$$

We note that from Equation (2-5) and (2-8) one can obtain the equation

$$\frac{\partial\rho_{m\pi}}{\partial t} + \nabla\cdot\mathbf{J}_{ms} = 0,$$

which has the form of the conservation law of induced mass.

2D. ***Equation of entropy balance.*** The general form of the entropy balance equation is [de Groot and Mazur 1962]

$$\frac{d}{dt}\int_{(V)} \rho s\, dV = -\oint_{(\Sigma)} \mathbf{J}_s \cdot \mathbf{n}\, d\Sigma - \oint_{(\Sigma)} \rho s\mathbf{v}\cdot\mathbf{n}\, d\Sigma + \int_{(V)} \sigma_s\, dV + \int_{(V)} \rho\frac{\Re}{T}\, dV, \tag{2-9}$$

where s is the specific entropy (entropy per unit mass), $\mathbf{J}_s$ is the density of entropy flux, T is the absolute temperature, σ_s is the strength of the entropy source, or the entropy production per unit volume and unit time, and $\Re$ denotes the distributed thermal sources.

In the local form Equation (2-9) is given by

$$\rho T \frac{ds}{dt} = -\nabla \cdot \mathbf{J}_q + \frac{1}{T}\mathbf{J}_q \cdot \nabla T + T\sigma_s + \rho\Re. \tag{2-10}$$

Here $\mathbf{J}_q = T\mathbf{J}_s$ is the density of heat flux and $d/dt = \partial/\partial t + \mathbf{v}\cdot\nabla$ is the substantive derivative.

2E. ***Equation of the balance of electromagnetic field energy.*** From the Maxwell equations (2-1), the equation, which is known as the energy balance equation of the electromagnetic field [Bredov et al. 1985; Landau and Lifshitz 1984; Tamm 1979], follows, namely

$$\frac{\partial U_e}{\partial t} + \nabla \cdot \mathbf{S}_e + \left(\mathbf{J}_e + \frac{\partial \mathbf{P}}{\partial t}\right)\cdot \mathbf{E} = 0, \tag{2-11}$$

where $U_e = \left(\varepsilon_0 \mathbf{E}^2 + \mu_0 \mathbf{H}^2\right)/2$ is the energy density of the electromagnetic field and $\mathbf{S}_e = \mathbf{E}\times\mathbf{H}$ is the flux density of its energy.

Let us rewrite the last term in Equation (2-11) in such a way that it contains the specific polarization $\mathbf{p} = \mathbf{P}/\rho$, the vectors $\mathbf{E}_*$, $\mathbf{P}_*$, $\mathbf{J}_{e*}$ of the electromagnetic field and density of electric current in the reference frame of the center of mass moving with speed $\mathbf{v}$ relatively to the laboratory reference frame, that is,

$$\mathbf{E}_* = \mathbf{E} + \mathbf{v}\times\mathbf{B}, \quad \mathbf{P}_* = \mathbf{P} + \varepsilon_0\mu_0 \mathbf{v}\times\mathbf{M}, \quad \mathbf{J}_{e*} = \mathbf{J}_e - \rho_e \mathbf{v}.$$

Here $\mathbf{M}$ denotes the magnetization vector (in the nonmagnetic case the magnetization vector is zero) and $\mathbf{J}_{e*}$ is the conduction current density. Using the mass conservation law (2-6), we can rewrite the balance equation of energy of electromagnetic field (2-11) as

$$\frac{\partial U_e}{\partial t} + \nabla\cdot\mathbf{S}_e + \mathbf{J}_{e*}\cdot\mathbf{E}_* + \left[\rho_e\mathbf{E}_* + \left(\mathbf{J}_{e*} + \frac{\partial(\rho\mathbf{p})}{\partial t}\right)\times\mathbf{B} + \rho\left(\nabla\mathbf{E}_*\right)\cdot\mathbf{p}\right]\cdot\mathbf{v} + \rho\mathbf{E}_*\cdot\frac{d\mathbf{p}}{dt} - \nabla\cdot\left[\rho\left(\mathbf{E}_*\cdot\mathbf{p}\right)\mathbf{I}\cdot\mathbf{v}\right] = 0. \tag{2-12}$$

2F. ***Equation of balance of energy for system body — electromagnetic field.*** We assume that for an arbitrary moment of time, the total energy of system is the sum of internal energy ρu (u is the specific internal energy), kinetic $\rho\mathbf{v}^2/2$ energy, and the energy of the electromagnetic field U_e. On the other hand, the total energy change is the result of the convective energy transport $\rho\left(u + \mathbf{v}^2/2\right)$ through the surface, the work $\hat{\boldsymbol{\sigma}}\cdot\mathbf{v}$ of surface forces; the heat flux $\mathbf{J}_q$, the electromagnetic energy flux $\mathbf{S}_e$, the work $\mu\mathbf{J}_m$ connected to the mass transport relative to the center of mass of the body (here $\mathbf{J}_m = \rho(\mathbf{v}_* - \mathbf{v})$); the work $\mu_\pi \partial\boldsymbol{\Pi}_m/\partial t$ related with structure ordering (local mass displacement), and the action of mass forces $\mathbf{F}$ and distributed thermal sources $\Re$. One therefore has

$$\frac{d}{dt}\int_{(V)}\left(\rho u + U_e + \tfrac{1}{2}\rho\mathbf{v}^2\right)dV = -\oint_{(\Sigma)}\left[\rho\left(u + \tfrac{1}{2}\mathbf{v}^2\right)\mathbf{v} - \hat{\boldsymbol{\sigma}}\cdot\mathbf{v} + \mathbf{S}_e + \mathbf{J}_q + \mu\mathbf{J}_m + \mu_\pi\frac{\partial\boldsymbol{\Pi}_m}{\partial t}\right]\cdot\mathbf{n}\,d\Sigma + \int_{(V)}\left(\rho\mathbf{F}\cdot\mathbf{v} + \rho\Re\right)dV, \tag{2-13}$$

where $\hat{\boldsymbol{\sigma}}$ is the Cauchy's stress tensor, μ is the chemical potential, and μ_π is the energy measure of the influence of the mass displacement on the internal energy.

By the use of the Ostrogradsky–Gauss theorem, the balance equations of mass (2-6), the electromagnetic energy equation (2-12), and the entropy equation (2-10), we obtain from Equation (2-13)

$$\rho\frac{du}{dt}=\rho T\frac{ds}{dt}+\left[\hat{\boldsymbol{\sigma}}-\rho\left(\mathbf{E}_*\cdot\mathbf{p}\right)\hat{\mathbf{I}}\right]:\frac{d\hat{\mathbf{e}}}{dt}+\rho\mathbf{E}_*\cdot\frac{d\mathbf{p}}{dt}-\mu'_\pi\frac{\partial\nabla\cdot\boldsymbol{\Pi}_m}{\partial t}-\nabla\mu'_\pi\cdot\frac{\partial\boldsymbol{\Pi}_m}{\partial t}+\rho_e\mathbf{E}_*-\mathbf{J}_{e*}\cdot\mathbf{E}_*$$
$$-\mathbf{J}_q\cdot\frac{\nabla T}{T}-T\sigma_s+\mathbf{v}\cdot\left\{-\rho\frac{d\mathbf{v}}{dt}+\nabla\cdot\left[\hat{\boldsymbol{\sigma}}-\rho\left(\mathbf{E}_*\cdot\mathbf{p}\right)\hat{\mathbf{I}}\right]+\left(\mathbf{J}_{e*}+\frac{\partial(\rho\mathbf{p})}{\partial t}\right)\times\mathbf{B}+\rho\left(\nabla\mathbf{E}_*\right)\cdot\mathbf{p}+\rho\mathbf{F}\right\},\quad (2\text{-}14)$$

where $\mu'_\pi=\mu_\pi-\mu$, $\hat{\mathbf{e}}=\left[\nabla\mathbf{u}+(\nabla\mathbf{u})^T\right]/2$ is the strain tensor, $\mathbf{u}$ is the displacement vector, and an upper index T denotes a transposed tensor.

Introducing the specific values $\boldsymbol{\pi}_m=\boldsymbol{\Pi}_m/\rho$ and $\rho_m=\rho_{m\pi}/\rho$ and taking into account the mass balance Equation (2-6), we obtain the following balance equation for the internal energy using Equation (2-14)

$$\rho\frac{du}{dt}=\rho T\frac{ds}{dt}+\hat{\boldsymbol{\sigma}}_*:\frac{d\hat{\mathbf{e}}}{dt}+\rho\mathbf{E}_*\cdot\frac{d\mathbf{p}}{dt}+\rho\mu'_\pi\frac{d\rho_m}{dt}-\rho\nabla\mu'_\pi\cdot\frac{d\boldsymbol{\pi}_m}{dt}+\mathbf{J}_{e*}\cdot\mathbf{E}_*-\mathbf{J}_q\cdot\frac{\nabla T}{T}$$
$$-T\sigma_s+\mathbf{v}\cdot\left\{-\rho\frac{d\mathbf{v}}{dt}+\nabla\cdot\hat{\boldsymbol{\sigma}}_*+\rho_e\mathbf{E}_*+\left(\mathbf{J}_{e*}+\frac{\partial(\rho\mathbf{p})}{\partial t}\right)\times\mathbf{B}+\rho\left(\nabla\mathbf{E}_*\right)\cdot\mathbf{p}+\rho\mathbf{F}_*\right\},\quad (2\text{-}15)$$

where

$$\hat{\boldsymbol{\sigma}}_*=\hat{\boldsymbol{\sigma}}-\rho\left(\mathbf{E}_*\cdot\mathbf{p}-\rho_m\mu'_\pi-\boldsymbol{\pi}_m\cdot\nabla\mu'_\pi\right)\hat{\mathbf{I}},\quad \mathbf{F}_*=\mathbf{F}+\rho_m\nabla\mu'_\pi-\boldsymbol{\pi}_m\cdot\nabla\nabla\mu'_\pi.$$

Furthermore, when using the new thermodynamical function of the generalized Helmholtz free energy $f=u-Ts-\mathbf{E}_*\cdot\mathbf{p}+\nabla\mu'_\pi\cdot\boldsymbol{\pi}_m$, we obtain from Equation (2-15)

$$\rho\frac{df}{dt}=-\rho s\frac{dT}{dt}+\hat{\boldsymbol{\sigma}}_*:\frac{d\hat{\mathbf{e}}}{dt}-\rho\mathbf{p}\cdot\frac{d\mathbf{E}_*}{dt}+\rho\mu'_\pi\frac{d\rho_m}{dt}+\rho\boldsymbol{\pi}_m\cdot\frac{d\nabla\mu'_\pi}{dt}+\mathbf{J}_{e*}\cdot\mathbf{E}_*-\mathbf{J}_q\cdot\frac{\nabla T}{T}$$
$$-T\sigma_s+\mathbf{v}\cdot\left\{-\rho\frac{d\mathbf{v}}{dt}+\nabla\cdot\hat{\boldsymbol{\sigma}}_*+\rho_e\mathbf{E}_*+\left(\mathbf{J}_{e*}+\frac{\partial(\rho\mathbf{p})}{\partial t}\right)\times\mathbf{B}+\rho\left(\nabla\mathbf{E}_*\right)\cdot\mathbf{p}+\rho\mathbf{F}_*\right\}.\quad (2\text{-}16)$$

From the requirement that Equation (2-16) is invariant with respect to translations and assuming that the free energy f is the function of scalar quantities T, ρ_m, vector quantities $\mathbf{E}_*$, $\nabla\mu'_\pi$, and tensor quantity $\hat{\mathbf{e}}$ (all of them are independent parameters), we obtain the generalized Gibbs equation

$$df=-s\,dT+\rho^{-1}\hat{\boldsymbol{\sigma}}_*:d\hat{\mathbf{e}}-\mathbf{p}\cdot d\mathbf{E}_*+\mu'_\pi d\rho_m+\boldsymbol{\pi}_m\cdot d\nabla\mu'_\pi,\quad (2\text{-}17)$$

a relation for the entropy production

$$\sigma_s=\mathbf{J}_{e*}\cdot\frac{\mathbf{E}_*}{T}-\mathbf{J}_q\cdot\frac{\nabla T}{T^2},\quad (2\text{-}18)$$

and the momentum equation

$$\rho\frac{d\mathbf{v}}{dt}=\nabla\cdot\hat{\boldsymbol{\sigma}}_*+\mathbf{F}_e+\rho\mathbf{F}_*,\quad (2\text{-}19)$$

where $\mathbf{F}_e=\rho_e\mathbf{E}_*+\left(\mathbf{J}_{e*}+\partial(\rho\mathbf{p})/\partial t\right)\times\mathbf{B}+\rho\left(\nabla\mathbf{E}_*\right)\cdot\mathbf{p}$ is the ponderomotive force.

In this case the free energy depends not only on temperature T, strain tensor $\hat{\mathbf{e}}$ and electric field $\mathbf{E}_*$, but also on the parameters related to the mass displacement, namely, ρ_m and $\nabla\mu'_\pi$. Note that the introduction of both the local mass displacement and the electric charge displacement leads to additional (ponderomotive) forces in Equation (2-19) and to the redefinition of the stress tensor.

2G. ***Constitutive relations.*** Since the parameters T, ρ_m, $\mathbf{E}_*$, $\nabla\mu'_\pi$, and $\hat{\mathbf{e}}$ are independent, we obtain the following relations from the Gibbs Equation (2-17)

$$s = -\frac{\partial f}{\partial T}\bigg|_{\hat{\mathbf{e}},\,\rho_m,\,\nabla\mu'_\pi,\mathbf{E}_*}, \qquad \hat{\boldsymbol{\sigma}}_* = \rho\frac{\partial f}{\partial \hat{\mathbf{e}}}\bigg|_{T,\,\rho_m,\nabla\mu'_\pi,\mathbf{E}_*}, \qquad \mu'_\pi = \frac{\partial f}{\partial \rho_m}\bigg|_{T,\,\hat{\mathbf{e}},\,\nabla\mu'_\pi,\mathbf{E}_*},$$
$$\mathbf{p} = -\frac{\partial f}{\partial \mathbf{E}_*}\bigg|_{T,\,\hat{\mathbf{e}},\,\rho_m,\nabla\mu'_\pi}, \qquad \boldsymbol{\pi}_m = \frac{\partial f}{\partial(\nabla\mu'_\pi)}\bigg|_{T,\,\hat{\mathbf{e}},\,\rho_m,\,\mathbf{E}_*}. \tag{2-20}$$

Let $\hat{\mathbf{e}} = 0$, $T = T_0$, $\rho_m = 0$, $\mathbf{E}_* = 0$, $\nabla\mu'_\pi = 0$, $s = s_0$, $\hat{\boldsymbol{\sigma}}_* = 0$, $\mu'_\pi = \mu'_{\pi 0}$, $\mathbf{p} = 0$, and $\boldsymbol{\pi}_m = 0$ in the reference state, then in the linear approximation, Equation (2-20) may be written in the form

$$s = s_0 - [a^s_T(T - T_0) + \rho_0^{-1} a_{eT} e + a_{\rho T}\rho_m],$$
$$\hat{\boldsymbol{\sigma}}_* = 2a^\sigma_2 \hat{\mathbf{e}} + [a^\sigma_1 e + a_{eT}(T - T_0) + a_{e\rho}\rho_m]\,\hat{\mathbf{I}},$$
$$\mu'_\pi = \mu'_{\pi 0} + a^\mu_\rho \rho_m + \rho_0^{-1} a_{e\rho} e + a_{\rho T}(T - T_0),$$
$$\mathbf{p} = -a^p_E \mathbf{E}_* - a_{E\mu}\nabla\mu'_\pi, \qquad \boldsymbol{\pi}_m = a^\pi_\mu \nabla\mu'_\pi + a_{E\mu}\mathbf{E}_*, \tag{2-21}$$

where $e \equiv \hat{\mathbf{e}} : \hat{\mathbf{I}}$ is the first invariant of the strain tensor; a^σ_1, a^σ_2, a^s_T, a^μ_ρ, a^p_E, a^π_μ, a_{eT}, $a_{\rho T}$, $a_{e\rho}$, and $a_{E\mu}$ are the characteristics of material; and s_0 and $\mu'_{\pi 0}$ are the entropy and the reduced potential μ'_π, respectively, in the reference state.

An analysis of the Gibbs equation reveals that our model requires two additional pairs of parameters for enabling us to describe the local thermodynamic state of the body:

(i) the induced mass $\rho_m = -(\nabla \cdot \boldsymbol{\Pi}_m)/\rho$ and the energy measure μ'_π of the influence of the mass displacement on internal energy;

(ii) the vector of density of the mass displacement $\boldsymbol{\pi}_m = \boldsymbol{\Pi}_m/\rho$ and the gradient of μ'_π.

These parameters are related to the local displacement of the mass. Such an extension of the state parameters space allows one to describe near-surface inhomogeneity of the stress-strained state and electrical polarization [Hrytsyna and Kondrat 2006; Burak et al. 1991; Hrytsyna et al. 2006]. Indeed, according to the chosen model the electric polarization is caused not solely by the electric field but also by the gradient of μ'_π. In the near-surface region the value of $|\nabla\mu'_\pi|$ can be sufficiently large to induce essential surface polarization. This can be important in studies of electromagnetic emission caused by the formation of a new surface within the body or an electromagnetic response of the body towards external dynamic influence on its surface [Fursa et al. 2003]. The details of such phenomena are considered below where we describe the near-surface inhomogeneity in an infinite polarized layer.

By using the Onsager principle and Equation (2-18) for the entropy production, one finds in the linear approximation [de Groot and Mazur 1962]

$$\mathbf{J}_{e*} = \sigma_e \mathbf{E}_* + \sigma_e \eta \nabla T, \qquad \mathbf{J}_q = -\lambda\nabla T + \pi_t \mathbf{J}_{e*},$$

where σ_e and λ are electric and thermal conductivity, respectively. Here coefficients η and π_t characterize thermoelectric phenomena and are related by $T\eta = -\pi_t$, respectively.

The obtained constitutive relations, the conservation laws of momentum, masses, and entropy, the equations of electrodynamics and geometrical relations form a complete set of equations of electro-magneto-thermo-mechanics of the polarized nonferromagnetic solids taking into account the local displacements of the mass and electric charge.

Let us take the displacement vector $\mathbf{u}$, temperature T, electric field $\mathbf{E}$, magnetic induction $\mathbf{B}$, and the induced mass ρ_m as a set of basic functions. Then in the linear approximation we obtain the following equation for ρ_m

$$\Delta\rho_m + \frac{1}{a_\mu^\pi a_\rho^\mu}\rho_m = -\frac{1}{a_\rho^\mu}\left[\frac{a_{e\rho}}{\rho_0}\Delta\big(\nabla\cdot\mathbf{u}\big) + a_{\rho T}\Delta T + \frac{a_{E\mu}}{a_\mu^\pi}\nabla\cdot\mathbf{E}\right].$$

It is possible to show that one may present ρ_m as a functional of functions $\mathbf{u}$, T, and $\mathbf{E}$, therefore ρ_m can be eliminated from the basic set of equations and also from the constitutive relations. In this case, the basic set of equations becomes integro-differential equations whereas the constitutive equations (2-21) become spatially nonlocal.

3. Example

We consider an elastic polarized layer of an ideal dielectric in the region $-l \le x \le l$. At time $t = 0$ the layer is cut from an infinite medium in such a way that at time $t > 0$ it is in contact with a medium which behaves as vacuum with regards to its electromagnetic properties. The effect of temperature is neglected while the processes of deformation, polarization, and displacement of mass are considered.

In this case the basic functions $\mathbf{f} = \{\mathbf{u}, \mathbf{E}, \mathbf{p}, \mathbf{D}, \boldsymbol{\pi}_m, \mathbf{E}_v\}$ and $g = \{\tilde{\mu}'_\pi, \rho_m\}$ are functions of the space coordinate x and time coordinate t such that $\mathbf{f} = (f, 0, 0)$, $f = \{u, E, p, D, \pi_m, E_v\}$, $f = f(x,t)$, and $g = g(x,t)$, where $\mathbf{E}_v$ is the vector of the electric field in vacuum, and $\tilde{\mu}'_\pi = \mu'_\pi - \mu'_{\pi 0}$.

With this notation our basic set of equations is reduced to the set of equations for the layer $-l \le x \le l$

$$\rho_0\frac{\partial^2 u}{\partial t^2} = \left(a_1^\sigma + 2a_2^\sigma - \frac{a_{e\rho}^2}{\rho_0 a_\rho^\mu}\right)\frac{\partial^2 u}{\partial x^2} + \frac{a_{e\rho}}{a_\rho^\mu}\frac{\partial\tilde{\mu}'_\pi}{\partial x}, \qquad \mu_0\left(\varepsilon_0 - \rho_0 a_E^p\right)\frac{\partial E}{\partial t} - \rho_0\mu_0 a_{E\mu}\frac{\partial^2\tilde{\mu}'_\pi}{\partial x\partial t} = 0,$$

$$\frac{\partial^2\tilde{\mu}'_\pi}{\partial x^2} + \frac{1}{a_\mu^\pi a_\rho^\mu}\tilde{\mu}'_\pi = \frac{1}{a_\mu^\pi a_\rho^\mu}\frac{a_{e\rho}}{\rho_0}\frac{\partial u}{\partial x} - \frac{a_{E\mu}}{a_\mu^\pi}\frac{\partial E}{\partial x},$$

where $E = -\partial\varphi/\partial x$, φ is the electric potential, and the set of equations for the potential φ_v of the electric field in vacuum ($x < -l$, $x > l$) is

$$\frac{\partial^2\varphi_v}{\partial x^2} - \varepsilon_0\mu_0\frac{\partial^2\varphi_v}{\partial t^2} = 0, \qquad E_v = -\frac{\partial\varphi_v}{\partial x}.$$

The surfaces of the layer are stress-free and the potential μ'_π at $x = \pm l$ is zero. Therefore, taking into account the continuity condition for electric potential, the boundary conditions and radiation conditions become

$$\left(a_1^\sigma + 2a_2^\sigma - \frac{a_{e\rho}^2}{\rho_0 a_\rho^\mu}\right)\frac{\partial u}{\partial x} + \frac{a_{e\rho}}{a_\rho^\mu}\tilde{\mu}'_\pi = 0, \qquad \lim_{x\to\pm\infty}\left(\frac{\partial\varphi_v}{\partial x} \pm \sqrt{\varepsilon_0\mu_0}\frac{\partial\varphi_v}{\partial t}\right) = 0,$$

$$\tilde{\mu}'_\pi = -\mu'_{\pi 0}, \qquad \varphi = \varphi_v.$$

We suppose that at $t = 0$ the sought functions are zero. Neglecting the inertial forces, the solution of this boundary problem is given by

$$\tilde{\mu}'_\pi = -\mu'_{\pi 0}\frac{\mathrm{ch}(\lambda x)}{\mathrm{ch}(\lambda l)}\theta(t), \qquad E = -\lambda\mu'_{\pi 0}\frac{\rho_0 a_{E\mu}}{\varepsilon_0 - \rho_0 a_E^p}\frac{\mathrm{sh}(\lambda x)}{\mathrm{ch}(\lambda l)}\theta(t),$$

$$\varphi = \mu'_{\pi 0}\frac{\rho_0 a_{E\mu}}{\varepsilon_0 - \rho_0 a_E^p}\frac{\mathrm{ch}(\lambda x)}{\mathrm{ch}(\lambda l)}\theta(t), \quad p = \lambda\mu'_{\pi 0}\frac{\varepsilon_0 a_{E\mu}}{\varepsilon_0 - \rho_0 a_E^p}\frac{\mathrm{sh}(\lambda x)}{\mathrm{ch}(\lambda l)}\theta(t),$$

$$\sigma_{yy} = \sigma_{zz} \equiv \sigma = -\mu'_{\pi 0}\frac{a_{e\rho}}{a_\rho^\mu}\left[1 - \frac{a_1^\sigma}{a_1^\sigma + 2a_2^\sigma - a_{e\rho}^2/\left(\rho_0 a_\rho^\mu\right)}\left(1 - \frac{a_{e\rho}^2}{\rho_0 a_\rho^\mu}\right)\right]\frac{\mathrm{ch}(\lambda x)}{\mathrm{ch}(\lambda l)}\theta(t)$$

for $-l \le x \le l$;

$$\varphi_v = \begin{cases} \mu'_{\pi 0}\dfrac{\rho_0 a_{E\mu}}{\varepsilon_0 - \rho_0 a_E^p}\theta\left(t + \sqrt{\varepsilon_0\mu_0}(x+l)\right), & \text{if } x < -l, \\ \mu'_{\pi 0}\dfrac{\rho_0 a_{E\mu}}{\varepsilon_0 - \rho_0 a_E^p}\theta\left(t - \sqrt{\varepsilon_0\mu_0}(x-l)\right), & \text{if } x > l, \end{cases}$$

where

$$\lambda^2 = -\frac{\rho_0(a_1^\sigma + 2a_2^\sigma)(\varepsilon_0 - \rho_0 a_E^p)}{\left[\rho_0 a_\rho^\mu(a_1^\sigma + 2a_2^\sigma) - a_{e\rho}^2\right]\left[a_\mu^\pi(\varepsilon_0 - \rho_0 a_E^p) + \rho_0 a_{E\mu}^2\right]}.$$

For the density of the bound surface charge $\sigma_{se}(\pm l) = \rho_0 p(\pm l)$ at $t > 0$ one has

$$\sigma_{se}(\pm l) = \mu'_{\pi 0}\lambda a_{E\mu}\frac{\varepsilon_0\rho_0}{\varepsilon_0 - \rho_0 a_E^p}\, th(\lambda l). \tag{3-1}$$

The analysis of the solution shows that the distributions of the stresses σ_{yy}, σ_{zz}, the reduced energy measure $\tilde{\mu}'_\pi$ and functions E, φ, and p exhibit inhomogeneities close to the surface. Figure 1 displays the distributions of the stresses σ/σ^*, the electric potential φ/φ^*, and electric polarization p/p^* in the layer where

$$\varphi^* = \mu'_{\pi 0}\frac{\rho_0 a_{E\mu}}{\varepsilon_0 - \rho_0 a_E^p}, \qquad p^* = \mu'_{\pi 0}\frac{\varepsilon_0\lambda a_{E\mu}}{\varepsilon_0 - \rho_0 a_E^p},$$

$$\sigma^* = -\mu'_{\pi 0}\frac{a_{e\rho}}{a_\rho^\mu}\left[1 - \frac{a_1^\sigma}{a_1^\sigma + 2a_2^\sigma - a_{e\rho}^2/(\rho_0 a_\rho^\mu)}\left(1 - \frac{a_{e\rho}^2}{\rho_0 a_\rho^\mu}\right)\right].$$

As one can see, thin layers (curves 1–3 in Figure 1 are characterized by the overlay of the near-surface inhomogeneities while there is a well-defined bulk region characterized by the uniform (constant) profile for thicker layers (curves 4 and 5). This effect manifests itself in the dependence of the surface charge density $\sigma_{se}/\sigma_{se}^*$, where $\sigma_{se}^* = \mu'_{\pi 0}\lambda a_{E\mu}\varepsilon_0\rho_0/(\varepsilon_0 - \rho_0 a_E^p)$, on the layer thickness (see Figure 2).

The bounded charge (3-1) is induced at the surfaces of the layer while in the vacuum the momentum of the electric field arises and propagates from $x = \pm l$ to $\pm\infty$. Thus the proposed model allows the description of the interface inhomogeneity of the stress-strained state and the surface polarization in dielectrics, the appearance of electrical charge at surfaces as well as an electromagnetic signal caused by the surface formation.

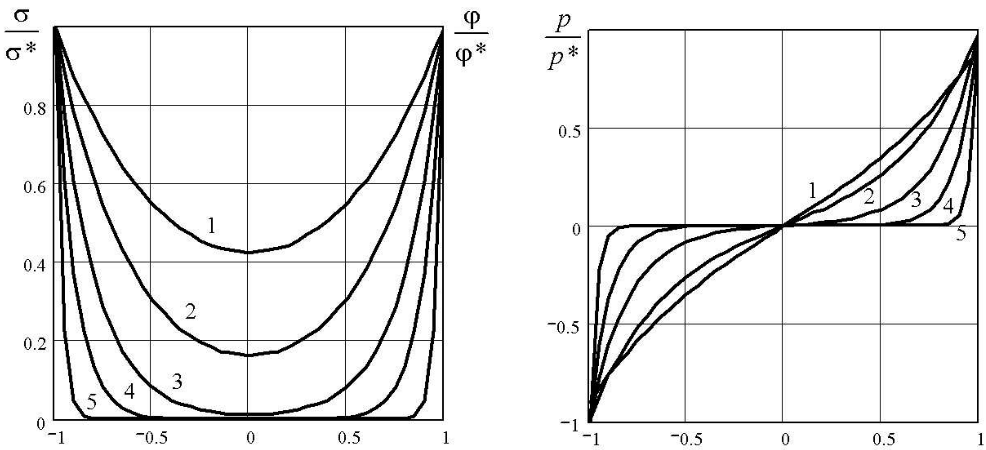

Figure 1. The distributions of the stresses σ/σ^* , the electric potential φ/φ^* , and electric polarization p/p^* in the layer for $\lambda l = 1.5, 2.5, 5, 10, 30$, that is, curves 1–5 respectively.

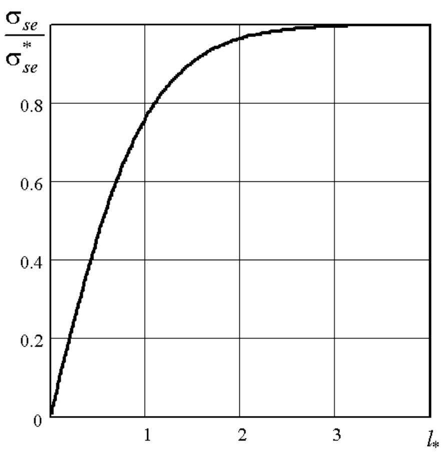

Figure 2. The dependence of the surface charge density $\sigma_{se}/\sigma_{se}^*$ on the layer thickness $l_* = \lambda l$.

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