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PROPAGATION OF A SURFACE WAVE IN A VORTEX ARRAY ALONG A SUPERCONDUCTING HETEROSTRUCTURE

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We analyze the propagation conditions and dispersion relations for *SH* surface waves (Love-like waves) running along a vortex array in a supercoducting heterostructure consisting of a layer and a half-space. Investigations allowed us to estimate a new interval for the wave phase velocity values different from the classical estimate and to show that the structure has filtering properties.

1. Introduction

Superconductors generally fall into two classes. A type-I superconductor expels magnetic flux from the material and hence is in the Meisser state. That is possible only at an applied magnetic field strength less than the determined critical value. In contrast a type-II superconductor behaves in another way. For an applied field less than the lower critical field a type-II superconductor will exhibit the usual Meisser effect. Applied fields greater than the upper critical field strength destroy the superconductivity altogether. In between the lower H_{c1} and upper H_{c2} magnetic field strengths the superconductor is in the mixed or vortex state. The second variable that determines the existence of that state is the temperature $T < T_c$, where T_c denotes the critical phase transition temperature [Tilley and Tilley 1974; Tinkham 1975; Orlando and Delin 1991; Cyrot and Pavuna 1992; Blatter et al. 1994; Brandt 1995; Lüthi 2005; Fossheim and Sudbø 2004]. Magnetic flux can penetrate a type-II superconductor in the form of Abrikosov vortices (also called flux lines, flux tubes, or fluxons) each carrying a quantum of magnetic flux. These tiny vortices of supercurrent tend to arrange themselves in a triangular or quadratic flux-line lattice [Cyrot and Pavuna 1992; Fossheim and Sudbø 2004] which is more or less pertubed by material inhomogeneties that pin the flux lines. Pinning is caused by imperfections of the crystal lattice, such as dislocations, point defects grain boundaries, etc. Hence a honeycomb-like pattern of the vortex array presents some thermomechanical properties.

In the natural state of any superconductor the thermomechanical field comes from atomic and/or molecular interactions both within crystalline (solid) and amorphous (fluid) states of the material in the presence of temperature changes. Such a situation transfers itself to the vortex state as well.

Since the vortices are formed by the applied magnetic field and the supercurrent flows around each vortex, there are also Lorenz force interactions among the vortices. Those interactions form an origin of an additional thermomechanical (stress) field occurring in the type-II superconductor. Near the lower critical magnetic intensity limit H_{c1}, this field has an elastic character. However, if the density of the supercurrent is above its critical value and/or the temperature is sufficiently high, a flow of vortex lines occurs in the superconducting body. Within such a situation vortices behave rather as a fluid than as an

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elastic lattice. The fluidity of the vortex array is also observed when the applied magnetic field tends to its upper critical limit H_{c2} (same references as on page 1097). In this way we meet a very interesting situation in a type-II superconduntor. We can say that there are two coexisting thermomechanical fields in the medium. One field is of a pure thermoelastic character coming from the mechanical properties of the crystal lattice of the superconductor. The second field comes from the vortex array, which keeps its thermoelastic character near the lower magnetic field strength limit H_{c1} and transfers smoothly into a "fluid" near the upper magnetic field strength limit H_{c2}. The above phenomenon (transfer and coexistence) occurs in the $\{(H(T), T) : H_{c1} < H < H_{c2}, T < T_c\}$ space. However, the vortex field also has a viscous character. The motion of vortices is damped by a force proportional to the vortex velocity. There are two reasons for that damping. The first reason comes from simultaneous interactions among magnetic, mechanical, and thermal fields. The second reason occurs because the resistivity in area of vortex creep is the same as the resistivity of a current which would flow inside the vortex core. Hence the viscosity coefficient reads, from [Cyrot and Pavuna 1992],

$$\eta = \frac{\Phi_0 \mu_0 H_{c2}}{\rho_n}, \tag{1}$$

where Φ_0 is the magnetic flux, μ_0 denotes the permeability of vacuum, and ρ_n is the resistivity in the normal state.

Since the vortices may be described within a macroscopic phenomenology, except for the description presented in [Blatter et al. 1994; Ketterson and Song 1999], an unconventional model of magnetothermomechanical processes running in the vortex array in a continuous manner has been proposed [Maruszewski and Restuccia 1999; Maruszewski 1998; 2007; Maruszewski et al. 2007]. Following that model, field equations have been obtained and their form shows that not only diffusion, creep, or flow of the vortices are possible in the superconducting material but also wave propagation (transmission of singnals) [Restuccia and Maruszewski 1998; 1999; Maruszewski and Restuccia 2001; Drzewiecki et al. 2002a; 2002b].

This paper deals with Love's wave propagation along the superconducting heterostructure consisting of two II-type superconductors (the layer on the half-space) placed in an external magnetic field perpendicular to characteristic planes of the structure. The analysis has been based on [Achenbach 1976; Maruszewski et al. 2007; Maruszewski and van de Ven 1995; Drzewiecki et al. 2004]. We focus only on magnetoelastic interactions within the vortex lattice (the applied magnetic field intensity is close to the lower magnetic field limit H_{c1}, satisfying the inequality $H^0 > H_{c1}$).

2. Magnetoelastic Love's waves

Let us consider a superconducting heterostructure consisting of a layer of thickness h and a half-space. That heterostructure is placed in an external constant magnetic field $\mathbf{H} = [H^0, 0, 0]$. Along the interface between the structure components Love's wave propagates with a velocity v. The propagation direction is x_2. The complete geometry of the problem is presented in Figure 1.

The general linearized equations describing the propagation of harmonic waves in the above heterostructure (solely in the vortex field) read as follows (see [Maruszewski and Restuccia 1999; Maruszewski 2007; Maruszewski et al. 2007; Maruszewski and van de Ven 1995; Restuccia and Maruszewski 1999;

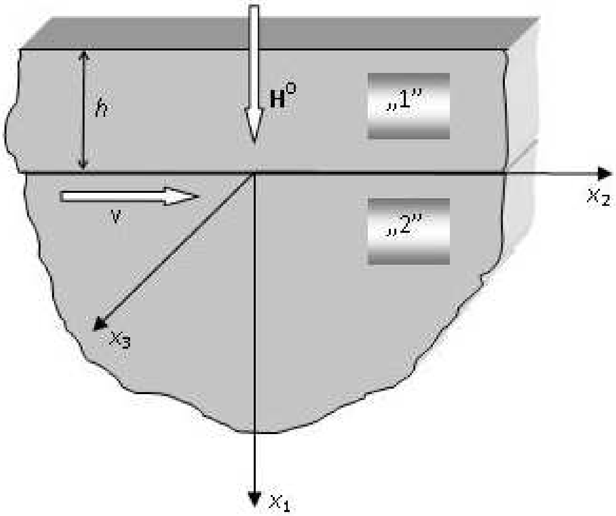

Figure 1. Geometry of the problem.

Drzewiecki et al. 2002a]):

$$\mu u_{i,jj} + \eta \dot{u}_{i,jj} + (\lambda + \mu) u_{j,ij} + \tfrac{1}{3}\eta \dot{u}_{j,ij} + \mu_0 (h_{r,i} - h_{i,r}) H_r^0 - \rho \ddot{u}_i = 0,$$
$$\lambda_0^2 h_{i,kk} - h_i + u_{i,k} H_k^0 - u_{k,k} H_i^0 = 0. \tag{2}$$

Since the viscosity coefficient (1) is very small we neglect the damping features in the vortex field in the sequel. The linearization has been done assuming the total magnetic field in the structure of the form

$$\mathbf{H} = \mathbf{H}^0 + \mathbf{h}, \quad |\mathbf{h}| \ll \left|\mathbf{H}^0\right|, \quad \mathbf{H}^0 = \left[H_1^0, 0, 0\right], \quad H_1^0 = \text{const}, \tag{3}$$

where $\mathbf{h}$ is the small contribution to the total magnetic field $\mathbf{H}$ coupled with the displacement vector $\mathbf{u}$. Lamé's constants, λ and μ, have been calculated from H^0 and H_{c1} [Blatter et al. 1994; Ketterson and Song 1999], μ_0 is the permeability of vacuum, and λ_0 is the London penetration depth. Note that Equations (2) are valid simultaneously for both arrays 1 and 2 in Figure 1.

Now assuming that the solutions of (2) in the geometry shown in Figure 1 are in the following form

$$f(x_1, x_2, t) = \overline{f}(x_1) \exp[\imath(\omega t - k x_2)], \tag{4}$$

where $f(x_1, x_2, t)$ stands for all fields in (2), that is,

$$f(x_1, x_2, t) = \left\{{}^0u_3, {}^0h_3\right\}(x_1, x_2, t), \tag{5}$$

where Love's mode concerns only the u_3 component, Equations (2) may be rewritten in the form (see [Achenbach 1976; Maruszewski and van de Ven 1995])

$$\mu_K u^K_{3,jj} - \mu_0 h^K_{3,1} H_1^0 - \rho_K \ddot{u}^K_3 = 0 \quad \text{and} \quad \lambda_0 h^0_{3,jj} - k^K_3 + u^K_{3,1} H_1^0 = 0 \qquad \text{both with } j = 1, 2, \tag{6}$$

where $K = 1, 2$ distinguishes the layer (1) from the half-space (2).

To facilitate the investigation of (6) and the analysis of its solutions, we convert the above formula to a dimensionless form with the help of the relations

$$
\begin{gathered}
x_1 = hx, \quad x_2 = hy, \quad x_3 = hz, \quad t = T\tau, \quad T = h\sqrt{\frac{\rho_1}{\mu_1}} = \frac{h}{v_{T1}}, \\
H_1^0 = H_{c1}H_0, \quad h_3^K = H_{c1}h_z^K, \quad u_3^K = hu_z^K, \quad \Omega = \omega T, \quad V = \frac{v}{v_{T1}}, \\
k = \frac{\omega}{v} = \frac{\Omega}{vT} = \frac{\Omega}{Vh}, \quad \widetilde{\rho}_K = \frac{\rho_K h^2}{T^2\mu_1}, \quad \widetilde{\lambda}_K = \frac{\lambda_K}{\mu_1}, \quad \widetilde{\mu}_K = \frac{\mu_K}{\mu_1}, \widetilde{\mu}_0 = \frac{\mu_0 H_{c1}^2}{\mu_1}, \quad \widetilde{\lambda}_{0K}^2 = \frac{\lambda_{0K}^2}{h^2},
\end{gathered}
\tag{7}
$$

where v_{TK} denotes the transverse elastic mode phase velocity in the layer and the substrate.

Recasting the set (6) dimensionless form using Equations (4), (5), and (7), we obtain

$$
\begin{gathered}
\widetilde{\mu}_K \frac{d^2u_z^K}{dx^2} + \frac{\Omega^2}{V^2}\left(V^2\widetilde{\rho}_K - \widetilde{\mu}_K\right)u_z^K + \widetilde{\mu}_0 H_0 \frac{dh_z^K}{dx} = 0, \\
\widetilde{\lambda}_{0K} \frac{d^2h_z^2}{dx^2} - \left(\widetilde{\lambda}_{0K}^2 \frac{\Omega^2}{V^2} + 1\right)h_z^K + H_0 \frac{du_z^K}{dx} = 0.
\end{gathered}
\tag{8}
$$

The boundary and jump conditions for the variables in (7) across the characteristic planes of the heterostructure are

$$
\text{at } x = -1: \quad \begin{cases} h_z^1 = 0 & \text{(continuity of the tangent component of the magnetic field)}, \\ u_{z,x}^1 = 0 & \text{(the plane is stress free)}, \end{cases}
$$

and

$$
\text{at } x = 0: \quad \begin{cases} [|h_z|] = h_z^1 - h_z^2 = 0, \\ [|u_z|] = u_z^1 - u_z^2 = 0 & \text{(continuity of displacements)}, \\ [|u_{z,x}|] = u_{z,x}^1 - u_{z,x}^2 = 0 & \text{(continuity of stress)}. \end{cases}
$$

The characteristic equation of (8) for both layer and substrate reads

$$
\lambda_{0K}^2 \mu_K p^4 + \left[\lambda_{0K}^2 B_K(\Omega, V) - F_K(\Omega, V)\mu_K - \mu_0 H_0^2\right]p^2 - F_K(\Omega, V)B_K(\Omega, V) = 0, \tag{9}
$$

where the solutions of (8) were assumed to be in the form

$$
\left\{u_z^K, h_z^K\right\} = \left\{{}^0u_z^K, {}^0h_z^K\right\}e^{px} \tag{10}
$$

and

$$
B_K(\Omega, V) = \frac{\Omega^2}{V^2}\left(V^2\widetilde{\rho}_K - \widetilde{\mu}_K\right), \quad F_K(\Omega, V) = \widetilde{\lambda}_{0K}^2 \frac{\Omega^2}{V^2} + 1 > 0.
$$

The waves under consideration propagate if the solutions of (10), u_z^1 and h_z^1, are convergent, that is, the squares of the roots p_1 and p_2 of the characteristic equation (9) in the layer are both real and p_3 and p_4 in the substrate are of opposite signs. To avoid divergence of solutions (10) in the substrate, we assume additionally that u_z^2 and h_z^2 vanish if $x \to \infty$. The requirements above for p_1–p_4 are satisfied, if for

$$
\begin{gathered}
p_1, p_2 : B_1(\Omega, V) < 0 \to V^2 < \tilde{\mu}_1/\tilde{\rho}_1, \\
p_3, p_4 : B_2(\Omega, V) > 0 \to V^2 > \tilde{\mu}_2/\tilde{\rho}_2.
\end{gathered}
$$

Hence we obtain a very important condition for Love's phase velocity wave if its propagation is possible

$$\tilde{\mu}_2/\tilde{\rho}_2 < V^2 < \tilde{\mu}_1/\tilde{\rho}_1 \quad \text{(dimensionless form)}$$

or

$$v_{T2} < v < v_{T1} \quad \text{(dimensional form).} \tag{11}$$

That is a new result and it differs from the classical result for the elastic Love's wave propagation condition which run along interface between two elastic materials (layer and substrate); see [Achenbach 1976]. For the latter case the inequality (11) is reciprocal.

As a result, the solutions (10) for the layer are, in detailed form,

$$u_z^1 = S_1 e^{p_1 x} + S_2 e^{-p_1 x} + S_3 e^{p_2 x} + S_4 e^{-p_2 x} \tag{12}$$

and

$$h_z^1 = -M(p_1, \Omega, V) S_1 e^{p_1 x} + M(p_1, \Omega, V) S_2 e^{-p_1 x} - M(p_2, \Omega, V) S_3 e^{p_2 x} - M(p_2, \Omega, V) S_4 e^{-p_2 x}, \tag{13}$$

where

$$M(p_i, \Omega, V) = \frac{p_i}{\tilde{\mu}_0 H_0} + \frac{\Omega^2 (V^2 - 1)}{V^2 H_0 p_i}, \quad i = 1, 2. \tag{14}$$

For the substrate the solutions are

$$u_z^2 = S_5 e^{-p_3 x}, \quad h_z^1 = N\big(p_3, \Omega, V\big) S_5 e^{-p_3 x}, \text{ where } N(p_3, \Omega, V) = \frac{\tilde{\mu}_2 p_3}{\tilde{\mu}_0 H_0} + \frac{\Omega^2 (V^2 \tilde{\rho}_2 - \tilde{\mu}_2)}{V^2 \tilde{\mu}_0 H_0 p_3}.$$

Now using solutions (12)–(15) for the boundary and jump conditions, we arrive at the homogeneous algebraic equations

$$W_{mn}(\Omega, V) S_n = 0, \quad m, n = 1, \ldots, 5. \tag{15}$$

Equation (15) has nontrivial solutions only if its determinant satisfies the relation below

$$\det W_{mn}(\Omega, V) = 0. \tag{16}$$

We have thus proved that Love's waves can propagate in a superconducting heterostructure and that their dispersion relation is given by (16).

3. Numerical results

The numerical analysis of the problem considered in the paper has been done for the superconducting heterostructure consisting of two ceramics, $YBa_2Cu_3O_{6+x}$ (YBCO) as the layer and $La_{1-x}Sr_xCuO_4$ as the half-space. All the necessary data are collected in Table 1. The results of using these data in the dispersion relation (16) are presented in Figures 2–3.

The first very important result from Equation (16) is that the waves considered are able to propagate only if the thickness of the layer satisfies

$$10^{-7} < h < 10^{-5}. \tag{17}$$

Then from Figures 2–3 it is seen that there are two frequency regions where waves are nondispersive. This means that they can be stably modulated in order to transmit signals carrying information. Between

Quantity	$YBa_2Cu_3O_{6+x}$	$La_{1-x}Sr_xCuO_4$	Unit
λ_0	$4\cdot 10^{-7}$	$2.5\cdot 10^{-7}$	m
ρ	10^{-6}	$5\cdot 10^{-6}$	kg/m^3
H_{c1}	$0.01/\mu_0$	$0.01/\mu_0$	A/m
H_{c2}	$120/\mu_0$	$120/\mu_0$	A/m
ξ	10^{-9}	$1.5\cdot 10^{-9}$	m
H_c	$H_{c2}\xi/(\lambda_0\sqrt{2})$	$H_{c2}\xi/(\lambda_0\sqrt{2})$	A/m
c_{11}	$\mu_0 H_1^{02}/4\pi$	$\mu_0 H_1^{02}/4\pi$	N/m^2
c_{66}	$(H_c^2/16\pi)(1-0.29b)(1-b)^2 b$	$(H_c^2/16\pi)(1-0.29b)(1-b)^2 b$	N/m^2
b	$\mu_0 H_1^0/H_{c2}$	$\mu_0 H_1^0/H_{c2}$	Vs/Am
μ	c_{66}	c_{66}	N/m^2
λ	$c_{11}-2c_{66}$	$c_{11}-2c_{66}$	N/m^2
μ_0	$4\pi\cdot 10^{-7}$	$4\pi\cdot 10^{-7}$	Vs/Am

Table 1. Data for the superconducting heterostructure.

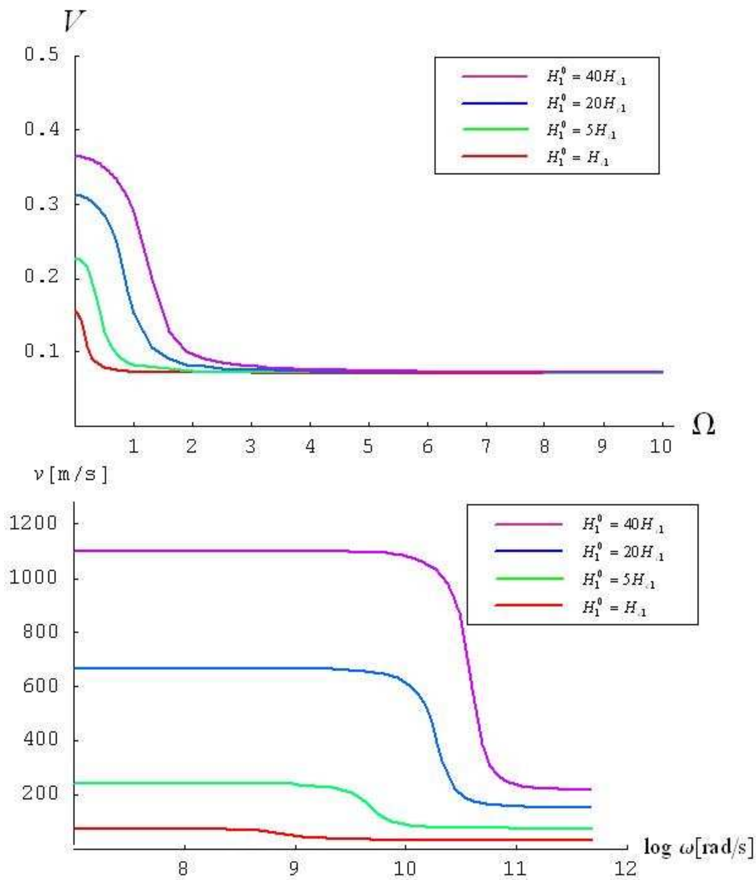

Figure 2. Dispersion for various intensities of applied magnetic field and fixed layer thickness $h = 10^{-7}$ m for the dimensionless (top) and dimensional (bottom) version.

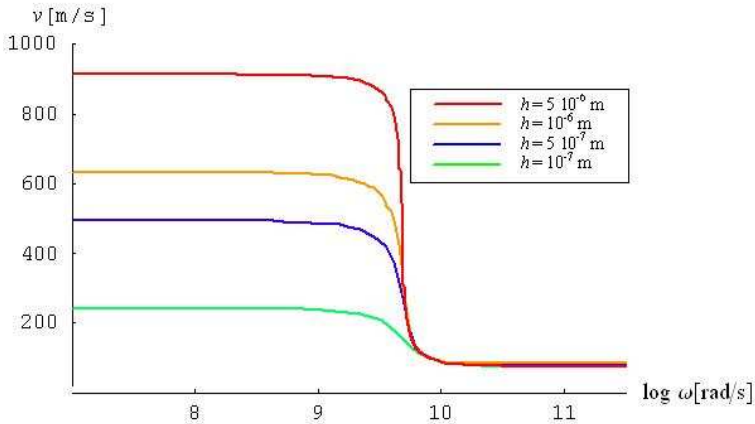

Figure 3. Dispersion for various layer thicknesses and fixed magnetic field intensities $H_1^0 = 20H_{c1}$.

those regions there is a forbidden interval (for frequencies)

$$10^8 < \omega < 10^{12}, \tag{18}$$

where strong dispersion of Love's wave is observed.

These properties are not typical if we compare them to those related to classical ones, concerning waves in a elastic material heterostructure.

4. Conclusions

(i) The paper proves that Love's waves can propagate within a vortex array existing in a superconducting heterostructure.

(ii) The anomalous range of the phase velocity of (11) indicates that in this case the layer should have a higher vortex density and the substrate should have a lower vortex density contrary to the classical elastic material case.

(iii) The thickness of the layer allowing wave propagation is limited; see Equation (17).

(iv) There are two dispersionless regions concerning Love's modes in the structure. The similar property has been observed in the case of bulk waves in the vortex array existing in the superconducting space [Drzewiecki et al. 2002a; 2004].

(v) There is a forbidden region where the dispersion is very high.

(vi) The waves under consideration propagate with an acoustic phase velocity and an optical wave frequency. This is another anomalous feature about them.

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