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PERFORMANCE AND PARAMETRIC STUDY OF ACTIVE MULTIPLE TUNED MASS DAMPERS FOR ASYMMETRIC STRUCTURES UNDER GROUND ACCELERATION

CHUNXIANG LI, JINHUA LI, ZHIQIANG YU AND YAN QU

The application of active multiple tuned mass dampers (AMTMD) for suppressing translational and torsional responses is addressed for a simplified two-degree of freedom structure, able to represent the dynamic characteristics of general asymmetric structures subject to ground motions. By employing the developed optimum parameter and effectiveness criteria of the AMTMD, the influences of the normalized eccentricity ratio and the torsional-to-translational frequency ratio of asymmetric structures on the optimum parameters and effectiveness of the AMTMD are investigated in detail. For comparison purposes, the results of a single active tuned mass damper also are taken into consideration.

A list of symbols can be found starting on page 584.

1. Introduction

Structural vibration control using passive, hybrid, semiactive, and active control strategies is a viable technology for enhancing structural functionality and safety against natural hazards such as strong earthquakes and high wind gusts. Significant strides have been made in recent years toward the development and application of hybrid, semiactive, and active control schemes for vibration control of civil engineering structures (including accounting for nonlinearity) in seismic zones. A multiobjective optimal design of a hybrid control system, consisting of a tuned mass damper (TMD) and an active mass driver (AMD), has been proposed for seismically excited structures by Ahlawat and Ramaswamy [2002a]. To achieve response reductions in smart base isolated buildings in near-fault earthquakes, a new semiactive independently variable damper has been developed by Nagarajaiah and Narasimhan [2007]. For active control systems, either an active tuned mass damper (ATMD) or an AMD can be installed on the top floor of a tall building to alleviate the acceleration response under wind excitations [Yang et al. 2004] or to attenuate the seismic response [Spencer et al. 1998a; 1998b]. In particular, structural control technology aimed at nonlinear structures has received considerable attention from researchers in recent years [Ohtori et al. 2004; Nagarajaiah and Narasimhan 2006; Nagarajaiah et al. 2008; Narasimhan et al. 2006; 2008]. Thus the significance of structural control for inelastic structures under strong earthquakes is well recognized.

The TMD is one of the simplest and most reliable control devices. It consists of a mass, a spring, and a viscous damper attached to the structure. Its mechanism for attenuating undesirable oscillations of a

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structure is to transfer the vibration energy of the structure to the TMD and dissipate it there through damping.

To increase the dissipation energy in the TMD, it is very important to determine its optimum parameters. Conversely, a main drawback of TMDs is that performance may worsen due to mistuned frequency or off-optimum damping. A possible remedy for this is to use more than one TMD with different dynamic characteristics. Multiple tuned mass dampers (MTMD) with distributed natural frequencies were proposed by Xu and Igusa [1992], and investigated by many authors [Yamaguchi and Harnpornchai 1993; Abe and Fujino 1994; Kareem and Kline 1995; Jangid 1999; Li 2000; 2006, Gu et al. 2001; Park and Reed 2001; Chen and Wu 2003; Bakre and Jangid 2004; Kwon and Park 2004; Yau and Yang 2004a; 2004b; Hoang and Warnitchai 2005; Li and Li 2005; Li and Zhang 2005; Lin et al. 2005; Han and Li 2006]. MTMDs have been shown to be more effective in the mitigation of oscillations than TMDs.

The effectiveness of TMDs can be further enhanced by introducing an active force to act between the structure and the TMD; this is the principle of the ATMD [Chang and Soong 1980]. However, neither the robustness against change nor the estimation error in the structural natural frequency of ATMDs can be compared with that of MTMDs. Investigations in optimizing the feedback gains and damper characteristics of ATMDs in order to minimize structural displacements and/or accelerations have been carried out, for example, in [Chang and Yang 1995; Ankireddi and Yang 1996; Yan et al. 1999]. However, as a building gets taller and more massive, in order to achieve the required level of response reduction during strong earthquakes or typhoons, a heavier additional mass is required, requiring too much space to be economically practical. With an active control system, a large control force must be created and the power limitations of the actuator prevent this system from being implemented in large buildings. Thus it is of great practical interest to search for control systems that can relax the requirements for masses and control forces.

In view of this, active multiple tuned mass dampers (AMTMD) have been proposed [Li and Liu 2002; Li et al. 2003] to attenuate undesirable oscillations of structures under ground acceleration. In studies on AMTMD, for design purposes it is assumed that a structure vibrates in only one direction or in multiple directions independently, each with its fundamental modal properties, without considering transverse-torsional coupled effects. This assumption simplifies the analysis of a system and the synthesis of a controller. In real structures, however, this assumption is not always appropriate because structures generally possess multidirectional coupled vibration modes and the control performance of controllers will degrade due to parameter variation or spillover induced by the effects of their coupling. Furthermore, there exist not only transverse vibrations but also torsional vibrations in real structures, which generally possess coupling. A real structure is asymmetric to some degree, even with a nominally symmetric plan, and will undergo lateral as well as torsional vibrations simultaneously under purely translational excitations. Consequently, controller design must take into account the effects of transverse-torsional coupled vibration modes in such cases.

For representative studies of TMD, ATMD, MTMD and HMD (hybrid mass damper) design, taking into account the effects of transverse-torsional coupled vibration modes, see [Jangid and Datta 1997; Arfiadi and Hadi 2000; Lin et al. 2000a; 2000b; Ahlawat and Ramaswamy 2002b; 2003; Singh et al. 2002; Pansare and Jangid 2003; Wang and Lin 2005; Li and Qu 2006]. It is well known that structures where the center of mass and center of resistance do not coincide will develop a coupled lateral-torsional response when subjected to earthquake ground motions. For practical applications, it would be important to include the effects of torsional coupling in consideration when estimating the performance of an AMTMD. Recently, Li and Xiong [2008] investigated AMTMD performance for asymmetric structures using a simplified two-degree of freedom (2-DOF) structure able to represent the dynamic characteristics of general asymmetric structures subject to ground motions. This structure is a generalized 2-DOF system of an asymmetric structure with predominant translational and torsional responses under earthquake excitations using the mode reduced-order method. Depending on the torsional-to-translational eigenfrequency ratio λ_ω of the asymmetric structure, three cases can be distinguished: torsionally flexible structures (TFS), when $\lambda_\omega < 1.0$; torsionally intermediate stiff structures (TISS), when $\lambda_\omega \approx 1.0$; and torsionally stiff structures (TSS), when $\lambda_\omega > 1.0$.

The search criterion for optimum parameters of an AMTMD is the minimization of the minimum values of the maximum translational and torsional displacement dynamic magnification factors (DMF) of an asymmetric structure with an AMTMD. The criterion used for assessing the effectiveness of an AMTMD is the ratio of the minimization of the minimum values of the maximum translational and torsional displacement DMF of the asymmetric structure with an AMTMD to the maximum translational and torsional displacement DMF of the asymmetric structure without an AMTMD. By employing these criteria, a careful examination of the effects of the normalized eccentricity ratio on the effectiveness and robustness of the AMTMD is carried out in the mitigation of both the translational and torsional responses of the asymmetric structure for different values of λ_ω. Likewise, the effectiveness of a single ATMD with optimum parameters is presented and compared with that of an AMTMD.

Following the direction of [Li and Xiong 2008], we investigate further the optimum performance of AMTMD in attenuating the translational and torsional responses of asymmetric structures under ground acceleration. The chosen optimization criteria are the (separate) minimizations of the translational and torsional displacement variances of the AMTMD-endowed asymmetric structure. The measure of effectiveness we adopt for the AMTMD is the ratio of the minimum translational or torsional displacement variance of AMTMD-endowed structure to the same variances for the structure without an AMTMD.

Using these evaluation criteria, we quantitatively discuss and demonstrate the influence of the normalized eccentricity ratio E_R the and the torsional-to-translational eigenfrequency ratio λ_ω of an asymmetric structure on the optimum parameters and effectiveness of AMTMDs in the reduction of both the translational and torsional responses of asymmetric structures under ground acceleration.

2. Damping of asymmetric structures

We take the structure to be controlled with an AMTMD to be asymmetric, in the sense that the center of resistance (CR) of the structure does not coincide with the center of mass (CM), as shown in Figure 1. The two uncoupled frequencies of the asymmetric structure are defined as

$$\omega_s = \sqrt{\frac{k_s}{m_s}}, \qquad \omega_\theta = \sqrt{\frac{k_\theta}{m_s r^2}}, \tag{1}$$

in which m_s is the mode-generalized mass of the structure; $k_s = k_{s1} + k_{s2}$ is the mode-generalized lateral stiffness of the structure in the x direction, where k_{s1} and k_{s2} refer to the stiffnesses of the two resisting elements; $k_\theta = k_{s1} y_{s1}^2 + k_{s2} y_{s2}^2$ represents the mode-generalized torsional stiffness of the structure with respect to the CM, where y_{s1} and y_{s2} denote the distances from the CM to the two resisting elements;

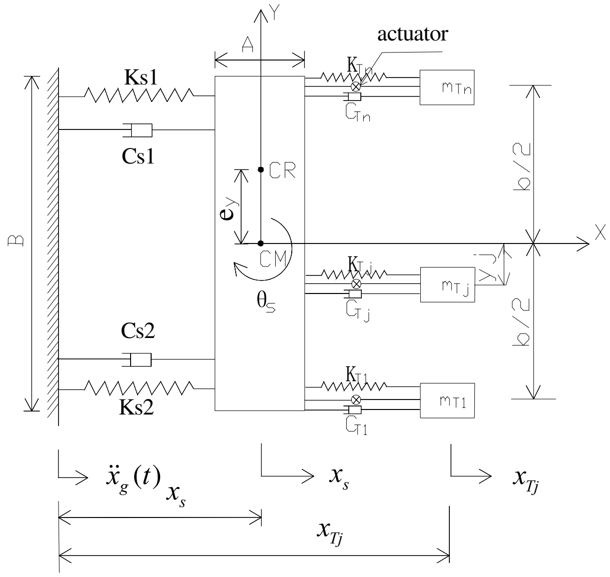

Figure 1. Generalized 2-DOF system of an asymmetric structure with the predominant translational and torsional responses set with the AMTMD.

and r represents the radius of gyration of the deck about the vertical axis through the CM. The equations of motion of the asymmetric structure can be written in matrix form as

$$\begin{bmatrix} m_s & 0 \\ 0 & m_s r^2 \end{bmatrix} \begin{bmatrix} \ddot{x}_s \\ \ddot{\theta}_s \end{bmatrix} + \begin{bmatrix} c_s & c_{s\theta} \\ c_{s\theta} & c_\theta \end{bmatrix} \begin{bmatrix} \dot{x}_s \\ \dot{\theta}_s \end{bmatrix} + \begin{bmatrix} k_s & k_s e_y \\ k_s e_y & k_\theta \end{bmatrix} \begin{bmatrix} x_s \\ \theta_s \end{bmatrix} = -\begin{bmatrix} m_s \\ 0 \end{bmatrix} \ddot{x}_g(t), \tag{2}$$

where c_s, $c_{s\theta}$, and c_θ denote the elements of the damping matrix, to be determined next; e_y is the eccentricity between the CR and CM, defined as $e_y = (k_{s1} y_{s1} - k_{s2} y_{s2})/(k_{s1} + k_{s2})$; and $\ddot{x}_g(t)$ is the ground acceleration.

We denote the fundamental and second natural frequencies by ω_{s1} and ω_{s2} (so $\omega_{s2} > \omega_{s1}$). They can be derived by solving the eigenvalue problem associated with (2) as

$$\frac{\omega_{s1}}{\omega_s} = \sqrt{\frac{1 + \lambda_\omega^2 - \sqrt{(\lambda_\omega^2 - 1)^2 + 4E_R^2}}{2}}, \qquad \frac{\omega_{s2}}{\omega_s} = \sqrt{\frac{1 + \lambda_\omega^2 + \sqrt{(\lambda_\omega^2 - 1)^2 + 4E_R^2}}{2}}, \tag{3}$$

in which $E_R = e_y/r$, or normalized eccentricity ratio (NER), is the ratio of the eccentricity to the radius of gyration of the deck, and $\lambda_\omega = \omega_\theta/\omega_s$, is the uncoupled torsional-to-translational frequency ratio (TTFR).

Hypothesizing the same damping ratio $\xi_{s1} = \xi_{s2} = \xi_s$ for the two modes (in this study, $\xi_s = 0.02$) and superposing the modal damping matrices, the damping matrix can be expressed in the form

$$\begin{bmatrix} c_s & c_{s\theta} \\ c_{s\theta} & c_\theta \end{bmatrix} = a_0 \begin{bmatrix} m_s & 0 \\ 0 & m_s r^2 \end{bmatrix} + b_0 \begin{bmatrix} k_s & k_s e_y \\ k_s e_y & k_\theta \end{bmatrix}, \tag{4}$$

where

$$a_0 = \frac{2(\xi_{s2}\omega_{s1} - \xi_{s1}\omega_{s2})}{\omega_{s1}^2 - \omega_{s2}^2}\,\omega_{s1}\omega_{s2}, \qquad b_0 = \frac{2(\xi_{s1}\omega_{s1} - \xi_{s2}\omega_{s2})}{\omega_{s1}^2 - \omega_{s2}^2}. \tag{5}$$

Rearranging (4) yields the elements of the damping matrix

$$c_s = 2a_s m_s \xi_s \omega_s, \qquad c_{s\theta} = 2a_{s\theta} m_s r \xi_s \omega_s, \qquad c_\theta = 2a_\theta m_s r^2 \xi_s \omega_s, \tag{6}$$

in which

$$a_s = \frac{\dfrac{\omega_{s1}}{\omega_s} \times \dfrac{\omega_{s2}}{\omega_s} + 1}{\dfrac{\omega_{s1}}{\omega_s} + \dfrac{\omega_{s2}}{\omega_s}}, \qquad a_{s\theta} = \frac{E_R}{\dfrac{\omega_{s1}}{\omega_s} + \dfrac{\omega_{s2}}{\omega_s}}, \qquad a_\theta = \frac{\dfrac{\omega_{s1}}{\omega_s} \times \dfrac{\omega_{s2}}{\omega_s} + \lambda_\omega^2}{\dfrac{\omega_{s1}}{\omega_s} + \dfrac{\omega_{s2}}{\omega_s}}. \tag{7}$$

3. State equations of the AMTMD asymmetric structure system

Referring again to Figure 1, consider an AMTMD evenly placed within the width b, with its center at the CM; we are interested in its effectiveness in reducing the translational and torsional responses of the asymmetric structure. The ordinate of each ATMD in the AMTMD can be determined by

$$y_j = \left(-\frac{1}{2} + \frac{j-1}{n-1}\right) b \qquad (j = 1, 2, \dots, n). \tag{8}$$

When the relative displacements of the structure (x_s) and of each ATMD (x_{Tj}) with reference to the ground are introduced, the equations of motion for the asymmetric structure with AMTMD under ground acceleration can be formulated as follows:

$$\begin{gathered} m_s\ddot{x}_s + c_s\dot{x}_s + k_s x_s + c_{s\theta}\dot{\theta}_s + k_s e_y \theta_s = -m_s \ddot{x}_g(t) + \sum_{j=1}^{n} F_j(t), \\ m_s r^2 \ddot{\theta}_s + c_\theta \dot{\theta}_s + k_\theta \theta_s + c_{s\theta}\dot{x}_s + k_s e_y x_s = \sum_{j=1}^{n} y_j F_j(t), \\ m_{Tj}(\ddot{x}_g(t) + \ddot{x}_{Tj}) + c_{Tj}\big(\dot{x}_{Tj} - (\dot{x}_s + y_j\dot{\theta}_s)\big) + k_{Tj}\big(x_{Tj} - (x_s + y_j\theta_s)\big) = u_j(t), \\ F_j(t) = c_{Tj}\big(\dot{x}_{Tj} - (\dot{x}_s + y_j\dot{\theta}_s)\big) + k_{Tj}\big(x_{Tj} - (x_s + y_j\theta_s)\big) - u_j(t). \end{gathered} \tag{9}$$

An active control algorithm is required in order to use the measured responses of the 2-DOF torsionally coupled structure-AMTMD system to calculate an active control force to drive the mass block. The linear quadratic regulator algorithm developed by several authors [Chang and Soong 1980; Abe 1996; Ikeda 1997; Nagashima 2001] may be employed, but here we choose instead the frequency domain design method adopted by some others [Chang and Yang 1995; Ankireddi and Yang 1996; Yan et al. 1999]. In this method one starts with the optimum MTMD configuration, obtained earlier. It is expected that the inertial force of the AMTMD can be increased by feeding back the acceleration of the structure. However, this acceleration feedback will disturb the performance of the optimum MTMD. An alternative approach to this problem is to move the optimum MTMD over to another optimum operating point and feed back the displacement and velocity of the MTMD. In view of this, the active control force can be explicitly expressed as

$$u_j(t) = -m_{tj}\ddot{x}_s - c_{tj}\big(\dot{x}_{Tj} - (\dot{x}_s + y_j\dot{\theta}_s)\big) + k_{tj}\big(x_{Tj} - (x_s + y_j\theta_s)\big), \tag{10}$$

in which m_{Tj}, c_{Tj}, and k_{Tj} are the mass, damping, and stiffness, respectively of the j-th ATMD; and m_{tj}, c_{tj}, and k_{tj} the gains of the acceleration, velocity, and displacement feedback of the j-th ATMD.

Here we make the stiffness and damping for all the ATMDs ($k_{Tj} = k_T$ and $c_{Tj} = c_T$ for each j) but let their masses m_{Tj} be different. Likewise, the control forces of the AMTMD are generated assuming the displacement and velocity feedback gains are all the same ($k_{tj} = k_t$ and $c_{tj} = c_t$ for each j) while the acceleration feedback gains m_{tj} are allowed to differ.

We now define the normalized acceleration feedback gain factor $\alpha_j = m_{tj}/m_{Tj} = \alpha$, assumed the same for each j, and we introduce the further notation

$$\omega_j^2 = \frac{k_{Tj} + k_{tj}}{m_{Tj}} = \frac{k_T + k_t}{m_{Tj}}, \qquad \mu_T = \frac{m_T}{m_s} = \sum_{j=1}^{n} \frac{m_{Tj}}{m_s} = \sum_{j=1}^{n} \mu_{Tj}, \qquad \xi_j = \frac{c_{Tj} + c_{tj}}{2m_{Tj}\omega_j} = \frac{c_T + c_t}{2m_{Tj}\omega_j},$$

Defining ω_T, the average natural frequency of the AMTMD, by $\omega_T = \sum_{j=1}^{n}(\omega_j/n)$, we choose the natural frequency of each ATMD as

$$\omega_j = \omega_T\left(1 + \left(j - \frac{n+1}{2}\right)\frac{\beta}{n-1}\right) = f\omega_s\left(1 + \left(j - \frac{n+1}{2}\right)\frac{\beta}{n-1}\right), \tag{11}$$

where $\beta = (\omega_n - \omega_1)/\omega_T$ is a nondimensional frequency spacing parameter and $f = \omega_T/\omega_s$ is the tuning frequency ratio of the AMTMD.

From (11), the ratio of the natural frequency of each ATMD to the controlled natural frequency of the structure is seen to be

$$r_j = \frac{\omega_j}{\omega_s} = f\left(1 + \left(j - \frac{n+1}{2}\right)\frac{\beta}{n-1}\right). \tag{12}$$

Since the damping ratios of the ATMDs are unequal, it is necessary to introduce the average damping ratio $\xi_T = \sum_{j=1}^{n} \xi_j/n$. The total mass ratio and the damping ratio of each ATMD is then

$$\mu_T = \sum_{j=1}^{n} \mu_{Tj} = \left(\sum_{j=1}^{n} \frac{1}{r_j^2}\right)\mu_{T1} r_1^2 = \left(\sum_{j=1}^{n} \frac{1}{r_j^2}\right)\mu_{T2} r_2^2 = \cdots = \left(\sum_{j=1}^{n} \frac{1}{r_j^2}\right)\mu_{Tn} r_n^2, \tag{13}$$

$$\xi_T = \sum_{j=1}^{n} \frac{\xi_j}{n} = \xi_1 r_1^{-1} f = \xi_2 r_2^{-1} f = \cdots = \xi_n r_n^{-1} f; \tag{14}$$

see [Li 2000; Li and Qu 2006]. For the deduction of the state equations, we rewrite (9) and (10) in matrix form as

$$M\ddot{x} + C\dot{x} + Kx = \Gamma \ddot{x}_g(t), \tag{15}$$

where we have set

$$x = \begin{bmatrix} x_s & r\theta_s & x_{T1} & \dots & x_{Tj} & \dots & x_{Tn} \end{bmatrix}^T, \qquad \dot{x} = \begin{bmatrix} \dot{x}_s & r\dot{\theta}_s & \dot{x}_{T1} & \dots & \dot{x}_{Tj} & \dots & \dot{x}_{Tn} \end{bmatrix}^T,$$

$$\ddot{x} = \begin{bmatrix} \ddot{x}_s & r\ddot{\theta}_s & \ddot{x}_{T1} & \dots & \ddot{x}_{Tj} & \dots & \ddot{x}_{Tn} \end{bmatrix}^T, \qquad \Gamma = \begin{bmatrix} -1 & 0 & -\mu_{T1} & \dots & -\mu_{Tj} & \dots & -\mu_{Tn} \end{bmatrix}^T,$$

$$M = \begin{bmatrix} A_1 & 0 & 0_{1\times n} \\ B_1 & B_2 & 0_{1\times n} \\ (C_1)_{n\times 1} & (C_2)_{n\times n} & 0_{n\times 1} \end{bmatrix}, \quad C = \begin{bmatrix} A_2 & A_3 & (A_4)_{1\times n} \\ B_3 & B_4 & (B_5)_{1\times n} \\ (C_3)_{n\times 1} & (C_4)_{n\times 1} & (C_5)_{n\times n} \end{bmatrix}, \quad K = \begin{bmatrix} A_5 & A_6 & (A_7)_{1\times n} \\ B_6 & B_7 & (B_8)_{1\times n} \\ (C_6)_{1\times n} & (C_7)_{n\times 1} & (C_8)_{n\times n} \end{bmatrix},$$

with

$$A_1 = 1 - \sum_{j=1}^{n} \alpha \mu_{Tj}, \qquad A_2 = 2\omega_s\left(\alpha_s\xi_s + \sum_{j=1}^{n}\mu_{Tj}\xi_j f\left(1+\left(j-\frac{n+1}{2}\right)\frac{\beta}{n-1}\right)\right),$$

$$A_3 = 2\omega_s\left(a_{s\theta}\xi_s + \sum_{j=1}^{n}\mu_{Tj}\xi_j f\left(\frac{y_j}{r}\right)\left(1+\left(j-\frac{n+1}{2}\right)\frac{\beta}{n-1}\right)\right),$$

$$A_4 = -2\omega_s\left[\mu_{T1}\xi_1 f\left(1-\frac{\beta}{2}\right)\ \ldots\ \mu_{Tj}\xi_j f\left(1+\left(j-\frac{n+1}{2}\right)\frac{\beta}{n-1}\right)\ \ldots\ \mu_{Tn}\xi_n f\left(1+\frac{\beta}{2}\right)\right],$$

$$B_1 = -\sum_{j=1}^{n}\alpha\mu_{Tj}\left(\frac{y_j}{r}\right), \qquad B_2 = 1, \qquad B_3 = A_3,$$

$$B_4 = 2\omega_s\left(a_\theta\xi_s + \sum_{j=1}^{n}\mu_{Tj}\xi_j f\left(\frac{y_j}{r}\right)^2\left(1+\left(j-\frac{n+1}{2}\right)\frac{\beta}{n-1}\right)\right),$$

$$B_5 = -2\omega_s\left[\mu_{T1}\xi_1 f\left(\frac{y_j}{r}\right)\left(1-\frac{\beta}{2}\right)\ \ldots\ \mu_{Tj}\xi_j f\left(\frac{y_j}{r}\right)\left(1+\left(j-\frac{n+1}{2}\right)\frac{\beta}{n-1}\right)\ \ldots\ \mu_{Tn}\xi_n f\left(\frac{y_j}{r}\right)\left(1+\frac{\beta}{2}\right)\right],$$

$$C_1 = \left[\alpha\mu_{T1}\ \ldots\ \alpha\mu_{Tj}\ \ldots\ \alpha\mu_{Tn}\right]^T,$$

$$C_2 = \mathrm{diag}\left[\mu_{T1}\ \ldots\ \mu_{Tj}\ \ldots\ \mu_{Tn}\right], \qquad C_3 = A_4^T, \qquad C_4 = B_5^T,$$

$$C_5 = 2\omega_s\,\mathrm{diag}\left[\mu_{T1}\xi_1 f\left(1-\frac{\beta}{2}\right)\ \ldots\ \mu_{Tj}\xi_j f\left(1+\left(j-\frac{n+1}{2}\right)\frac{\beta}{n-1}\right)\ \ldots\ \mu_{Tn}\xi_n f\left(1+\frac{\beta}{2}\right)\right],$$

$$A_5 = \omega_s^2\left(1+\sum_{j=1}^{n}\mu_{Tj} f^2\left(1+\left(j-\frac{n+1}{2}\right)\frac{\beta}{n-1}\right)^2\right),$$

$$A_6 = B_6 = \omega_s^2\left(E_R+\sum_{j=1}^{n}\mu_{Tj} f^2\left(\frac{y_j}{r}\right)\left(1+\left(j-\frac{n+1}{2}\right)\frac{\beta}{n-1}\right)^2\right),$$

$$A_7 = -\omega_s^2\left[\mu_{T1} f^2\left(1-\frac{\beta}{2}\right)^2\ \ldots\ \mu_{Tj} f^2\left(1+\left(j-\frac{n+1}{2}\right)\frac{\beta}{n-1}\right)^2\ \ldots\ \mu_{Tn} f^2\left(1+\frac{\beta}{2}\right)^2\right],$$

$$B_7 = \omega_s^2\left(\lambda_\omega^2+\sum_{j=1}^{n}\mu_{Tj} f^2\left(\frac{y_j}{r}\right)^2\left(1+\left(j-\frac{n+1}{2}\right)\frac{\beta}{n-1}\right)^2\right),$$

$$B_8 = -\omega_s^2\left[\mu_{T1} f^2\left(\frac{y_j}{r}\right)\left(1-\frac{\beta}{2}\right)^2\ \ldots\ \mu_{Tj} f^2\left(\frac{y_j}{r}\right)\left(1+\left(j-\frac{n+1}{2}\right)\frac{\beta}{n-1}\right)^2\ \ldots\ \mu_{Tn} f^2\left(\frac{y_j}{r}\right)\left(1+\frac{\beta}{2}\right)^2\right],$$

$$C_6 = A_7^T, \qquad C_7 = B_8^T,$$

$$C_8 = \omega_s^2\,\mathrm{diag}\left[\mu_{T1} f^2\left(1-\frac{\beta}{2}\right)^2\ \ldots\ \mu_{Tj} f^2\left(1+\left(j-\frac{n+1}{2}\right)\frac{\beta}{n-1}\right)^2\ \ldots\ \mu_{Tn} f^2\left(1+\frac{\beta}{2}\right)^2\right].$$

(Here of course $\mathrm{diag}[\cdots]$ stands for the $n\times n$ diagonal matrix with the given entries.)

Equation (15) can further be transformed into the state equations

$$\dot{X} = AX + Bw, \qquad Y = CX + Dw, \tag{16}$$

where the $2(n+2) \times 1$ state vector X, the state matrix A, the input matrix B, the output matrix C, the output vector Y, and the input w are as follows ($E_{(n+2)\times(n+2)}$ being the identity matrix of rank $n+2$):

$$X = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \quad A = \begin{bmatrix} 0_{(n+2)\times(n+2)} & E_{(n+2)\times(n+2)} \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0_{(n+2)\times 1} \\ M^{-1}\Gamma \end{bmatrix},$$

$$C = \begin{bmatrix} 0_{(n+2)\times(n+2)} & 0_{(n+2)\times(n+2)} \\ 0_{(n+2)\times(n+2)} & E_{(n+2)\times(n+2)} \end{bmatrix}, \quad D = \begin{bmatrix} 0_{(n+2)\times 1} \\ 0_{(n+2)\times 1} \end{bmatrix}, \quad Y = \begin{bmatrix} 0_{(n+2)\times 1} \\ x_{(n+2)\times 1} \end{bmatrix}, \quad w = \ddot{x}_g(t),$$

4. Optimum AMTMD criteria for asymmetric structures

We now assumed fixed the number n of ATMDs, the normalized width b/r and structural damping ratio ξ_s of the structure, the normalized eccentricity ratio E_R, the torsional-to-translational frequency ratio λ_ω, the normalized acceleration feedback gain factor α, and the total mass ratio μ_T of the AMTMD.

The optimization of the three parameters f, ξ_T, and β is done numerically by the gradient search method (GSM), an iterative method where the objective function and its partial derivatives are evaluated at each step. We now select appropriate objective functions for the optimization.

Based on a single-frequency oscillation $\ddot{x}_g(t) = X_g e^{-i\omega t}$, where X_g represents the displacement transfer function of earthquake ground motions, the translational and torsional displacements of the structure are given by

$$x_s = H_{x_s}(-i\omega)e^{-i\omega t} \quad \text{and} \quad r\theta_s = H_{\theta_s}(-i\omega)e^{-i\omega t},$$

leading to the translational and torsional displacement variances:

$$\sigma_{x_s}^2 = \int_{-\infty}^{+\infty} \left|H_{x_s}(i\omega)\right|^2 \left|X_g(i\omega)\right|^2 d\omega, \qquad \sigma_{r\theta_s}^2 = \int_{-\infty}^{+\infty} \left|H_\theta(i\omega)\right|^2 \left|X_g(i\omega)\right|^2 d\omega. \tag{17}$$

We can also define nondimensionalized variances

$$\sigma_{x_s}^{*\,2} \quad \text{and} \quad {\sigma_{r\theta_s}^{*}}^{2} \tag{18}$$

by dividing by the corresponding quantities for the same structure without an AMTMD.

These four variances are the objective functions to be minimized. We thus distinguish four minima:

$$R_I = \min_{f,\xi_T,\beta} \sigma_{x_s}^2, \quad R_{II} = \min_{f,\xi_T,\beta} \sigma_{r\theta_s}^2, \quad R_{III} = \min_{f,\xi_T,\beta} \sigma_{x_s}^{*\,2}, \quad R_{IV} = \min_{f,\xi_T,\beta} {\sigma_{r\theta_s}^{*}}^{2}. \tag{19}$$

We will primarily use R_{III} and R_{IV} as our quantitative estimates of the effectiveness of the AMTMD in controlling the translational and torsional displacements of asymmetric structures, respectively.

Note that the normalized acceleration feedback gain factor α gets smaller as n increases. The smaller α is, the less control force is required. Thus the AMTMD is more easily implemented than an ATMD of the same effectiveness, when a large control force must be created.

5. Numerical results

The minimization procedure was carried out with the following choices for the parameters: $n = 5$; $b/r = 1.0$; $\xi_s = 0.02$; $\alpha = 4$ or 8; $E_R = 0, 0.05, 0.1, 0.2, 0.3, 0.4$; and $\mu_T = 0.01, 0.02, 0.03, 0.04, 0.05$. Further we choose $\lambda_\omega = 0.5$, 1.0 and 2.0 to represent respectively TFS, TISS and TSS (see page 573). E_R and λ_ω are the key parameters in quantitatively assessing the effectiveness and robustness of the AMTMD for asymmetric structures.

Translational response. Figure 2 displays the optimum parameters and the R_{III} (translational) effectiveness of the AMTMD for a TFS ($\lambda_\omega = 0.5$), in terms of the values of α, E_R, and μ_T just listed. The figure suggests that the influence of E_R on the optimum tuning frequency ratio is not significant, regardless of the value of α. The effect of E_R on the optimum frequency spacing (robustness) of the AMTMD is also not remarkable, regardless of α. But, in the case of $\alpha = 4.0$, a pronounced difference in the optimum average damping ratio can be detected when the total mass ratio is below 0.03.

Comparison of the cases $\alpha = 4.0$ and $\alpha = 8.0$ shows that the interrelation between the effectiveness of the AMTMD and the total mass ratio has the same trend:

(1) For $E_R = 0.1$, the effectiveness of the AMTMD for asymmetric structures with $\lambda_\omega = 0.5$ is almost the same as for symmetric structures ($E_R = 0$).

(2) For $E_R > 0.1$, the effectiveness falls with increasing E_R, and drastically so when $E_R = 0.4$.

(3) Increasing α may enhance the effectiveness for asymmetric structures with $\lambda_\omega = 0.5$.

(4) As the total mass ratio increases, the effectiveness initially increases, then remains almost invariant when the total mass ratio is above 0.02.

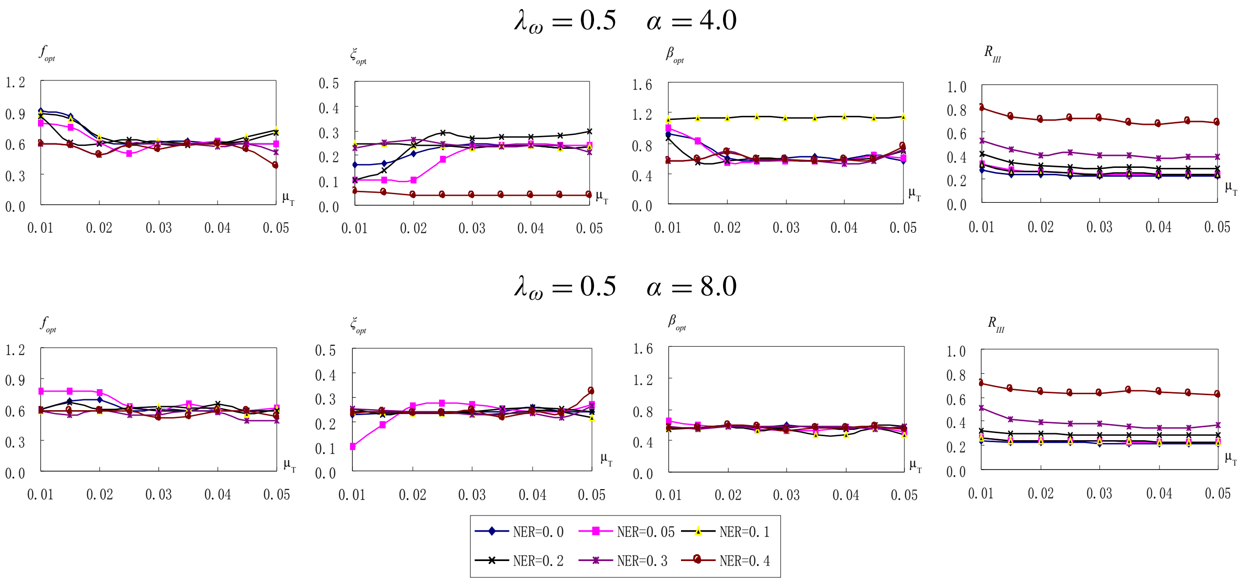

Figure 2. Optimum parameters f_{opt}, ξ_{opt}, β_{opt} and translational effectiveness R_{III} as functions of μ_T, E_R, and α, for a TFS ($\lambda_\omega = 0.5$). The total number n of AMTDs is 5 for all graphs up to Figure 7.

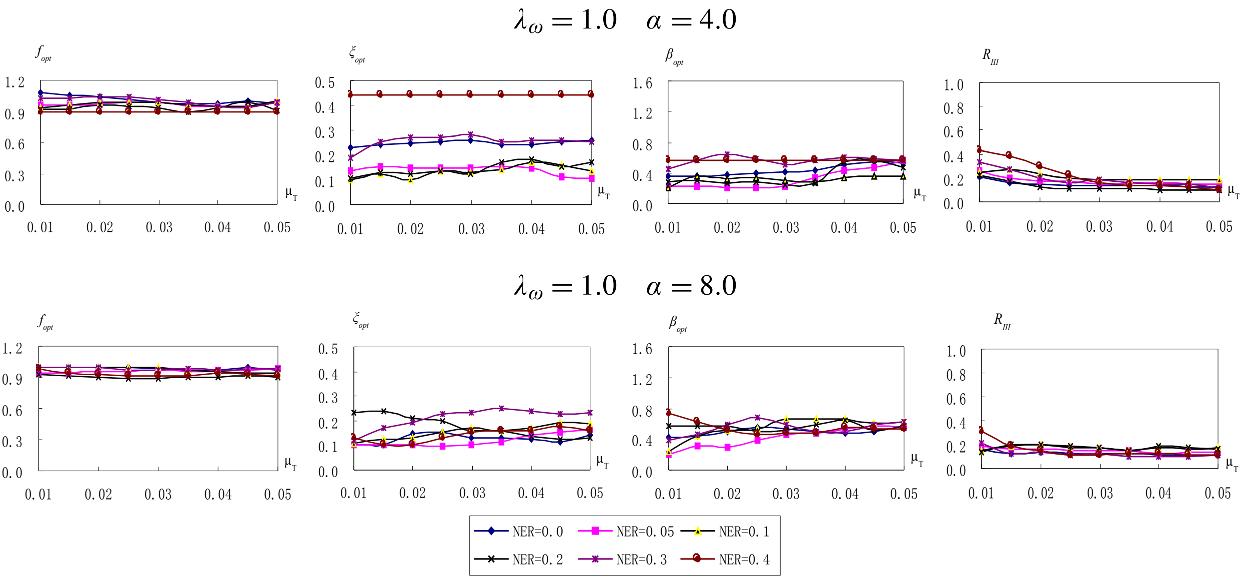

Figure 3. Optimum parameters f_{opt}, ξ_{opt}, β_{opt} and translational effectiveness R_{III} as functions of μ_T, E_R, and α, for a TISS ($\lambda_\omega = 1.0$).

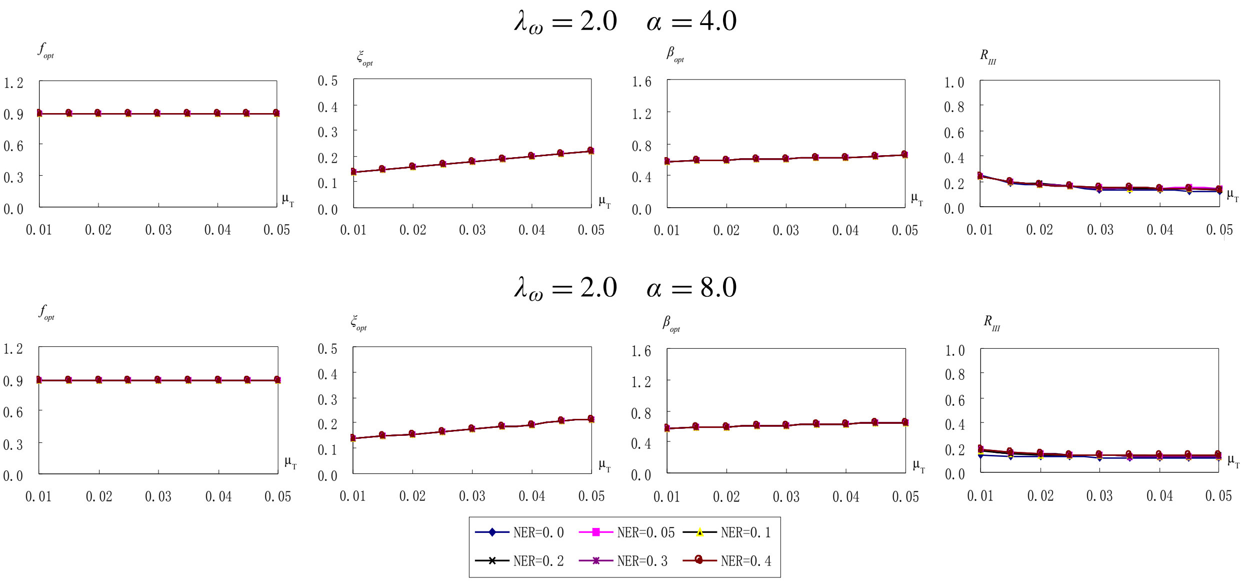

Figure 4. Optimum parameters f_{opt}, ξ_{opt}, β_{opt} and translational effectiveness R_{III} as functions of μ_T, E_R, and α, for a TSS ($\lambda_\omega = 2.0$).

Figure 3 shows the corresponding results for a TISS ($\lambda_\omega = 1.0$). We see that E_R has an influence on the optimum tuning frequency ratio and the optimum frequency spacing, but it is not great. The value of E_R has a significant effect on the optimum average damping ratio of the AMTMD, especially for the smaller value of α (4.0). The effectiveness of the AMTMD for the TFS and the TISS show similar trends, but it is less influenced by E_R in the TFS.

Figure 4 reports the corresponding results for a TSS ($\lambda_\omega = 2.0$). Here E_R seems to have very little influence on the optimum parameters and effectiveness.

Torsional response. Figure 5 shows the optimum parameters and the R_{IV} (rotational) effectiveness of the AMTMD for a TFS ($\lambda_\omega = 0.5$), for the same values of α, E_R, and μ_T as before (apart from $E_R = 0$).

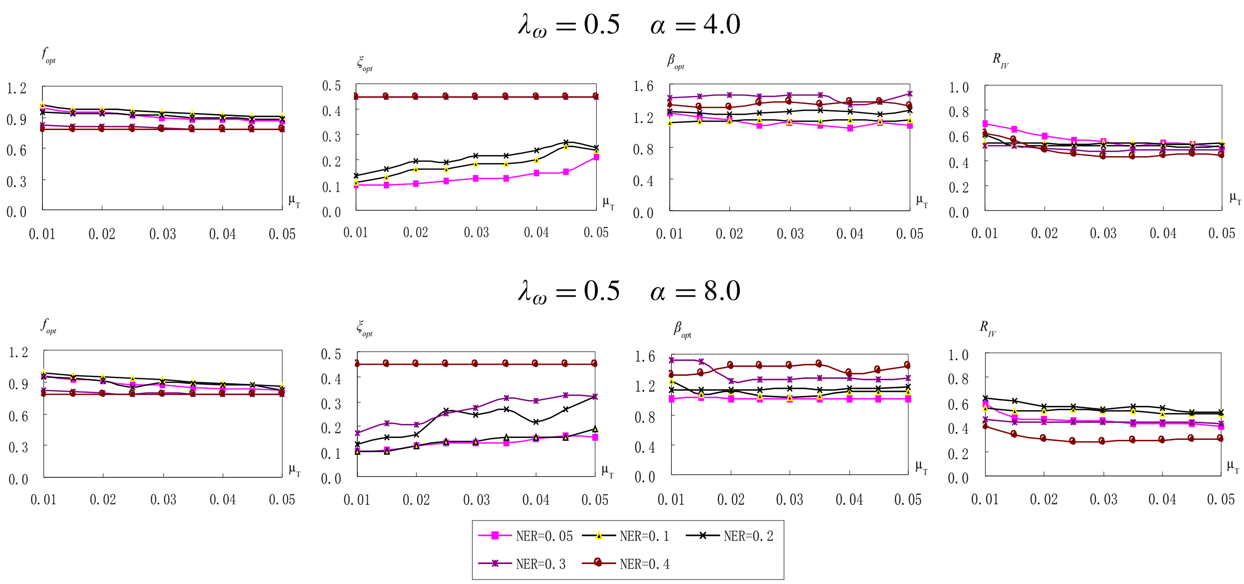

Figure 5. Optimum parameters f_{opt}, ξ_{opt}, β_{opt} and torsional effectiveness R_{IV} as functions of μ_T, E_R, and α, for a TFS ($\lambda_\omega = 0.5$).

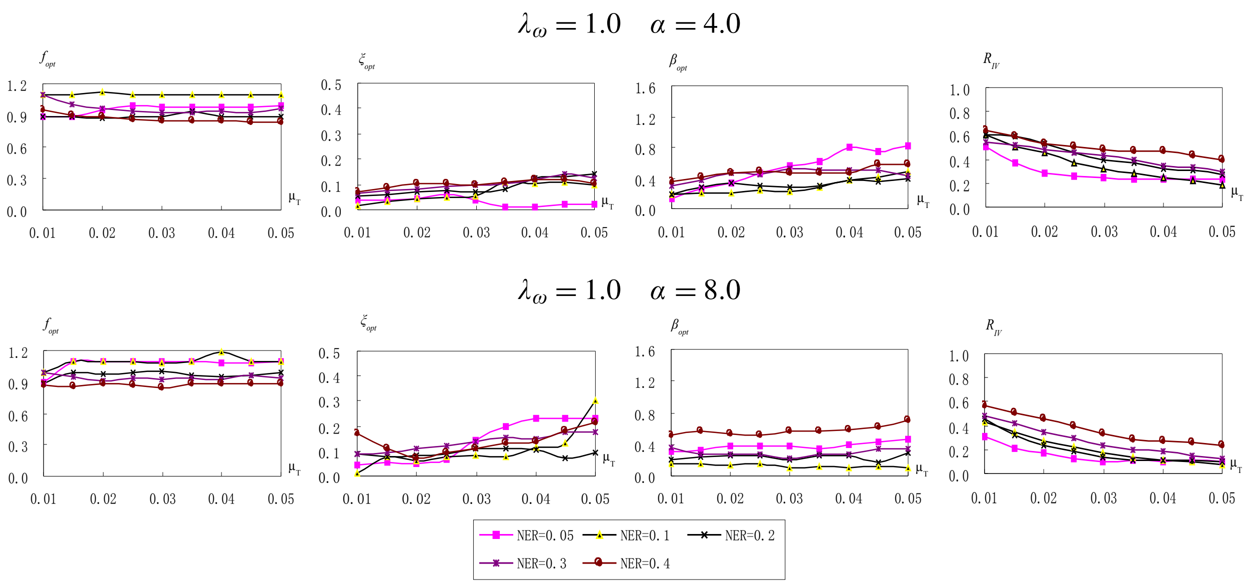

Figure 6. Optimum parameters f_{opt}, ξ_{opt}, β_{opt} and torsional effectiveness R_{IV} as functions of μ_T, E_R, and α, for a TISS ($\lambda_\omega = 1.0$).

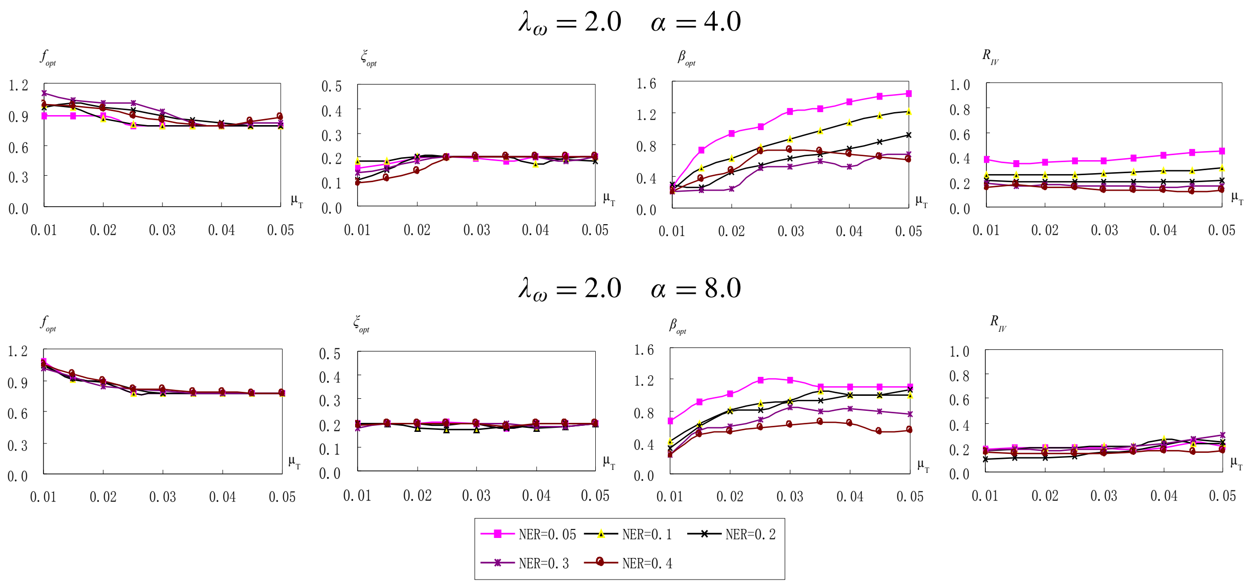

Figure 7. Optimum parameters f_{opt}, ξ_{opt}, β_{opt} and torsional effectiveness R_{IV} as functions of μ_T, E_R, and α, for a TSS ($\lambda_\omega = 2.0$).

It can be seen that the optimum frequency spacing β_{opt} generally increases somewhat with E_R. So does the optimal average damping ratio ξ_{opt}, albeit in an irregular fashion.

With the lower value of α (4.0), the effectiveness is somewhat higher than for $\alpha = 8.0$, and not very sensitive to E_R. With the higher α, the effectiveness varies irregularly with increasing E_R, decreasing while $E_R \leq 0.2$ and increasing again when $E_R \geq 0.3$.

Figure 6 shows the corresponding data for a TISS ($\lambda_\omega = 1$). In this case E_R significantly affects the optimum parameters and the effectiveness of the AMTMD. With the exception of the case of $E_R = 0.4$ and $\alpha = 8.0$, the optimum frequency spacing of the AMTMD tends to be greatest for $E_R = 0.05$. More importantly, the torsional effectiveness takes on an irregular pattern; it decreases with increasing E_R.

Figure 7 is the corresponding display for a TSS ($\lambda_\omega = 2.0$). The influence of E_R on the optimum tuning frequency ratio and average damping ratio is rather negligible here, especially in the case of $\alpha = 8.0$. However, E_R significantly affects the optimum frequency spacing; the two numbers move in opposite directions. The effectiveness decreases with increasing E_R for $\alpha = 4.0$, but is very little affected by E_R for $\alpha = 8.0$.

Comparison with a single ATMD. Figure 8 presents the effectivenesses R_{III} and R_{IV} of a single ATMD over the same ranges of μ_T, λ_ω, α, and E_R that we used for the AMTMD calculations. By comparing Figure 8 with the corresponding graphs in Figures 2–7, one sees that, in comparison with a single ATMD, the AMTMD is slightly more effective in reducing the translational response of asymmetric structures, but slightly less effective in controlling the tosional response of asymmetric structures,

6. Conclusions

The following major conclusions can be drawn:

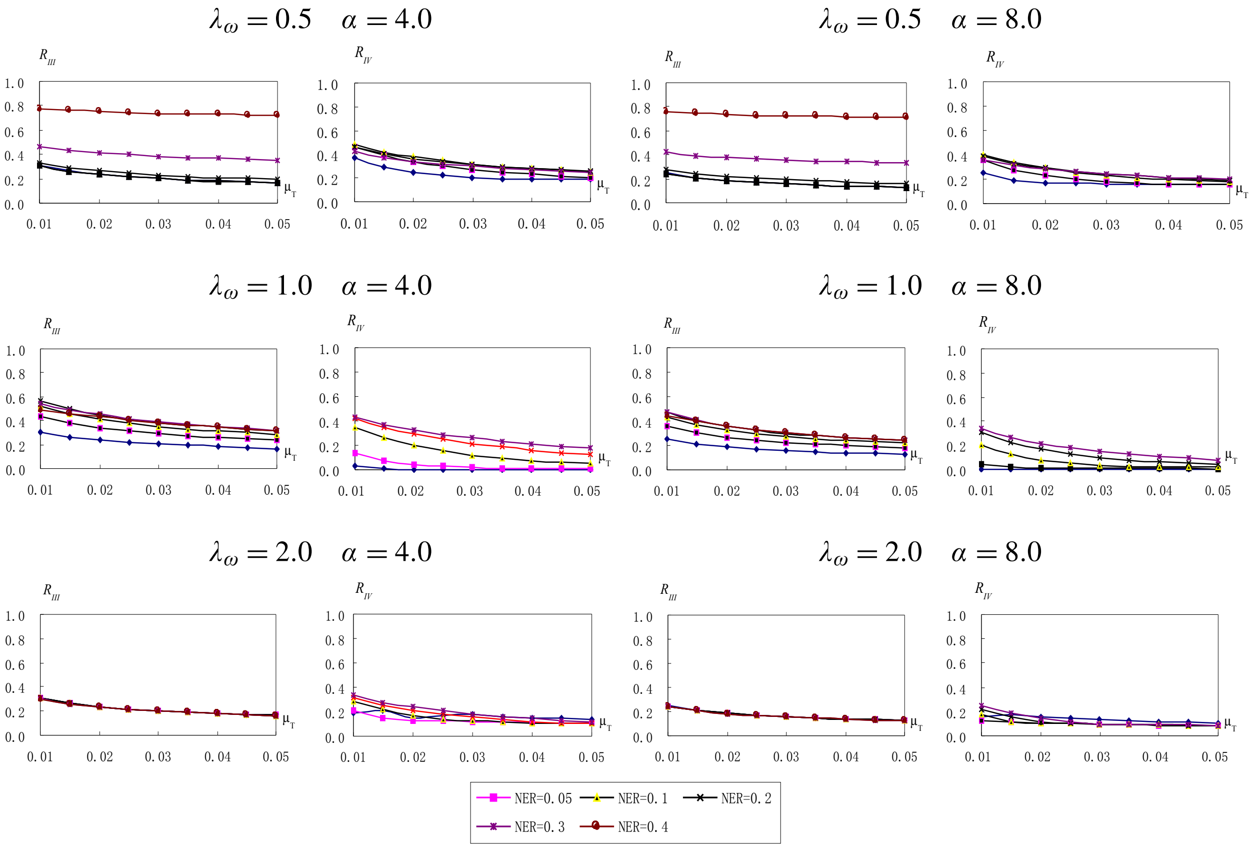

Figure 8. Translational effectiveness R_{III} (first and third columns) and torsional effectiveness R_{IV} (second and fourth columns) as functions of μ_T, E_R, and α for a single ATMD.

(1) In attenuating the translational response of TFS ($\lambda_\omega = 0.5$), the effect of E_R on the optimum frequency spacing (robustness) of the AMTMD is not significant; the effectiveness of the AMTMD for asymmetric structures with E_R smaller than 0.1 is practically equal to that for symmetric structures ($E_R = 0$) and reduces as E_R increases above 0.1.

(2) In attenuating the torsional response of TFS, the optimum frequency spacing of the AMTMD generally increases with increasing E_R. With smaller α, the effectiveness of the AMTMD improves with increasing E_R, though this is not very obvious. With higher α, the effectiveness of the AMTMD decreases with increasing E_R when $E_R \leq 0.2$, and increases with increasing E_R when $E_R \geq 0.3$.

(3) In controlling the translational response of TISS ($\lambda_\omega = 1.0$), E_R has influence on the optimum frequency spacing of the AMTMD, but it is not that obvious; the effectiveness of the AMTMD for TISS has similar trends as that for TFS.

(4) In controlling the torsional response of TISS, the optimum frequency spacing and effectiveness of the AMTMD generally decreases with increasing E_R.

(5) In mitigating the translational response of TSS, the AMTMD will take on the same optimum parameters, robustness, and effectiveness, regardless of different E_R values.

(6) In reducing the torsional response of TSS, the optimum frequency spacing of the AMTMD generally decreases with increasing E_R. The effectiveness of the AMTMD with a smaller α, such as $\alpha = 4.0$, decreases as E_R increases. The influence of E_R on the effectiveness of the AMTMD with higher α, such as $\alpha = 8.0$, is rather negligible.

(7) The AMTMD provides slightly higher effectiveness than a single ATMD in reducing the translational response of asymmetric structures.

(8) The AMTMD are slightly less effective than a single ATMD in controlling the tosional response of asymmetric structures.

We point out that by employing the present approach, which falls into the frequency domain design method of control, Li et al. [2007] have numerically investigated the earthquake resistant performance of AMTMD for asymmetric buildings, so as to further validate the effectiveness and robustness of AMTMD in reducing the translational and torsional responses of asymmetric buildings in the time domain. The SIMULINK analysis has been implemented on a three-story asymmetric steel structure building under various earthquakes, taking into account both the certainty and uncertainty in the structural stiffness. The numerical simulations indicate that AMTMD can effectively control the translational and torsional responses of asymmetric buildings subjected to earthquakes. Likewise, AMTMD generally have better performance than an ATMD for seismically excited asymmetric buildings. More recently, in order to further validate the control force decentralization of AMTMD for the control of wind-induced vibrations of tall buildings in the time-domain, a 22-story steel-frame building is chosen as an example problem [Li et al. 2009]. The numerical results in the time-domain indicate that a large control force can indeed be decentralized into many smaller control forces when using AMTMD. Simultaneously, the effectiveness of AMTMD is a little larger than that of an ATMD based on the DRF and ARF criteria [Li et al. 2009].

List of symbols

AMTMD	active multiple tuned mass dampers
α	normalized acceleration feedback gain factor (NAFGF)
b/r	normalized width of an asymmetric structure, here set equal to 1.0
β	nondimensional frequency spacing
CM	center of mass
CR	center of resistance
c_s	mode-generalized damping coefficient
c_T	constant damping coefficient of the AMTMD
c_{Tj}	damping coefficient of the j-th ATMD in the AMTMD
c_{tj}	velocity feedback of the j-th ATMD in the AMTMD
c_t	constant velocity feedback of the j-th ATMD in the AMTMD
E_R	normalized eccentricity ratio (ratio between eccentricity and gyration radius of the deck)
e_y	eccentricity between the CR and CM
f	tuning frequency ratio of the AMTMD

$H_{x_s}(-i\omega)$	transfer function for translational displacement
$H_{\theta_s}(-i\omega)$	transfer function for torsional displacement
j	number of ATMDs in the AMTMD
k_s	mode-generalized lateral stiffness of an asymmetric structure in the translational x direction
k_T	constant spring stiffness of the AMTMD
k_{Tj}	spring stiffness of the j-th ATMD in the AMTMD
k_t	constant displacement feedback of the j-th ATMD in the AMTMD
k_{tj}	displacement feedback of the j-th ATMD in the AMTMD
k_θ	mode-generalized torsional stiffness of an asymmetric structure with respect to the CM
λ_ω	uncoupled torsional-to-translational frequency ratio (TTFR)
MTMD	multiple tuned mass dampers
m_s	mode-generalized mass of an asymmetric structure
m_{Tj}	mass of the j-th ATMD in the AMTMD
m_{tj}	acceleration feedback of the j-th ATMD in the AMTMD
μ_{Tj}	mass ratio of the j-th ATMD in the AMTMD
μ_T	total mass ratio of the AMTMD
n	total number of ATMDs in the AMTMD
R_I	minimum of translational displacement variance over f, ξ_T, β
R_{II}	minimum of torsional displacement variance over f, ξ_T, β
R_{III}	effectiveness of the AMTMD in attenuating the structure's translational response; see (19)
R_{IV}	effectiveness of the AMTMD in attenuating the structure's torsional response; see (19)
r	radius of gyration of the deck about the vertical axis through the CM
r_j	ratio of the natural frequency of the j-th ATMD to the uncoupled translational natural frequency of an asymmetric structure
θ_s	torsional displacement of an asymmetric structure
$\ddot{x}_g(t)$	ground acceleration
x_s	translational displacement of an asymmetric structure with respect to the ground
x_{Tj}	translational displacement of each ATMD with reference to the ground
ξ_j	damping ratio of the j-th ATMD in the AMTMD
ξ_s	structural damping ratio, which is set equal to 0.02 in this study
ξ_T	average damping ratio of the AMTMD
y_j	translational displacement of each ATMD with reference to the ground
$y_{(n+1)/2}$	center of the AMTMD, placement of the $(n+1)/2$-th ATMD in the AMTMD
ω	external excitation frequency
ω_j	natural frequency of the j-th ATMD in the AMTMD
ω_s	uncoupled translational natural frequency of an asymmetric structure
ω_{s1}	coupled fundamental natural frequency of an asymmetric structure
ω_{s2}	coupled second natural frequency of an asymmetric structure
ω_θ	uncoupled torsional natural frequency of an asymmetric structure
ω_T	average natural frequency of the AMTMD

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