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A TWO-TEMPERATURE GENERALIZED THERMOELASTIC MEDIUM SUBJECTED TO A MOVING HEAT SOURCE AND RAMP-TYPE HEATING: A STATE-SPACE APPROACH

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We construct a model of two-temperature generalized thermoelasticity for an elastic half-space with constant elastic parameters. The Laplace transform and state-space techniques are used to obtain the general solution for any set of boundary conditions. The general solution obtained is applied to the specific problem of a half-space subjected to a moving heat source with constant velocity and ramp-type heating. The inverse Laplace transforms are computed numerically. The effects of different values of the heat source velocity, the two-temperature parameter, and the ramping time parameter are compared.

A list of symbols can be found on page 1648.

1. Introduction

P. J. Chen and collaborators [Chen and Gurtin 1968; Chen and Williams 1968; Chen et al. 1969] formulated a theory of heat conduction in deformable bodies, which depends upon two temperatures: the conductive temperature φ and the dynamical temperature T. For time-independent situations, the difference between these two temperatures is proportional to the heat supply. In the absence of any heat supply, the two temperatures are identical [Chen and Gurtin 1968]. For time-dependent problems, however, and for wave propagation problems in particular, the two temperatures can be different regardless of the presence of a heat supply. The two temperatures, T and φ, and the strain are found to have representations in the form of a traveling wave plus a response, which occurs instantaneously throughout the body [Boley and Tolins 1962].

Warren and Chen [1973] investigated the wave propagation in the two-temperature theory of thermoelasticity. In [Youssef 2006b] we investigated this theory in the context of the generalized theory of thermoelasticity.

In most earlier studies, mechanical or thermal loading on the bounding surface is considered to be in the form of a shock. However, the sudden jump in the load is merely an idealized situation, because it is impossible to realize a pulse described mathematically by a step function; even a very rapid rise time (on the order of 10^{-9} s) may be slow in terms of the continuum. This is particularly true in the case of second sound effects when the thermal relaxation times for typical metals are less than 10^{-9} s. It is thus felt that a finite rise time of the external load (mechanical or thermal) applied on the surface should be considered while studying a practical problem of this nature. Considering this aspect of rise time, Misra et al. [1991a; 1991b; 1992] solved some problems involving ramp-type heating. In [Youssef 2005] we used the state-space approach to solve the generalized thermoelasticity problem of an infinite material with a spherical

Keywords: generalized thermoelasticity, two-temperature, heat source, ramp type.

cavity and variable thermal conductivity subjected to ramp-type heating. Later we found the solutions of the problem of a generalized thermoelastic infinite medium with a cylindrical cavity subjected to a ramp-type heating and loading [2006a] and the two-dimensional generalized thermoelasticity problem for a half-space subjected to ramp-type heating [2006c]. In [Youssef and Al-Lehaibi 2007] we used the state-space approach in the problem of two-temperature generalized thermoelasticity while in [Bassiouny and Youssef 2008] we solved the two-temperature generalized thermopiezoelasticity problem of a finite rod subjected to different types of thermal loading. In [Youssef 2008] solved the two-dimensional problem of a two-temperature generalized thermoelastic half-space subjected to ramp-type heating. Al-Huniti et al. [2001] discussed the dynamic response of a rod due to a moving heat source under the hyperbolic heat conduction model.

Here we consider a half-space filled with an elastic material with constant elastic parameters. The governing equations are written in the context of two-temperature generalized thermoelasticity theory. A moving heat source with constant velocity is applied to the medium. Laplace transforms and state-space techniques are used to obtain the general solution for any set of boundary conditions. The general solution obtained is applied to a half-space subjected to ramp-type heating with a traction-free bounding plane. The inverse Laplace transforms are computed numerically using the Riemann sum approximation method. The effects of the heat source velocity, the two-temperature parameter, and the ramping time parameter are estimated.

1.1. ***Formulation of the problem.*** According to our model, the heat conduction equation takes the form [Youssef 2006b]

$$K\varphi,_{ii} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)(\rho C_E \theta + \gamma T_0 e) - \left(1 + \tau_0 \frac{\partial}{\partial t}\right) Q, \qquad i = 1, 2, 3. \tag{1}$$

The constitutive equations take the form

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e_{kk}\delta_{ij} - \gamma\theta\delta_{ij}, \qquad i = 1, 2, 3, \tag{2}$$

where δ_{ij} is the Kronecker delta function

The equations of motion without body forces take the form

$$\sigma_{ij},_j = \rho \ddot{u}_i, \qquad i = 1, 2, 3. \tag{3}$$

The relation between the heat conduction and the thermodynamic heat takes the form

$$\varphi - T = a\varphi,_{ii}, \qquad i = 1, 2, 3, \tag{4}$$

where a is a nonnegative parameter called the two-temperature parameter [Youssef 2006b].

Now, we will suppose an elastic and homogeneous half-space $x \geq 0$ which obeys Equations (1)–(4) and is initially quiescent, where all the state functions depend only on the dimension x and the time t.

The displacement components for a one-dimensional medium have the form

$$u_x = u(x, t), \qquad u_y = u_z = 0. \tag{5}$$

The strain component takes the form

$$e = e_{xx} = \frac{\partial u}{\partial x}. \tag{6}$$

The heat conduction equation takes the form

$$K\frac{\partial^2\varphi}{\partial x^2}=\Big(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\Big)(\rho C_E T+\gamma T_0 e)-\Big(1+\tau_0\frac{\partial}{\partial t}\Big)Q. \tag{7}$$

The constitutive equation takes the form

$$\sigma_{xx}=\sigma=(2\mu+\lambda)e-\gamma(T-T_0). \tag{8}$$

The equation of motion takes the form

$$\frac{\partial^2\sigma}{\partial x^2}=\rho\frac{\partial^2 e}{\partial t^2}. \tag{9}$$

The relation between the heat conduction and the thermodynamic heat takes the form

$$\varphi-T=a\frac{\partial^2\varphi}{\partial x^2}. \tag{10}$$

For simplicity, we will use the nondimensional variables

$$x\leftarrow c_0\eta x,\quad \iota\leftarrow c_0^2\eta\iota,\quad \tau_0\leftarrow c_0^2\eta\tau_0,\quad \theta\leftarrow\frac{\theta}{T_0},\quad \varphi\leftarrow\frac{\varphi}{T_0},\quad \sigma\leftarrow\frac{\sigma}{2\mu+\lambda},\quad Q\leftarrow\frac{Q}{K_0c_0^2\eta^2T_0},$$

where

$$c_0^2=\frac{2\mu+\lambda}{\rho},\qquad \eta=\frac{\rho C_E}{K}.$$

Hence, we have the system of equations

$$\frac{\partial^2\varphi}{\partial x^2}=\Big(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\Big)(\theta+\varepsilon e)-\Big(1+\tau_0\frac{\partial}{\partial t}\Big)Q,\quad \sigma=e-b\theta,\quad \frac{\partial^2\sigma}{\partial x^2}=\frac{\partial^2 e}{\partial t^2},\quad \varphi-\theta=\beta\frac{\partial^2\varphi}{\partial x^2}, \tag{11}$$

where

$$\varepsilon=\frac{\gamma}{\rho C_E},\qquad b=\frac{\gamma T_0}{\lambda+2\mu},\qquad \beta=ac_0^2\eta^2.$$

Applying the Laplace transform

$$\bar f(s)=\int_0^\infty f(t)e^{-st}dt$$

to the equations in (11), we obtain

$$\frac{d^2\bar\varphi}{dx^2}=(s+\tau_0 s^2)\bar\theta+(s+\tau_0 s^2)\varepsilon\bar e-(1+\tau_0 s)\bar Q. \tag{12}$$

We consider that the medium is subjected to a moving heat source of constant strength releasing its energy continuously while moving along the x-axis in the positive direction with a constant velocity υ. This moving heat source is assumed to be of the nondimensional form [Al-Huniti et al. 2001]

$$Q=Q_0\delta(x-\upsilon t), \tag{13}$$

where Q_0 is the constant heat source strength and δ is the delta function.

After using a Laplace transformation, we get

$$\bar Q=\ell\exp\Big(\frac{s}{\upsilon}x\Big),\qquad \ell=\frac{Q_0}{\upsilon}. \tag{14}$$

To simplify the notation we set $h = \dfrac{s}{\upsilon}$. Then we have

$$\frac{d^2\bar{\varphi}}{dx^2} = (s+\tau_0 s^2)\bar{\theta} + (s+\tau_0 s^2)\varepsilon\bar{e} - (1+\tau_0 s)\ell e^{-hx}, \tag{15}$$

$$\bar{\sigma} = \bar{e} - b\bar{\theta}, \qquad \frac{d^2\bar{\sigma}}{dx^2} = s^2\bar{e}, \qquad \bar{\theta} = \bar{\varphi} - \beta\frac{d^2\bar{\varphi}}{dx^2}, \tag{16}$$

where all the initial state functions are equal to zero.

Eliminating $\bar{e}$ and $\bar{\theta}$ from these equations, we obtain

$$\frac{d^2\bar{\varphi}}{dx^2} = (1+\varepsilon b)s\alpha_1\bar{\varphi} + s\varepsilon\alpha_1\bar{\sigma} - \ell\alpha_1 e^{-hx}, \qquad \text{where } \alpha_1 = \frac{1+\tau_0 s}{1+\beta(s+\tau_0 s^2)(1+b\varepsilon)}, \tag{17}$$

and

$$\frac{d^2\bar{\sigma}}{dx^2} = \alpha_2\bar{\sigma} + \alpha_3\bar{\varphi} + \alpha_4\ell e^{-hx}, \tag{18}$$

where

$$\alpha_2 = s^2(1-\beta\varepsilon s b\alpha_1), \qquad \alpha_3 = s^2 b\big(1-\beta s\alpha_1(1+b\varepsilon)\big), \qquad \alpha_4 = s^2 b\beta\alpha_1.$$

Then, we have

$$\bar{\theta} = \big(1-\beta s\alpha_1(1+b\varepsilon)\big)\bar{\varphi} - \beta\varepsilon s\alpha_1\bar{\sigma} + \beta\alpha_1\ell e^{-hx}. \tag{19}$$

2. State-space approach

Choosing as state variables the temperature of heat conduction $\bar{\varphi}$ and the stress component $\bar{\sigma}$ in the x-direction, equations (18) and (19) can be written in matrix form as

$$\frac{d^2\overline{V}(x,s)}{dx^2} = A(s)\overline{V}(x,s) + F(s)e^{-hx}, \tag{20}$$

where

$$\overline{V}(x,s) = \begin{bmatrix}\bar{\varphi}(x,s)\\ \bar{\sigma}(x,s)\end{bmatrix}, \qquad A(s) = \begin{bmatrix} s(1+b\varepsilon)\alpha_1 & s\varepsilon\alpha_1 \\ \alpha_3 & \alpha_2\end{bmatrix}, \qquad F(s) = \begin{bmatrix}-\ell\alpha_1\\ \ell\alpha_4\end{bmatrix}.$$

Solutions of (20) that remain bounded for large x (that is, not involving diverging exponentials) can be written as

$$\overline{V}(x,s) = \exp\big(-\sqrt{A(s)}\,x\big)C(s) + D(s)e^{-hx}, \tag{21}$$

where $C(s) = \begin{bmatrix}C_1(s)\\ C_2(s)\end{bmatrix}$ is to be determined, and $D(s) = \begin{bmatrix}D_1\\ D_2\end{bmatrix} = \big(h^2 I - A(s)\big)^{-1}F(s)$, with $I = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$.

We will use the Cayley–Hamilton theorem to find the matrix $\exp\big(-\sqrt{A(s)}\,x\big)$. The characteristic equation of $A(s)$ is

$$k^2 - k\big(s(1+b\varepsilon)\alpha_1 + \alpha_2\big) + \alpha_1 s\big((1+b\varepsilon)\alpha_2 - \varepsilon\alpha_3\big) = 0; \tag{22}$$

that is, the characteristic roots k_1 and k_2 satisfy

$$k_1 + k_2 = s(1+b\varepsilon)\alpha_1 + \alpha_2, \qquad k_1k_2 = s(1+b\varepsilon)\alpha_1\alpha_2 - s\varepsilon\alpha_1\alpha_3. \tag{23}$$

Next we write the spectral decomposition of $A(s)$ in terms of the projectors E_1 and E_2 of $A(s)$ (see [Cullen 1972] for details):

$$A(s) = k_1 E_1 + k_2 E_2 \tag{24}$$

By definition, the projectors satisfy $E_1 + E_2 = I$, $E_1 E_2 = E_2 E_1 = 0$, and $E_i^2 = E_i$ for $i = 1, 2$. Thus

$$AE_1 = k_1 E_1^2 + k_2 E_2 E_1 = k_1 E_1. \tag{25}$$

Similarly, $AE_2 = k_2 E_2$, so $A(I - E_1) = k_2 I - k_2 E_1$. Adding this latter equation to (25) we obtain $A = k_2 I + (k_1 - k_2)E_1$, which is to say, $E_1 = (A - k_2 I)/(k_1 - k_2)$. Taking into account (23) to achieve simplifications, and following a similar reasoning for E_2, we reach the explicit form of the projectors:

$$E_1 = \frac{1}{k_1 - k_2}\begin{bmatrix} k_1 - \alpha_2 & s\varepsilon\alpha_1 \\ \dfrac{(\alpha_2 - k_2)(k_1 - \alpha_2)}{s\varepsilon\alpha_1} & \alpha_2 - k_2 \end{bmatrix}, \qquad E_2 = \frac{1}{k_1 - k_2}\begin{bmatrix} \alpha_2 - k_2 & -s\varepsilon\alpha_1 \\ \dfrac{(\alpha_2 - k_2)(\alpha_2 - k_1)}{s\varepsilon\alpha_1} & k_1 - \alpha_2 \end{bmatrix}. \tag{26}$$

The matrix $\sqrt{A(s)}$ has the same projectors as $A(s)$ and its characteristic roots p_1, p_2 are given by $p_1 = \sqrt{k_1}$ and $p_2 = \sqrt{k_2}$. That is,

$$B(s) := \sqrt{A(s)} = \sqrt{k_1} E_1 + \sqrt{k_2} E_2 = \frac{A + \sqrt{k_1 k_2} I}{\sqrt{k_1} + \sqrt{k_2}} = \frac{1}{\sqrt{k_1} + \sqrt{k_2}} \begin{bmatrix} \sqrt{k_1 k_2} + s(1 + b\varepsilon)\alpha_1 & s\varepsilon\alpha_1 \\ \alpha_3 & \sqrt{k_1 k_2} + \alpha_2 \end{bmatrix}.$$

Thus the matrix exponential in (21) is given by

$$\exp\bigl(-\sqrt{A(s)}\, x\bigr) = \exp\bigl(-B(s)\, x\bigr) = \sum_{n=0}^{\infty} \frac{\bigl(-B(s)x\bigr)^n}{n!}. \tag{27}$$

By the Cayley–Hamilton theorem, the positive powers of B are linear combinations of I and B. Thus, the infinite series in (27) is of the form

$$\exp\bigl(-B(s)\bigr) = b_0(x, s)\, I + b_1(x, s)\, B(s), \tag{28}$$

where b_0 and b_1 are coefficients depending on s and x. To find these coefficients, note that the characteristic roots p_1 and p_2 of B satisfy

$$e^{-p_1 x} = b_0 + b_1 p_1, \qquad e^{-p_2 x} = b_0 + b_1 p_2. \tag{29}$$

Solving this linear system, we get $b_0 = \dfrac{1}{p_1 - p_2}\bigl(p_1 e^{-p_2 x} - k_2 e^{-p_1 x}\bigr)$ and $b_1 = \dfrac{1}{p_1 - p_2}\bigl(e^{-p_1 x} - e^{-p_2 x}\bigr)$. Hence the entries of the matrix

$$\exp\bigl(-B(s)\, x\bigr) = L_{ij}(x, s) \qquad i, j = 1, 2,$$

are given by

$$L_{11} = \frac{(k_1 - \alpha_2)e^{-\sqrt{k_1}\, x} - (k_2 - \alpha_2)e^{-\sqrt{k_2}\, x}}{k_1 - k_2}, \qquad L_{12} = \frac{s\varepsilon\alpha_1(e^{-\sqrt{k_1}\, x} - e^{-\sqrt{k_2}\, x})}{k_1 - k_2},$$

$$L_{22} = \frac{e^{-\sqrt{k_1}\, x}(\alpha_2 - k_2) - e^{-\sqrt{k_2}\, x}(\alpha_2 - k_1)}{k_1 - k_2}, \qquad L_{21} = \frac{\alpha_3(e^{-\sqrt{k_1}\, x} - e^{-\sqrt{k_2}\, x})}{k_1 - k_2}.$$

Similarly,

$$D_1 = \frac{\ell\alpha_1(\alpha_2 + s\varepsilon\alpha_4 - h^2)}{(h^2-k_1)(h^2-k_2)}, \qquad D_2 = \frac{\ell\big(h^2\alpha_4 - \alpha_1\alpha_3 - \alpha_1\alpha_4 s(1+b\varepsilon)\big)}{(h^2-k_1)(h^2-k_2)}.$$

We can write the solution (21) in the form

$$\begin{bmatrix}\bar\varphi(x,s)\\ \bar\sigma(x,s)\end{bmatrix} = \begin{bmatrix}L_{11}(x,s) & L_{12}(x,s)\\ L_{21}(x,s) & L_{22}(x,s)\end{bmatrix}\begin{bmatrix}C_1(s)\\ C_2(s)\end{bmatrix} + \begin{bmatrix}D_1(s)\\ D_2(s)\end{bmatrix}e^{-hx}. \tag{30}$$

To get C_1 and C_2 we set $x=0$ on the last equation, and we get

$$\begin{bmatrix}\bar\varphi(0,s)\\ \bar\sigma(0,s)\end{bmatrix} = \begin{bmatrix}L_{11}(0,s) & L_{12}(0,s)\\ L_{21}(0,s) & L_{22}(0,s)\end{bmatrix}\begin{bmatrix}C_1(s)\\ C_2(s)\end{bmatrix} + \begin{bmatrix}D_1(s)\\ D_2(s)\end{bmatrix},$$

which gives

$$\begin{bmatrix}C_1(s)\\ C_2(s)\end{bmatrix} = \begin{bmatrix}\bar\varphi(0,s)\\ \bar\sigma(0,s)\end{bmatrix} - \begin{bmatrix}D_1(s)\\ D_2(s)\end{bmatrix}. \tag{31}$$

Hence, for any set of boundary conditions, we have

$$\begin{aligned}\bar\varphi(x,s) &= \big(\bar\varphi(0,s)-D_1\big)L_{11}(x,s) + \big(\bar\sigma(0,s)-D_2\big)L_{12}(x,s) + D_1e^{-hx},\\ \bar\sigma(x,s) &= \big(\bar\varphi(0,s)-D_1\big)L_{21}(x,s) + \big(\bar\sigma(0,s)-D_2\big)L_{22}(x,s) + D_2e^{-hx}.\end{aligned} \tag{32}$$

3. Application

We now consider the boundary conditions on the boundary plane $x=0$, which are of two forms:

(1) *Thermal boundary condition.* We suppose that the boundary plane $x=0$ is subjected to ramp-type heating as follows [Youssef 2005]:

$$\varphi(0,t) = \begin{cases} 0 & \text{if } t \le 0,\\ \varphi_0\dfrac{t}{t_0} & \text{if } 0 < t < t_0,\\ \varphi_0 & \text{if } t \ge t_0,\end{cases} \tag{33}$$

where t_0 is called the ramping parameter and φ_0 is constant. After Laplace transformation, we get

$$\bar\varphi(0,s) = \frac{\varphi_0(1-e^{-st_0})}{s^2t_0}. \tag{34}$$

(2) *Mechanical boundary condition.* We consider the boundary plane $x=0$ traction-free, so $\sigma(0,t)=0$, which gives, after Laplace transformation,

$$\bar\sigma(0,s) = 0. \tag{35}$$

Applying (34) and (35) to (32) we get the solution for the heat conduction and stress x-component in the Laplace transform domain:

$$\begin{aligned}\bar\varphi(x,s) &= \varphi_1(s)e^{-\sqrt{k_1}\,x} - \varphi_2(s)e^{-\sqrt{k_2}\,x} + D_1(s)e^{-hx},\\ \bar\sigma(x,s) &= \sigma_1(s)e^{-\sqrt{k_1}\,x} - \sigma_2(s)e^{-\sqrt{k_2}\,x} + D_2(s)e^{-hx},\end{aligned} \tag{36}$$

where

$$\varphi_1(s)=\frac{1}{k_1-k_2}\left[\left(\frac{\varphi_0(1-e^{-st_0})}{s^2t_0}-D_1\right)(k_1-\alpha_2)-s\varepsilon\alpha_1 D_2\right],$$

$$\varphi_2(s)=\frac{1}{k_1-k_2}\left[\left(\frac{\varphi_0(1-e^{-st_0})}{s^2t_0}-D_1\right)(k_2-\alpha_2)-s\varepsilon\alpha_1 D_2\right],$$

$$\sigma_1(s)=\frac{1}{k_1-k_2}\left[\left(\frac{\varphi_0(1-e^{-st_0})}{s^2t_0}-D_1\right)\alpha_3-D_2(\alpha_2-k_2)\right],$$

$$\sigma_2(s)=\frac{1}{k_1-k_2}\left[\left(\frac{\varphi_0(1-e^{-st_0})}{s^2t_0}-D_1\right)\alpha_3-D_2(\alpha_2-k_1)\right].$$

By substituting the expressions (36) into (19), we obtain

$$\bar{\theta}(x,s)=(1-\beta k_1)\varphi_1(s)e^{-\sqrt{k_1}\,x}-(1-\beta k_2)\varphi_2(s)e^{-\sqrt{k_2}\,x}+(1-\beta h^2)D_1(s)e^{-hx}. \tag{37}$$

From $(16)_2$ and by using $(36)_2$, we obtain the displacement:

$$\bar{u}(x,s)=-\frac{1}{s^2}\big(\sigma_1(s)\sqrt{k_1}e^{-\sqrt{k_1}\,x}-\sigma_2(s)\sqrt{k_2}e^{-\sqrt{k_2}\,x}+D_2(s)he^{-hx}\big). \tag{38}$$

This completes the solution in the Laplace transform domain.

4. Numerical inversion of the Laplace transform

To determine numerically the conductive and thermal temperature, displacement, and stress distributions in the time domain, we used the Riemann sum approximation method. In this method, a function in the Laplace domain is inverted to the time domain through the sum

$$f(t)=\frac{e^{\kappa t}}{t}\left[\frac{1}{2}\bar{f}(\kappa)+\mathrm{Re}\sum_{n=1}^{N}(-1)^n\bar{f}\left(\kappa+\frac{in\pi}{t}\right)\right], \tag{39}$$

where Re is the real part and i is the imaginary number unit. For faster convergence, numerical experiments have shown that the value of κ should satisfy the relation $\kappa t\approx 4.7$ [Tzou 1997].

5. Numerical results and discussion

Copper was chosen as the material for the numerical evaluations. The constants of the problem (see [Bassiouny and Youssef 2008]) were as follows:

$$K=386\,\mathrm{N/K\,sec},\quad \alpha_T=1.78\times10^{-5}\,\mathrm{K}^{-1},\quad C_E=383.1\,\mathrm{m^2/K},\quad \eta=8886.73\,\mathrm{m/sec^2},$$
$$\mu=3.86\times10^{10}\,\mathrm{N/m^2},\quad \lambda=7.76\times10^{10}\,\mathrm{N/m^2},\quad \rho=8954\,\mathrm{kg/m^3},\quad \tau_0=0.02\,\mathrm{sec},$$
$$T_0=293\,\mathrm{K},\quad \varepsilon=1.618,\quad \beta=0.01,\quad b=0.01041.$$

The computations were carried out for $t=0.2$ and $\varphi_0=1.0$. The conductive temperature, the dynamical temperature, the stress and the displacement distributions are represented graphically with respect to x.

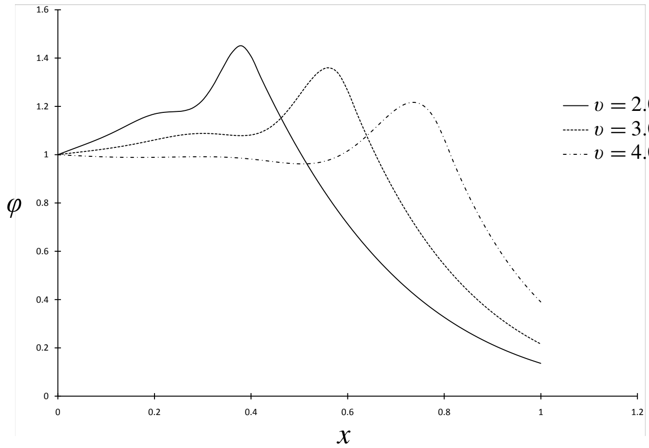

Figure 1. The conductive heat distribution at different values of the heat source velocity.

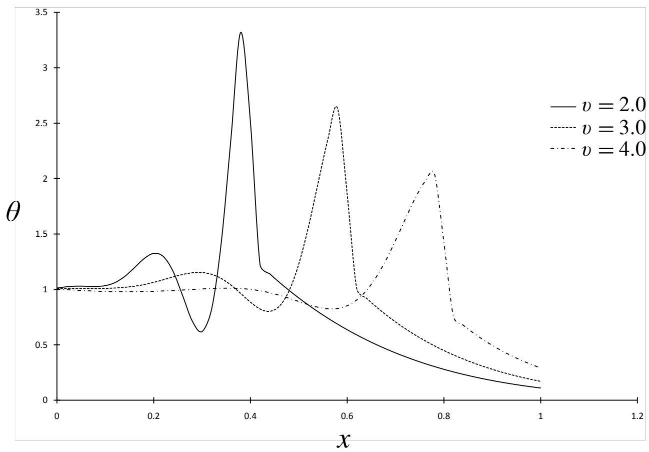

Figure 2. The thermodynamic heat distribution at different values of the heat source velocity.

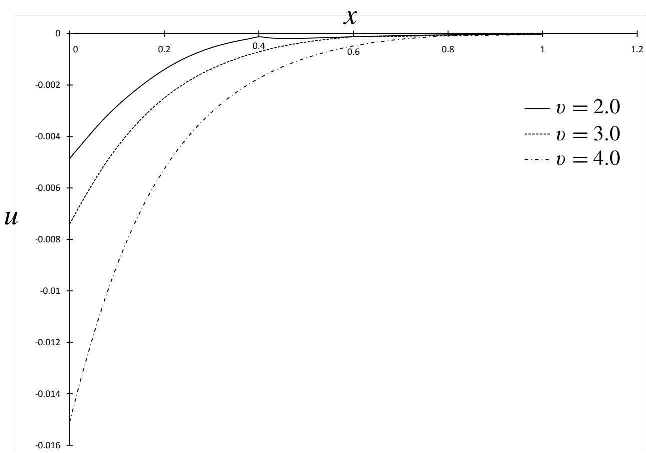

Figure 3. The displacement distribution at different values of the heat source velocity.

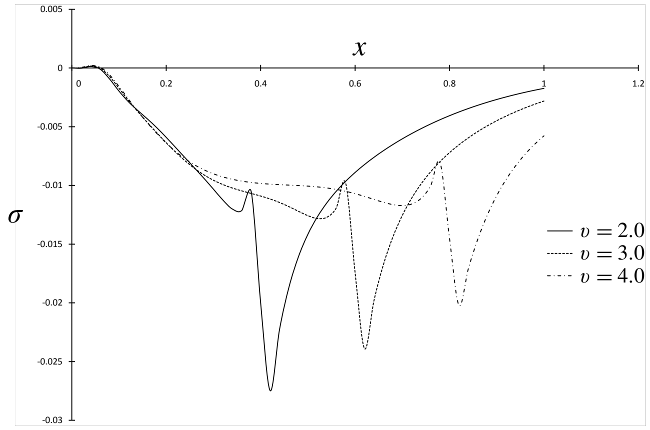

Figure 4. The stress distribution at different values of the heat source velocity.

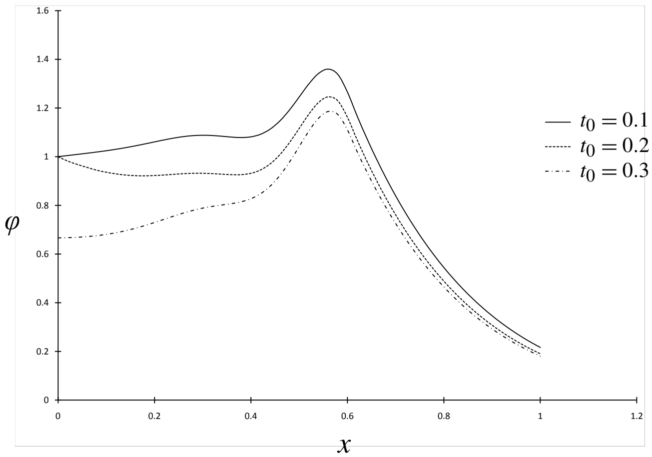

Figure 5. The conductive heat distribution at different values of the ramp time parameter.

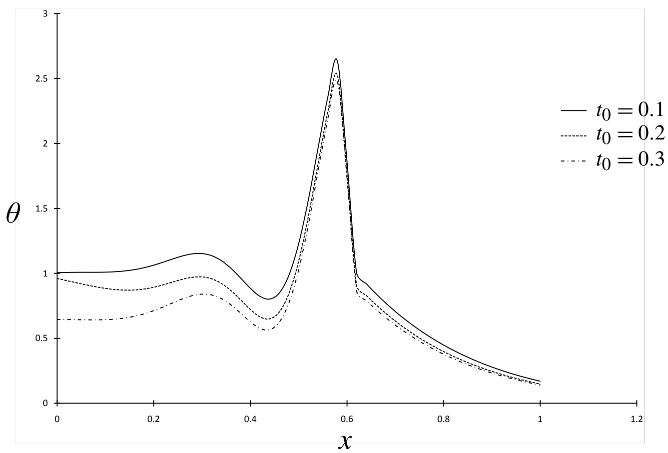

Figure 6. The thermodynamic heat distribution at different values of the ramp time parameter.

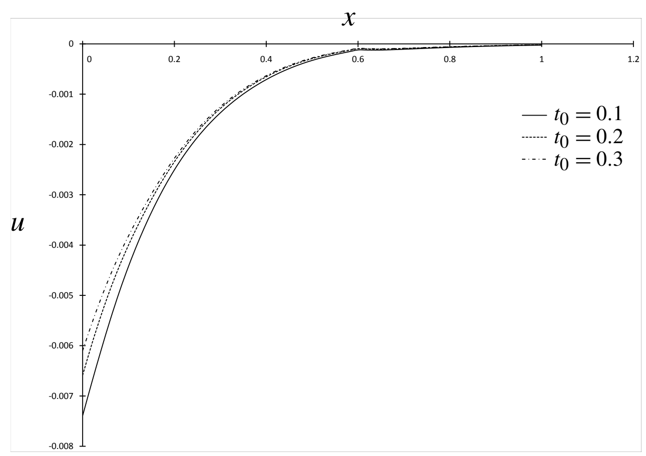

Figure 7. The displacement distribution at different values of the ramp time parameter.

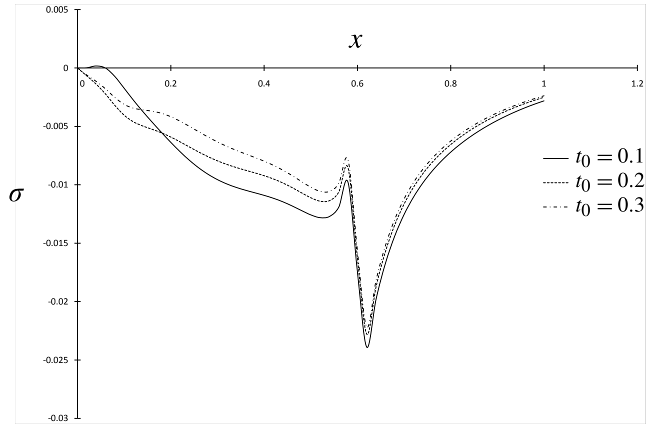

Figure 8. The stress distribution at different values of the ramp time parameter.

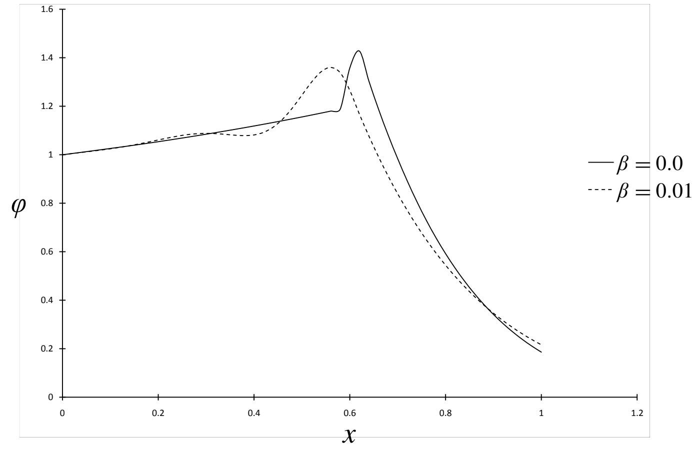

Figure 9. The conductive heat distribution for the L–S and Youssef models.

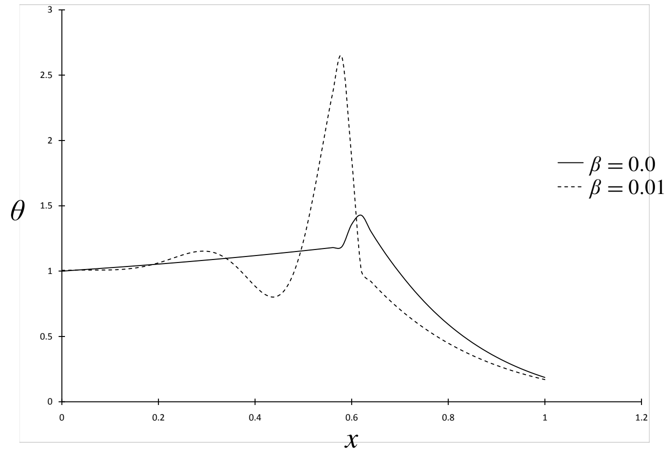

Figure 10. The thermodynamic heat distribution for the L–S and Youssef models.

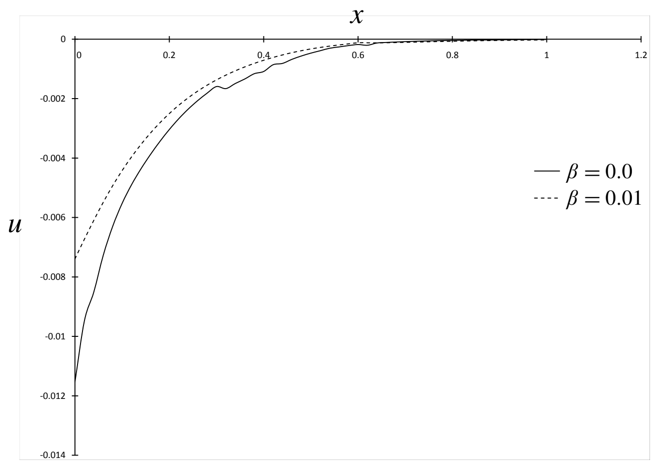

Figure 11. The displacement distribution for the L–S and Youssef models.

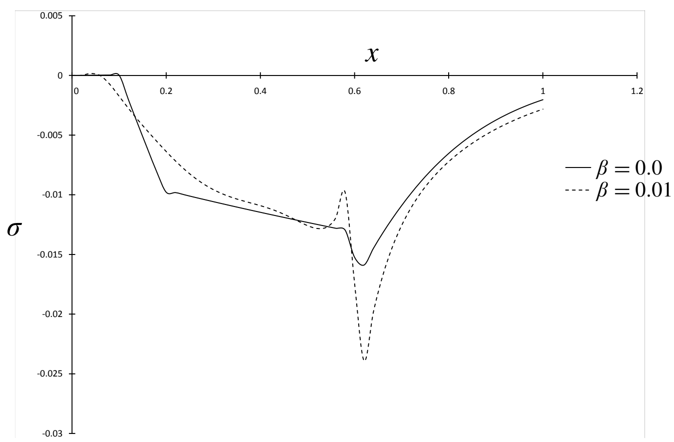

Figure 12. The stress distribution for the L–S and Youssef models.

Figures 1–4 display the conductive heat, the thermodynamic heat, the displacement, and the stress distributions at different values of heat source velocity υ ($\upsilon = 2.0$, $\upsilon = 3.0$ and $\upsilon = 4.0$) to show its effect, where we have noticed that the heat source velocity parameter v has a significant effect on all the fields. The peak values of the conductive heat, the thermodynamic heat, and the stress are found at the points when $x = \upsilon t$ ($x = 4.0$, $x = 6.0$ and $x = 8.0$) which mean that the heat source releases its maximum energy at the point $x = \upsilon t$ and just after this point the values of that fields decrease with high speed.

Figures 5–8 display the conductive heat, the thermodynamic heat, the displacement, and the stress distributions at constant velocity of heat source $\upsilon = 3.0$ and different values of the ramping time parameter t_0 ($t_0 = 0.1$, $t_0 = 0.2$, and $t_0 = 0.3$). The figures show that this parameter has significant effect on all the fields. The conductive heat and the thermodynamic heat decrease when the value of t_0 increases and the absolute values of the displacement and the stress also decrease when the value of t_0 increases. This gives this type of heating real character, more than the thermal shocks in previous works.

Figures 9–12 display the conductive heat, the thermodynamic heat, the displacement, and the stress distributions at constant velocity of heat source $\upsilon = 3.0$ and constant value of the ramping time parameter $t_0 = 0.1$ but with different values of the nondimensional two-temperature parameter β ($\beta = 0.0$ and $\beta = 0.01$). This shows the difference between the one temperature generalized thermoelasticity of Lord and Shulman (L–S) and the two-temperature generalized thermoelasticity of Youssef. We can see the significant effect of that parameter on all the fields.

The phenomenon of finite speeds of propagation is manifest in all these figures. This is expected, since the thermal wave travels with a finite velocity. It should be mentioned that in Figures 1, 2, 5, 6, 9 and 10 the effects of the ramp-type heating on $x = 0$ of the half-space remain in a bounded region of space in the two generalized theories (Youssef and L–S) and do not reach infinity instantaneously.

Nomenclature

λ, μ	Lamé constants
ρ	density
C_E	specific heat at constant strain
t	time
T	dynamical temperature
T_0	reference temperature
θ	dynamical temperature increment (= $T - T_0$)
φ	conductive temperature
α_T	coefficient of linear thermal expansion
γ	equal to $\alpha_T(3\lambda + 2\mu)$
σ_{ij}	components of stress tensor
e_{ij}	components of strain tensor
u_i	components of displacement vector
K	thermal conductivity
τ_0	relaxation times
c_0	longitudinal wave speed (= $\sqrt{(\lambda + 2\mu)/\rho}$)
η	thermal viscosity, = $\rho C_E/K$
ε	dimensionless thermoelastic coupling constant (= $\gamma/(\rho C_E)$)
a	two-temperature parameter, $a > 0$
β	dimensionless two-temperature parameter (= $a c_0^2 \eta^2$)
b	dimensionless mechanical coupling constant (= $\gamma T_0/(\lambda + 2\mu)$)
t_0	ramping parameter
υ	heat source velocity

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