Appendix to

ELASTIC ANALYSIS OF CLOSED-FORM SOLUTIONS FOR ADHESIVE STRESSES IN BONDED SINGLE-STRAP BUTT JOINTS BY GANG LI*

Details on the derivation of the following parameters, integral constants and functions are very lengthy and can be found elsewhere [Li 2008]. Only the final expressions for relevant parameters in the determination of adhesive stresses are provided in the following.

A.1. Determination of the seven integral constants in the Eq. (7a)

Submitting the shear stress into the boundary conditions, Eq. (7b), seven simultaneous equations written in terms of the seven constants, C_i ($i = 0$ to 6), can be obtained as:

$$\begin{cases} C_0 c + C_1 C_{10} + C_3 C_{30} + C_6 C_{60} = f_0 \\ -C_1 C_{11} + C_2 C_{21} - C_3 C_{31} + C_4 C_{41} + C_5 C_{51} - C_6 C_{61} = f_1 \\ C_1 C_{11} + C_2 C_{21} + C_3 C_{31} + C_4 C_{41} + C_5 C_{51} + C_6 C_{61} = f_2 \\ a_1 C_0 + C_1 C_{12} - C_2 C_{22} + C_3 C_{32} - C_4 C_{42} - C_5 C_{52} + C_6 C_{62} = f_3 \\ a_1 C_0 + C_1 C_{12} + C_2 C_{22} + C_3 C_{32} + C_4 C_{42} + C_5 C_{52} + C_6 C_{62} = f_4 \\ -C_1 C_{13} + C_2 C_{23} - C_3 C_{33} + C_4 C_{43} + C_5 C_{53} - C_6 C_{63} = f_5 \\ C_1 C_{13} + C_2 C_{23} + C_3 C_{33} + C_4 C_{43} + C_5 C_{53} + C_6 C_{63} = f_6 \end{cases} \tag{A.1.1a}$$

For the above linear systems of equations, the corresponding augmented matrix is [Derrick and Grossman 1987]:

*PhD, Associate Research Officer, Structures and Materials Performance Laboratory, Institute for Aerospace Research, National Research Council Canada, M-3, 1200 Montreal Road, Ottawa, Ontario, Canada K1A 0R6. Tel. 1-613-990-4989, Fax: 1-613-998-8609, E-mail: Gang.Li@nrc-cnrc.gc.ca.

$$\begin{bmatrix} c & C_{10} & 0 & C_{30} & 0 & 0 & C_{60} & f_0 \\ 0 & -C_{11} & C_{21} & -C_{31} & C_{41} & C_{51} & -C_{61} & f_1 \\ 0 & C_{11} & C_{21} & C_{31} & C_{41} & C_{51} & C_{61} & f_2 \\ a_1 & C_{12} & -C_{22} & C_{32} & -C_{42} & -C_{52} & C_{62} & f_3 \\ a_1 & C_{12} & C_{22} & C_{32} & C_{42} & C_{52} & C_{62} & f_4 \\ 0 & -C_{13} & C_{23} & -C_{33} & C_{43} & C_{53} & -C_{63} & f_5 \\ 0 & C_{13} & C_{23} & C_{33} & C_{43} & C_{53} & C_{63} & f_6 \end{bmatrix} \tag{A.1.1b}$$

The constants can be determined using Gauss elimination [Derrick and Grossman 1987; Kreyszig 1993], and the final expressions are:

$$C_6 = \frac{\dfrac{f_3+f_4}{2} - f_0\dfrac{a_1}{c} + \dfrac{f_1-f_2}{2}\dfrac{C_{12}-C_{10}\dfrac{a_1}{c}}{C_{11}} - \left(\dfrac{f_6-f_5}{2} + \dfrac{f_1-f_2}{2}\dfrac{C_{13}}{C_{11}}\right)\left(\dfrac{C_{32}-\dfrac{a_1}{c}C_{30}-C_{31}\dfrac{C_{12}-C_{10}\dfrac{a_1}{c}}{C_{11}}}{C_{33}-C_{31}\dfrac{C_{13}}{C_{11}}}\right)}{C_{62}-\dfrac{a_1}{c}C_{60}-C_{61}\dfrac{C_{12}-C_{10}\dfrac{a_1}{c}}{C_{11}} - \left(C_{63}-C_{61}\dfrac{C_{13}}{C_{11}}\right)\left(\dfrac{C_{32}-\dfrac{a_1}{c}C_{30}-C_{31}\dfrac{C_{12}-C_{10}\dfrac{a_1}{c}}{C_{11}}}{C_{33}-C_{31}\dfrac{C_{13}}{C_{11}}}\right)}$$

$$C_5 = \frac{\dfrac{f_5+f_6}{2} - \dfrac{f_1+f_2}{2}\dfrac{C_{23}}{C_{21}} - \left(\dfrac{f_4-f_3}{2} - \dfrac{f_1+f_2}{2}\dfrac{C_{22}}{C_{21}}\right)\left(\dfrac{C_{43}-C_{41}\dfrac{C_{23}}{C_{21}}}{C_{42}-C_{41}\dfrac{C_{22}}{C_{21}}}\right)}{C_{53}-C_{51}\dfrac{C_{23}}{C_{21}} - \left(C_{52}-C_{51}\dfrac{C_{22}}{C_{21}}\right)\left(\dfrac{C_{43}-C_{41}\dfrac{C_{23}}{C_{21}}}{C_{42}-C_{41}\dfrac{C_{22}}{C_{21}}}\right)}$$

$$C_4 = \frac{1}{C_{42}-C_{41}\dfrac{C_{22}}{C_{21}}}\left(\frac{f_4-f_3}{2} - \frac{f_1+f_2}{2}\frac{C_{22}}{C_{21}} - C_5\left(C_{52}-C_{51}\frac{C_{22}}{C_{21}}\right)\right)$$

$$C_3 = \frac{1}{C_{33} - C_{31}\dfrac{C_{13}}{C_{11}}}\left(\frac{f_6 - f_5}{2} + \frac{f_1 - f_2}{2}\frac{C_{13}}{C_{11}} - C_6\left(C_{63} - C_{61}\frac{C_{13}}{C_{11}}\right)\right) \tag{A.1.1c}$$

$$C_2 = \frac{1}{C_{21}}\left(\frac{f_1 + f_2}{2} - C_{41}C_4 - C_{51}C_5\right)$$

$$C_1 = -\frac{1}{C_{11}}\left(\frac{f_1 - f_2}{2} + C_{31}C_3 + C_{61}C_6\right)$$

$$C_0 = \frac{1}{c}\left(f_0 - C_{10}C_1 - C_{30}C_3 - C_{60}C_6\right)$$

where:

$$C_{10} = \frac{\sinh\ c\sqrt{\gamma_1 - \dfrac{a_1}{3}}}{\sqrt{\gamma_1 - \dfrac{a_1}{3}}}$$

$$C_{30} = \frac{\sin\dfrac{\beta}{2}}{|\phi|^{\frac{1}{2}}}\left(\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) + \frac{\cos\dfrac{\beta}{2}}{\sin\dfrac{\beta}{2}}\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\right)$$

$$C_{60} = \frac{\sin\dfrac{\beta}{2}}{|\phi|^{\frac{1}{2}}}\left(-\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) + \frac{\cos\dfrac{\beta}{2}}{\sin\dfrac{\beta}{2}}\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\right)$$

$$C_{11} = \sqrt{\gamma_1 - \frac{a_1}{3}}\sinh\ c\sqrt{\gamma_1 - \frac{a_1}{3}}\,, \quad C_{21} = \sqrt{\gamma_1 - \frac{a_1}{3}}\cosh\ c\sqrt{\gamma_1 - \frac{a_1}{3}}$$

$$C_{31} = |\phi|^{\frac{1}{2}}\left(\cos\frac{\beta}{2}\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) - \sin\frac{\beta}{2}\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\right)$$

$$C_{41} = |\phi|^{\frac{1}{2}}\left(\cos\frac{\beta}{2}\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) - \sin\frac{\beta}{2}\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\right)$$

$$C_{51} = |\phi|^{\frac{1}{2}}\left(\cos\frac{\beta}{2}\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) + \sin\frac{\beta}{2}\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\right)$$

$$C_{61} = |\phi|^{\frac{1}{2}}\left(\cos\frac{\beta}{2}\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) + \sin\frac{\beta}{2}\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\right)$$

$$C_{12} = \left(\gamma_1 + \frac{2a_1}{3}\right)\cosh c\sqrt{\gamma_1 - \frac{a_1}{3}}\,, \qquad C_{22} = \left(\gamma_1 + \frac{2a_1}{3}\right)\sinh c\sqrt{\gamma_1 - \frac{a_1}{3}}$$

$$\begin{aligned} C_{32} = {} & \left(|\phi|\cos^2\frac{\beta}{2} - |\phi|\sin^2\frac{\beta}{2} + a_1\right)\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) \\ & - 2|\phi|\sin\frac{\beta}{2}\cos\frac{\beta}{2}\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) \end{aligned}$$

$$\begin{aligned} C_{42} = {} & \left(|\phi|\cos^2\frac{\beta}{2} - |\phi|\sin^2\frac{\beta}{2} + a_1\right)\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) \\ & - 2|\phi|\sin\frac{\beta}{2}\cos\frac{\beta}{2}\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) \end{aligned}$$

$$\begin{aligned} C_{52} = {} & \left(|\phi|\cos^2\frac{\beta}{2} - |\phi|\sin^2\frac{\beta}{2} + a_1\right)\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) \\ & + 2|\phi|\sin\frac{\beta}{2}\cos\frac{\beta}{2}\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) \end{aligned}$$

$$\begin{aligned} C_{62} = {} & \left(|\phi|\cos^2\frac{\beta}{2} - |\phi|\sin^2\frac{\beta}{2} + a_1\right)\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) \\ & + 2|\phi|\sin\frac{\beta}{2}\cos\frac{\beta}{2}\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) \end{aligned}$$

$$C_{13} = \left(\gamma_1 + \frac{2a_1}{3}\right)\left(\gamma_1 - \frac{a_1}{3}\right)^{\frac{3}{2}}\sinh c\sqrt{\gamma_1 - \frac{a_1}{3}}\;, \qquad C_{23} = \left(\gamma_1 + \frac{2a_1}{3}\right)\left(\gamma_1 - \frac{a_1}{3}\right)^{\frac{3}{2}}\cosh c\sqrt{\gamma_1 - \frac{a_1}{3}}$$

$$\begin{aligned} C_{33} = {} & \left(|\phi|^{\frac{5}{2}}\cos^5\frac{\beta}{2} - 10|\phi|^{\frac{5}{2}}\cos^3\frac{\beta}{2}\sin^2\frac{\beta}{2} + 5|\phi|^{\frac{5}{2}}\cos\frac{\beta}{2}\sin^4\frac{\beta}{2} + a_1|\phi|^{\frac{3}{2}}\cos^3\frac{\beta}{2} - 3a_1|\phi|^{\frac{3}{2}}\cos\frac{\beta}{2}\sin^2\frac{\beta}{2}\right)\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) \\ & - \left(|\phi|^{\frac{5}{2}}\sin^5\frac{\beta}{2} - 10|\phi|^{\frac{5}{2}}\cos^2\frac{\beta}{2}\sin^3\frac{\beta}{2} + 5|\phi|^{\frac{5}{2}}\cos^4\frac{\beta}{2}\sin\frac{\beta}{2} - a_1|\phi|^{\frac{3}{2}}\sin^3\frac{\beta}{2} + 3a_1|\phi|^{\frac{3}{2}}\cos^2\frac{\beta}{2}\sin\frac{\beta}{2}\right)\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) \end{aligned}$$

$$\begin{aligned} C_{43} = {} & \left(|\phi|^{\frac{5}{2}}\cos^5\frac{\beta}{2} - 10|\phi|^{\frac{5}{2}}\cos^3\frac{\beta}{2}\sin^2\frac{\beta}{2} + 5|\phi|^{\frac{5}{2}}\cos\frac{\beta}{2}\sin^4\frac{\beta}{2} + a_1|\phi|^{\frac{3}{2}}\cos^3\frac{\beta}{2} - 3a_1|\phi|^{\frac{3}{2}}\cos\frac{\beta}{2}\sin^2\frac{\beta}{2}\right)\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) \\ & - \left(|\phi|^{\frac{5}{2}}\sin^5\frac{\beta}{2} - 10|\phi|^{\frac{5}{2}}\cos^2\frac{\beta}{2}\sin^3\frac{\beta}{2} + 5|\phi|^{\frac{5}{2}}\cos^4\frac{\beta}{2}\sin\frac{\beta}{2} - a_1|\phi|^{\frac{3}{2}}\sin^3\frac{\beta}{2} + 3a_1|\phi|^{\frac{3}{2}}\cos^2\frac{\beta}{2}\sin\frac{\beta}{2}\right)\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right) \end{aligned}$$

$$C_{53}=\left(|\phi|^{\frac{5}{2}}\cos^5\frac{\beta}{2}-10|\phi|^{\frac{5}{2}}\cos^3\frac{\beta}{2}\sin^2\frac{\beta}{2}+5|\phi|^{\frac{5}{2}}\cos\frac{\beta}{2}\sin^4\frac{\beta}{2}+a_1|\phi|^{\frac{3}{2}}\cos^3\frac{\beta}{2}-3a_1|\phi|^{\frac{3}{2}}\cos\frac{\beta}{2}\sin^2\frac{\beta}{2}\right)\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)$$
$$+\left(|\phi|^{\frac{5}{2}}\sin^5\frac{\beta}{2}-10|\phi|^{\frac{5}{2}}\cos^2\frac{\beta}{2}\sin^3\frac{\beta}{2}+5|\phi|^{\frac{5}{2}}\cos^4\frac{\beta}{2}\sin\frac{\beta}{2}-a_1|\phi|^{\frac{3}{2}}\sin^3\frac{\beta}{2}+3a_1|\phi|^{\frac{3}{2}}\cos^2\frac{\beta}{2}\sin\frac{\beta}{2}\right)\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)$$

$$C_{63}=\left(|\phi|^{\frac{5}{2}}\cos^5\frac{\beta}{2}-10|\phi|^{\frac{5}{2}}\cos^3\frac{\beta}{2}\sin^2\frac{\beta}{2}+5|\phi|^{\frac{5}{2}}\cos\frac{\beta}{2}\sin^4\frac{\beta}{2}+a_1|\phi|^{\frac{3}{2}}\cos^3\frac{\beta}{2}-3a_1|\phi|^{\frac{3}{2}}\cos\frac{\beta}{2}\sin^2\frac{\beta}{2}\right)\cosh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)$$
$$+\left(|\phi|^{\frac{5}{2}}\sin^5\frac{\beta}{2}-10|\phi|^{\frac{5}{2}}\cos^2\frac{\beta}{2}\sin^3\frac{\beta}{2}+5|\phi|^{\frac{5}{2}}\cos^4\frac{\beta}{2}\sin\frac{\beta}{2}-a_1|\phi|^{\frac{3}{2}}\sin^3\frac{\beta}{2}+3a_1|\phi|^{\frac{3}{2}}\cos^2\frac{\beta}{2}\sin\frac{\beta}{2}\right)\sinh\left(c|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos\left(c|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)$$

$$f_0=-\frac{T}{2}\quad,\qquad f_1=\frac{G_a}{\eta}\left(\frac{T}{E_u^{'}t_1}+\frac{t_1}{2}\frac{M_0}{D_u}\right),\qquad f_2=\frac{G_a}{\eta}\left(-\frac{T}{E_d^{'}t_2}+\frac{t_2}{2}\frac{M_1}{D_d}\right)$$

$$f_3=\frac{G_a}{\eta}\frac{t_1}{2}\frac{V_0}{D_u},\qquad f_4=\frac{G_a}{\eta}\frac{t_2}{2}\frac{V_1}{D_d},\qquad f_5=-a_2\frac{E_a}{\eta}\frac{M_0}{D_u}$$

$$f_6=a_2\frac{E_a}{\eta}\frac{M_1}{D_d}$$

A.2. The functions $G_{ip}(x)$ ($i=1$ to 4) in the Eq. (8b)

$$G_{1p}(x)=\int G_{1p}^{'}(x)dx$$
$$=-\frac{b_2}{4\left(\frac{b_1}{4}\right)^{\frac{3}{4}}}\int\frac{d\tau_a}{dx}\left(\sinh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)-\cosh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)dx$$
$$=-\frac{b_2}{4\left(\frac{b_1}{4}\right)^{\frac{3}{4}}}\tau_a\left(\sinh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)-\cosh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)$$
$$+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_0G_{10}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_1G_{11}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_2G_{12}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_3G_{13}(x)$$
$$+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_4G_{14}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_5G_{15}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_6G_{16}(x)$$

$$
\begin{aligned}
G_{2p}(x) &= \int G_{2p}^{'}(x)dx \\
&= -\frac{b_2}{4\left(\frac{b_1}{4}\right)^{\frac{3}{4}}}\int \frac{d\tau_a}{dx}\left(\sinh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)-\cosh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)dx \\
&= -\frac{b_2}{4\left(\frac{b_1}{4}\right)^{\frac{3}{4}}}\tau_a\left(\sinh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)-\cosh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\right) \\
&\quad +\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_0G_{20}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_1G_{21}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_2G_{22}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_3G_{23}(x) \\
&\quad +\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_4G_{24}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_5G_{25}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_6G_{26}(x)
\end{aligned}
\tag{A.2.1}
$$

$$
\begin{aligned}
G_{3p}(x) &= \int G_{3p}^{'}(x)dx \\
&= -\frac{b_2}{4\left(\frac{b_1}{4}\right)^{\frac{3}{4}}}\int \frac{d\tau_a}{dx}\left(\sinh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\cosh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)dx \\
&= -\frac{b_2}{4\left(\frac{b_1}{4}\right)^{\frac{3}{4}}}\tau_a\left(\sinh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\cosh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\right) \\
&\quad +\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_0G_{30}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_1G_{31}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_2G_{32}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_3G_{33}(x) \\
&\quad +\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_4G_{34}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_5G_{35}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_6G_{36}(x)
\end{aligned}
$$

$$
\begin{aligned}
G_{4p}(x) &= \int G_{4p}^{'}(x)dx \\
&= \frac{b_2}{4\left(\frac{b_1}{4}\right)^{\frac{3}{4}}}\int \frac{d\tau_a}{dx}\left(\sinh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\cosh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)dx \\
&= \frac{b_2}{4\left(\frac{b_1}{4}\right)^{\frac{3}{4}}}\tau_a\left(\sinh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\cosh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\right) \\
&\quad +\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_0G_{40}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_1G_{41}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_2G_{42}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_3G_{43}(x) \\
&\quad +\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_4G_{44}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_5G_{45}(x)+\frac{b_2}{2\left(\frac{b_1}{4}\right)^{\frac{2}{4}}}C_6G_{46}(x)
\end{aligned}
$$

where

$$G_{10}(x) = -\frac{1}{2\left(\sqrt[4]{\frac{b_1}{4}}\right)}\left(\cosh\, x\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin\, x\left(\sqrt[4]{\frac{b_1}{4}}\right) - \sinh\, x\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos\, x\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)$$

$$G_{11}(x) = -\frac{\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)-\sqrt[4]{\frac{b_1}{4}}\sinh x\left(\sqrt[4]{\frac{b_1}{4}}+\sqrt{\gamma_1-\frac{a_1}{3}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)^2\right)}$$

$$+\frac{\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)-\sqrt[4]{\frac{b_1}{4}}\sinh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)^2\right)}$$

$$G_{12}(x) = -\frac{\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh\, x\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\sin\, x\left(\sqrt[4]{\frac{b_1}{4}}\right)-\sqrt[4]{\frac{b_1}{4}}\cosh\, x\left(\sqrt[4]{\frac{b_1}{4}}+\sqrt{\gamma_1-\frac{a_1}{3}}\right)\cos\, x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)^2\right)}$$

$$+\frac{\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh\, x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\sin\, x\left(\sqrt[4]{\frac{b_1}{4}}\right)-\sqrt[4]{\frac{b_1}{4}}\cosh\, x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\cos\, x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)^2\right)}$$

$$G_{13}(x) = -\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)-\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cosh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cosh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)-\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\sinh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)}{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)^2+\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)^2}\right)$$

$$G_{14}(x)=-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)-\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)^2}\right)$$

$$+\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$+\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)-\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)^2}\right)$$

$$G_{15}(x)=\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)+\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)^2}\right)$$

$$+\frac{1}{4}\left(\frac{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cosh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cosh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)+\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\sinh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)}{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)^2+\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)^2}\right)$$

$$
\begin{aligned}
G_{16}(x) = {} & \frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right) \\
& -\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)+\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)^2}\right) \\
& -\frac{1}{4}\left(\frac{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sinh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right) \\
& +\frac{1}{4}\left(\frac{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sinh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)+\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)\cosh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)}{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)^2+\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}\right)^2}\right)
\end{aligned}
$$

$$
G_{20}(x) = \frac{1}{2\left(\sqrt[4]{\frac{b_1}{4}}\right)}\left(\sinh x\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)-\cosh x\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)
$$

$$
\begin{aligned}
G_{21}(x) = {} & \frac{-\sqrt[4]{\frac{b_1}{4}}\cosh x\left(\sqrt[4]{\frac{b_1}{4}}+\sqrt{\gamma_1-\frac{a_1}{3}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)^2\right)} \\
& +\frac{-\sqrt[4]{\frac{b_1}{4}}\cosh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
G_{22}(x) = {} & \frac{-\sqrt[4]{\frac{b_1}{4}}\sinh x\left(\sqrt[4]{\frac{b_1}{4}}+\sqrt{\gamma_1-\frac{a_1}{3}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)^2\right)} \\
& +\frac{-\sqrt[4]{\frac{b_1}{4}}\sinh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)^2\right)}
\end{aligned}
$$

$$
\begin{aligned}
G_{23}(x) = {} & \frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right) \\
& +\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right) \\
& -\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right) \\
& -\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
G_{24}(x) = {} & \frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right) \\
& +\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right) \\
& -\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right) \\
& -\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)
\end{aligned}
$$

$$G_{25}(x)=-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$+\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$+\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$G_{26}(x)=-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$+\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$+\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$G_{30}(x)=\frac{1}{2\left(\sqrt[4]{\frac{b_1}{4}}\right)}\left(\cosh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\sinh\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin\ x\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)$$

$$G_{31}(x)=\frac{\sqrt[4]{\frac{b_1}{4}}\sinh x\left(\sqrt[4]{\frac{b_1}{4}}+\sqrt{\gamma_1-\frac{a_1}{3}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)^2\right)}$$
$$+\frac{\sqrt[4]{\frac{b_1}{4}}\sinh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)^2\right)}$$

$$G_{32}(x)=\frac{\sqrt[4]{\frac{b_1}{4}}\cosh x\left(\sqrt[4]{\frac{b_1}{4}}+\sqrt{\gamma_1-\frac{a_1}{3}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)^2\right)}$$
$$-\frac{\sqrt[4]{\frac{b_1}{4}}\cosh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)^2\right)}$$

$$G_{33}(x)=\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$
$$+\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$
$$+\frac{1}{4}\left(\frac{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cosh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$
$$+\frac{1}{4}\left(\frac{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cosh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$G_{34}(x)=\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$+\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$G_{35}(x)=\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$+\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$+\frac{1}{4}\left(\frac{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cosh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$+\frac{1}{4}\left(\frac{\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cosh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(\sqrt[4]{\frac{b_1}{4}}-|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$G_{36}(x)=\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$+\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$G_{40}(x)=-\frac{1}{2\left(\sqrt[4]{\frac{b_1}{4}}\right)}\left(\sinh x\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\cosh x\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)$$

$$G_{41}(x)=-\frac{\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(\sqrt[4]{\frac{b_1}{4}}+\sqrt{\gamma_1-\frac{a_1}{3}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\sqrt[4]{\frac{b_1}{4}}\cosh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)^2\right)}$$

$$-\frac{\left(\sqrt[4]{\frac{b_1}{4}}-\sqrt{\gamma_1-\frac{a_1}{3}}\right)\sinh x\left(\sqrt[4]{\frac{b_1}{4}}-\sqrt{\gamma_1-\frac{a_1}{3}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\sqrt[4]{\frac{b_1}{4}}\cosh x\left(\sqrt[4]{\frac{b_1}{4}}-\sqrt{\gamma_1-\frac{a_1}{3}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt[4]{\frac{b_1}{4}}-\sqrt{\gamma_1-\frac{a_1}{3}}\right)^2\right)}$$

$$G_{42}(x)=-\frac{\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\sqrt[4]{\frac{b_1}{4}}\sinh x\left(\sqrt[4]{\frac{b_1}{4}}+\sqrt{\gamma_1-\frac{a_1}{3}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}+\sqrt[4]{\frac{b_1}{4}}\right)^2\right)}$$

$$-\frac{\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(\sqrt[4]{\frac{b_1}{4}}\right)+\left(\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sqrt{\frac{b_1}{4}}+\left(\sqrt{\gamma_1-\frac{a_1}{3}}-\sqrt[4]{\frac{b_1}{4}}\right)^2\right)}$$

$$G_{43}(x)=$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$G_{44}(x)=$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$$-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)$$

$G_{45}(x)=$

$$
\begin{aligned}
&-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)\\
&-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)\\
&-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)\\
&-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)
\end{aligned}
$$

$G_{46}(x)=$

$$
\begin{aligned}
&-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)\\
&-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)\\
&-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}+\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)\\
&-\frac{1}{4}\left(\frac{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cosh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sin x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)-\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\sinh x\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)\cos x\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)}{\left(|\phi|^{\frac{1}{2}}\cos\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2+\left(|\phi|^{\frac{1}{2}}\sin\frac{\beta}{2}-\sqrt[4]{\frac{b_1}{4}}\right)^2}\right)
\end{aligned}
$$

A.3. The four constants C_{iH} ($i=1$ to 4) in the Eqs. (8a) and (8d)

$$C_{1H}=\frac{\frac{E_a}{2\eta}\cosh c\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(\frac{\frac{V_0}{D_u}+\frac{V_1}{D_d}}{\sqrt[4]{\left(\frac{b_1}{4}\right)^3}}+\frac{\frac{M_0}{D_u}-\frac{M_1}{D_d}}{\sqrt{\frac{b_1}{4}}}\left(\tanh c\left(\sqrt[4]{\frac{b_1}{4}}\right)-\tan c\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)\right)}{\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)+\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}+H_1$$

$$C_{2H}=\frac{\frac{E_a}{2\eta}\sinh c\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(\frac{\frac{V_0}{D_u}-\frac{V_1}{D_d}}{\sqrt[4]{\left(\frac{b_1}{4}\right)^3}}+\frac{\frac{M_0}{D_u}+\frac{M_1}{D_d}}{\sqrt{\frac{b_1}{4}}}\left(\frac{1}{\tanh c\left(\sqrt[4]{\frac{b_1}{4}}\right)}-\tan c\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)\right)}{\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)-\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}+H_2 \qquad \text{(A.3.1)}$$

$$C_{3H}=\frac{\frac{E_a}{2\eta}\cosh c\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(\frac{\frac{V_0}{D_u}-\frac{V_1}{D_d}}{\sqrt[4]{\left(\frac{b_1}{4}\right)^3}}+\frac{\frac{M_0}{D_u}+\frac{M_1}{D_d}}{\sqrt{\frac{b_1}{4}}}\left(\tanh c\left(\sqrt[4]{\frac{b_1}{4}}\right)+\frac{1}{\tan c\left(\sqrt[4]{\frac{b_1}{4}}\right)}\right)\right)}{\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)-\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}+H_3$$

$$C_{4H}=\frac{\frac{E_a}{2\eta}\sinh c\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(\frac{\frac{V_0}{D_u}+\frac{V_1}{D_d}}{\sqrt[4]{\left(\frac{b_1}{4}\right)^3}}+\frac{\frac{M_0}{D_u}-\frac{M_1}{D_d}}{\sqrt{\frac{b_1}{4}}}\left(\frac{1}{\tanh c\left(\sqrt[4]{\frac{b_1}{4}}\right)}+\frac{1}{\tan c\left(\sqrt[4]{\frac{b_1}{4}}\right)}\right)\right)}{\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)+\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}$$

$$-\frac{1}{2}\frac{\sinh c\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin c\left(\sqrt[4]{\frac{b_1}{4}}\right)}{\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)+\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}\frac{b_2\left(\tau_a(-c)-\tau_a(c)\right)}{\sqrt[4]{\left(\frac{b_1}{4}\right)^3}}+H_4$$

where

$$H_1 = -\frac{1}{2}\left(G_{1p}(-c)+G_{1p}(c)\right)+\frac{\cosh^2 c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(G_{2p}(-c)-G_{2p}(c)\right)}{\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)+\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}$$

$$-\frac{\cos^2 c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(G_{3p}(-c)-G_{3p}(c)\right)}{\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)+\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}-\frac{\cosh c\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos c\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)+\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)}\frac{b_2\left(\tau_a(-c)-\tau_a(c)\right)}{\sqrt[4]{\left(\frac{b_1}{4}\right)^3}}$$

$$H_2 = \frac{\sinh^2 c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(G_{1p}(-c)-G_{1p}(c)\right)}{\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)-\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}-\frac{1}{2}\left(G_{2p}(-c)+G_{2p}(c)\right)$$

$$+\frac{\cos^2 c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(G_{4p}(-c)-G_{4p}(c)\right)}{\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)-\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}+\frac{\sinh c\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos c\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)-\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)}\frac{b_2\left(\tau_a(-c)+\tau_a(c)\right)}{\sqrt[4]{\left(\frac{b_1}{4}\right)^3}}$$

$$H_3 = \frac{\sin^2 c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(G_{1p}(-c)-G_{1p}(c)\right)}{\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)-\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}-\frac{1}{2}\left(G_{3p}(-c)+G_{3p}(c)\right)+\frac{\cosh^2 c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(G_{4p}(-c)-G_{4p}(c)\right)}{\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)-\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}$$

$$+\frac{\cosh c\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin c\left(\sqrt[4]{\frac{b_1}{4}}\right)}{2\left(\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)-\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)\right)}\frac{b_2\left(\tau_a(-c)+\tau_a(c)\right)}{\sqrt[4]{\left(\frac{b_1}{4}\right)^3}}$$

$$H_4 = -\frac{\sin^2 c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(G_{2p}(-c)-G_{2p}(c)\right)}{\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)+\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}+\frac{\sinh^2 c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(G_{3p}(-c)-G_{3p}(c)\right)}{\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)+\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}-\frac{1}{2}\left(G_{4p}(-c)+G_{4p}(c)\right).$$

A.4. Three constants C_{iS} ($i =$ 0 to 2) in the Eq. (10)

$$C_{0S} = \frac{1}{c}\left(-\frac{T}{2} + \frac{\frac{T}{E't} + \frac{t}{4}\frac{M_0 - M_1}{D}}{\frac{2}{E't} + \frac{t(t+\eta)}{2D}} \right),\quad C_{1S} = \frac{\sqrt{\frac{G_a}{\eta}}\left(-\frac{T}{E't} + \frac{t}{4}\frac{M_1 - M_0}{D} \right)}{\sqrt{\frac{2}{E't} + \frac{t(t+\eta)}{2D}}\ \sinh\left(c\sqrt{\frac{G_a}{\eta}\left(\frac{2}{E't} + \frac{t(t+\eta)}{2D} \right)} \right)}$$

$$C_{2S} = \frac{\sqrt{\frac{G_a}{\eta}}\ \frac{t}{4}\left(\frac{M_0 + M_1}{D} \right)}{\sqrt{\frac{2}{E't} + \frac{t(t+\eta)}{2D}}\ \cosh\left(c\sqrt{\frac{G_a}{\eta}\left(\frac{2}{E't} + \frac{t(t+\eta)}{2D} \right)} \right)} \tag{A.4.1}$$

A.5. Four integral constants C_{iS} ($i =$ 3 to 6) in the Eq. (11)

$$C_{3S} = \frac{\frac{E_a}{2\eta D}\cosh c\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(\frac{V_0 + V_1}{\sqrt[4]{\left(\frac{b_1}{4}\right)^3}} + \frac{M_0 - M_1}{\sqrt{\frac{b_1}{4}}}\left(\tanh c\left(\sqrt[4]{\frac{b_1}{4}}\right) - \tan c\left(\sqrt[4]{\frac{b_1}{4}}\right) \right) \right)}{\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right) + \sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}$$

$$C_{4S} = \frac{\frac{E_a}{2\eta D}\sinh c\left(\sqrt[4]{\frac{b_1}{4}}\right)\cos c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(\frac{V_0 - V_1}{\sqrt[4]{\left(\frac{b_1}{4}\right)^3}} + \frac{M_0 + M_1}{\sqrt{\frac{b_1}{4}}}\left(\frac{1}{\tanh c\left(\sqrt[4]{\frac{b_1}{4}}\right)} - \tan c\left(\sqrt[4]{\frac{b_1}{4}}\right) \right) \right)}{\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right) - \sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)} \tag{A.5.1}$$

$$C_{5S} = \frac{\frac{E_a}{2\eta D}\cosh c\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(\frac{V_0 - V_1}{\sqrt[4]{\left(\frac{b_1}{4}\right)^3}} + \frac{M_0 + M_1}{\sqrt{\frac{b_1}{4}}}\left(\tanh c\left(\sqrt[4]{\frac{b_1}{4}}\right) + \frac{1}{\tan c\left(\sqrt[4]{\frac{b_1}{4}}\right)} \right) \right)}{\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right) - \sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}$$

$$C_{6S} = \frac{\frac{E_a}{2\eta D}\sinh c\left(\sqrt[4]{\frac{b_1}{4}}\right)\sin c\left(\sqrt[4]{\frac{b_1}{4}}\right)\left(\frac{V_0+V_1}{\sqrt[4]{\left(\frac{b_1}{4}\right)^3}}+\frac{M_0-M_1}{\sqrt{\frac{b_1}{4}}}\left(\frac{1}{\tanh c\left(\sqrt[4]{\frac{b_1}{4}}\right)}+\frac{1}{\tan c\left(\sqrt[4]{\frac{b_1}{4}}\right)}\right)\right)}{\sin 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)+\sinh 2c\left(\sqrt[4]{\frac{b_1}{4}}\right)}$$