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We generalize the traditional Hamilton principle and give a complete nonlinear mathematical model of thermoelastic beams with voids based on this generalization, including the influences of the axial force, neutral layer inertia and rotation inertia. The differential quadrature method is used to discrete the nonlinear system on the spatial domain, and the Newton–Raphson method and Runge–Kutta method are adopted to solve the static and dynamical behaviors of the beam, respectively. The influences of the parameters on the nonlinear mechanical behavior of beam are studied in detail. The results show that the presence of voids enlarges beam deflection. And also one can see that the DQM has advantages of fewer workload, higher precision, better convergence, and so on.

1. Introduction

Thermoelastic materials with voids, which are common in various types of geological, biological and synthetic materials, are of practical utility in both structural and functional forms. Porous materials have extensive applications in aerospace, electronic communications, construction, metallurgy, nuclear energy, petrochemical, mechanical, medical and environmental protection due to their advantageous properties, such as low relative density, high specific strength and surface area, light weight, thermal and acoustical insulation and good permeability [Xi 2007].

Cowin and Nunziato [1983] proposed a linear theory of elastic materials with voids, which has practical utility for investigating various types of porous materials. Their theory is concerned with elastic materials consisting of a distribution of small voids, in which the void volume is included among the kinematic variables, and the theory reduces to the classical theory of elasticity in the limit case of the volume tending to zero. Iesan [1986] developed a linear theory of thermoelastic materials with voids. Puri and Cowin [1985] studied the behavior of plane waves in linear elastic materials with voids. Chiriţă and Scalia [2001] considered the spatial and temporal behaviors in linear thermoelasticity of materials with voids. Scalia et al. [2004] studied the steady time-harmonic oscillation in thermoelastic materials with voids. Chiriţă and Ciarletta [2008] discussed the structural stability of thermoelastic model of porous media. Cicco and Diaco [2002] developed a theory of thermoelastic materials with voids without energy dissipation. Ciarletta et al. [2007] studied thermoporoacoustic acceleration waves in elastic materials with voids without energy dissipation. Singh [2007] studied the wave propagation in a generalized

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thermoelastic material with voids. Kumar and Rani [2005] investigated the dynamic response of a homogeneous, isotropic, thermoelastic half-space with voids subjected to time harmonic normal force and thermal source.

In [Sheng and Cheng 2004; Cheng et al. 2006], the Gurtin-type variational principles of viscoelastic Timoshenko beams and thick plates with damage were established, damage being regarded as voids in materials. These works also studied quasistatic problems of viscoelastic beams and dynamical behaviors of viscoelastic plates. Bîrsan [2003] presented a bending theory of porous thermoelastic plates. Sharma et al. [2008] investigated the three-dimensional vibration of a thermoelastic cylindrical panel with voids.

As one kind of the most basic structural components, beams are widely applied to engineering and science, for example, bridges and beam-column systems of constructions, nanoscale bioprobes and piezoelectric devices, heat exchanger tubes in production equipment, titanium alloy artificial bones. Besides, rockets, missiles and other flying slender cylindrical structures can be approximated as a free-free beam to study the dynamical response and failure analysis under transient dynamical loads [Yu et al. 1996]. The structural elements mentioned above, which may be thermoelastic materials with voids, might be subject to a variety of static loads and external excitations, such as mechanical forces, seismic waves, shock waves, aerodynamic forces, thermal and nonthermal loads, etc. Therefore, it is very important to present a suitable nonlinear mathematical model of thermoelastic beams with voids.

For the linear thermoelastodynamics, there exist plenty of papers for variational principles and solving methods. Zhang [2007] presented a Gurtin-type variational formulation for functionally graded thermoviscoelastic beams by using the convolution bilinear form and the classical cartesian bilinear form. But for the thermoelastic problems with geometric nonlinearity, there are two difficulties to set up the corresponding Hamilton variational principles:

(1) The terms with the first-order time-derivative in the equation for the balance of entropy are not potential operators.

(2) The traditional Hamilton principle is used to characterize dynamical problems at the initial and final time.

Luo et al. [2002] presented unconventional Hamilton-type variational principles for nonlinear thermoelastodynamics, which can characterize this kind of the initial-boundary-value problems. However, there are seldom reports for the variational principles and numerical methods of thermoelastic beams with both voids and geometric nonlinearity.

In this paper, the traditional Hamilton principle is first generalized, a complete finite deformation theory of thermoelastic beams with voids is presented from the generalized Hamilton principle, in which, the influences of the axial force, neutral layer and rotation inertia are all considered. The theory exhibits a set of nonlinear equations about three displacements, one void moment and one thermal moment. As application, the plane bending of beams is studied. To improve the computational efficiency and accuracy, the differential quadrature method (DQM) is used to discrete the nonlinear system on the spatial domain, then the Newton–Raphson method and Runge–Kutta method are adopted to calculate and analyze the static and dynamical response of nonlinear systems of thermoelastic beams with voids, respectively. The mechanical behavior of beams is investigated under four cases, that is, thermoelastic materials with/without voids and elastic materials with/without voids. The influences of the parameters are studied in detail.

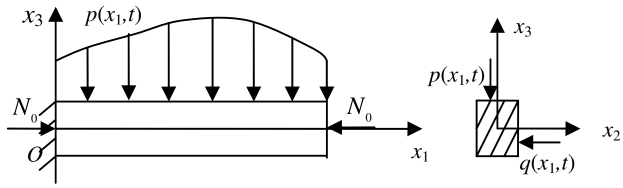

Figure 1. Model of a beam.

2. Basic equations of thermoelastic beams with voids

Consider a beam subjected to distributed transverse loads, as in Figure 1. Assume the beam's cross-section is uniform, the length is l, the width is b, and the thickness is h. Choose the x_1-axis so it contains the center of the section, the x_2- and x_3-axes as the orthogonal principal axes of the cross-section. Assume the displacements of the neutral axis are $u_i(x_1, t)$, for $i = 1, 2, 3$; we call $u_3(x_1, t)$ the deflection. Based on the Kirchhoff–Love hypothesis and finite deformation theory, we have the nonlinear geometry relations

$$\varepsilon_{11} = \varepsilon_{11}^0 - x_2 u_{2,11} - x_3 u_{3,11}, \quad \varepsilon_{11}^0 = u_{1,1} + (u_{2,1})^2/2 + (u_{3,1})^2/2, \tag{1}$$

where ε_{11} is the total strain component and ε_{11}^0 is the strain component along the neutral axis.

Following the theory presented in [Iesan 1986], the basic equations for thermoelastic materials with voids are given tensorially as follows:

$$\rho \ddot{u}_i = \sigma_{ji,j} + \rho f_i \quad \text{and} \quad \rho \chi \ddot{\varphi} = h_{i,i} + g + \rho l \tag{2}$$

are the equations of balance of linear momentum and equilibrated force, where σ_{ij} is the symmetric stress tensor, f_i is the body force vector, ρ is the density in the reference configuration, φ is the change in the volume fraction field, χ is the equilibrated inertia, h_i is the equilibrated stress vector, g is the intrinsic equilibrated body force, and ρl is the extrinsic equilibrated body force;

$$\rho T_0 \dot{\eta} = q_{i,i} + \rho S \tag{3}$$

is the equation for the balance of entropy, where S and η are the entropy and the external heat supply per unit initial mass, while q_i and T_0 are the heat flux and the absolute temperature in the reference configuration; and

$$\begin{gathered} \sigma_{11} = D_0 \varepsilon_{11} + b_v \varphi - \beta\theta, \quad g = -b_v \varepsilon_{11} - \xi_v \varphi + m_v \theta, \\ h_i = \alpha_v \varphi_{,i}, \quad \rho\eta = \beta\varepsilon_{11} + m_v \varphi + (\rho c_e/T_0)\theta, \quad q_i = K\theta_{,i} \end{gathered} \tag{4}$$

are the constitutive relations of isotropic thermoelastic beams with voids, where K the thermal conductivity, c_e the specific heat at the constant strain, σ_{11} be the beam's bending stress, α_v, b_v, ξ_v, m_v are parameters of the voids, θ denotes the absolute temperature with T_0 subtracted, D_0 is given by

$$D_0 = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}, \tag{5}$$

in which E, ν are the elastic modulus and Poisson's ratio, and finally $\beta = \alpha_t E/(1-2\nu)$, where α_t is the linear expansion coefficient.

3. The Hamilton principle and mathematical model of thermoelastic beams with voids

In order to present the nonlinear mathematical model of the problem, we first derive a generalized Hamilton principle for thermoelastic beams with voids under geometric nonlinearity.

Let

$$M_\varphi = \iint_A \varphi(x_i, t) x_2 x_3 \, dA \quad \text{and} \quad M_\theta = \iint_A \theta(x_i, t) x_2 x_3 \, dA \tag{6}$$

be the moments caused by the change of the volume fraction $\varphi(x_i, t)$ and temperature $\theta(x_i, t)$, respectively. Set

$$\begin{aligned} q_\varphi &= \alpha_v \iint_A x_2 x_3 (\varphi_{,22} + \varphi_{,33}) \, dA \\ &= \alpha_v \Big(\int_{-b/2}^{b/2} \big((x_3 \varphi_{,3})|_{-h/2}^{h/2} - \varphi|_{-h/2}^{h/2} \big) x_2 \, dx_2 + \int_{-h/2}^{h/2} \big((x_2 \varphi_{,2})|_{-b/2}^{b/2} - \varphi|_{-b/2}^{b/2} \big) x_3 \, dx_3 \Big), \\ q_\theta &= \frac{K}{T_0} \iint_A x_2 x_3 (\theta_{,22} + \theta_{,33}) \, dA \\ &= \frac{K}{T_0} \Big(\int_{-b/2}^{b/2} \big((x_3 \theta_{,3})|_{-h/2}^{h/2} - \theta|_{-h/2}^{h/2} \big) x_2 \, dx_2 + \int_{-h/2}^{h/2} \big((x_2 \theta_{,2})|_{-b/2}^{b/2} - \theta|_{-b/2}^{b/2} \big) x_3 \, dx_3 \Big). \end{aligned} \tag{7}$$

According to [Cowin and Nunziato 1983], on the surfaces of the beam, $\varphi(x_i, t)$ needs to satisfy the conditions $\varphi_{,2}|_{x_2=\pm b/2} = 0$ and $\varphi_{,3}|_{x_3=\pm h/2} = 0$, so the expression for q_φ simplifies to

$$q_\varphi = -\alpha_v \Big(\int_{-b/2}^{b/2} (\varphi|_{-h/2}^{h/2}) x_2 \, dx_2 + \int_{-h/2}^{h/2} (\varphi|_{-b/2}^{b/2}) x_3 \, dx_3 \Big) \tag{8}$$

The thermal boundary conditions of the beam may be given as

$$\begin{aligned} K\theta_{,2}|_{x_2=b/2} &= \hbar(T_\infty - \theta_{b/2}), \quad K\theta_{,2}|_{x_2=-b/2} = -\hbar(T_\infty - \theta_{-b/2}), \\ K\theta_{,3}|_{x_3=h/2} &= \hbar(T_\infty - \theta_{h/2}), \quad K\theta_{,3}|_{x_3=-h/2} = -\hbar(T_\infty - \theta_{-h/2}), \end{aligned} \tag{9}$$

where $\hbar$ is the heat transfer coefficient, $\theta_{b/2}$, $\theta_{-b/2}$, $\theta_{h/2}$, and $\theta_{-h/2}$ denote the temperatures on the surfaces $x_2 = \pm b/2$ and $x_3 = \pm h/2$, and T_∞ is the temperature of surrounding medium. If we assume that $T_\infty = 0$, the absolute temperature is equal to the reference temperature. The second expression of (7) then simplifies to

$$q_\theta = -\Big(\frac{h\hbar/2 + K}{T_0} \int_{-b/2}^{b/2} (\theta|_{-h/2}^{h/2}) x_2 \, dx_2 + \frac{b\hbar/2 + K}{T_0} \int_{-h/2}^{h/2} (\theta|_{-b/2}^{b/2}) x_3 \, dx_3 \Big). \tag{10}$$

To evaluate (8) and (10), we express the change of the volume fraction and temperature as the series

$$\varphi(x_i, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \varphi_{mn}(x_1, t) x_2^m x_3^n, \theta(x_i, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \vartheta_{mn}(x_1, t) x_2^m x_3^n \tag{11}$$

Substituting (11) into (6) yields

$$M_\varphi = \bar{A} \varphi_{11}(x_1, t) + o(b^3 h^3) \, , M_\theta = \bar{A} \vartheta_{11}(x_1, t) + o(b^3 h^3), \tag{12}$$

where $\varphi_{11}(x_1, t)$ and $\vartheta_{11}(x_1, t)$ are the terms of (11) when $m = n = 1$. The term $o(b^3h^3)$, indicating higher-order quantities than b^3h^3, can be omitted in the calculation.

From (8), (10), (11), (12), we have

$$q_\varphi \approx -\frac{(I_y + I_z)\alpha_v}{\bar{A}} M_\varphi, q_\theta \approx -\left(\frac{(h\hbar/2) + K}{T_0}\frac{I_z}{\bar{A}} + \frac{(b\hbar/2) + K}{T_0}\frac{I_y}{\bar{A}}\right) M_\theta, \tag{13}$$

where $\bar{A} = \iint_A (x_2x_3)^2\, dA = (b^3/12)(h^3/12)$, $I_y = \iint_A x_3^2\, dA = h^3b/12$, and $I_z = \iint_A x_2^2\, dA = b^3h/12$.

According to the theory of beams, the strain energy of a beam in terms of displacements, void moment and thermal moment may be expressed as

$$U = \frac{1}{2}\int_0^l \iint_A D_0\varepsilon_{11}\varepsilon_{11}\, dA\, dx_1 + \frac{1}{2(I_y + I_z)}\int_0^l (\alpha_v M_{\varphi,1}M_{\varphi,1} + \xi_v M_\varphi^2 - q_\varphi M_\varphi - \frac{\rho c_e}{T_0} M_\theta^2)\, dx_1$$
$$- \frac{1}{I_y + I_z}\int_0^l (b_v M_\varphi - \beta M_\theta)(I_z u_{2,11} + I_y u_{3,11})\, dx_1 - \frac{1}{I_y + I_z}\int_0^l m_v M_\varphi M_\theta\, dx_1. \tag{14}$$

Assume that the total kinetic energy of the beam is $T = T_1 + T_2$, in which T_1 is caused by displacements and void moment, T_2 is caused by rotation; these terms are given as

$$\begin{aligned} T_1 &= \frac{\rho h b}{2}\int_0^l (\dot{u}_1^2 + \dot{u}_2^2 + \dot{u}_3^2)\, dx_1 + \frac{1}{2(I_y + I_z)}\int_0^l \rho\chi \dot{M}_\varphi \dot{M}_\varphi\, dx_1, \\ T_2 &= \frac{\rho}{2}\int_0^l \iint_A \big((x_2\dot{u}_{2,1})^2 + (x_3\dot{u}_{3,1})^2\big)\, dA\, dx_1 = \frac{\rho}{2}\int_0^l \big(I_z(\dot{u}_{2,1})^2 + I_y(\dot{u}_{3,1})^2\big)\, dx_1. \end{aligned} \tag{15}$$

Define the symbol

$$D(M_\theta) = \frac{1}{2(I_y + I_z)}\int_0^l \int_0^{t_1} \left(\frac{K}{T_0} M_{\theta,1}M_{\theta,1} - q_\theta M_\theta\right) dt_1\, dx_1. \tag{16}$$

Assume that the beam is subjected to an arbitrary transverse distributed load q_i in the x_i-direction, that the two ends are subjected to an axial force N_0, and that all the forces are conservative. In the absence of external equilibrated force and heat source, the external work may be given as

$$W = \int_0^l q_i u_i\, dx_1 + \frac{N_0}{2}\int_0^l (u_{2,1}^2 + u_{3,1}^2)\, dx_1 + N_0 u_1(0) - N_0 u_1(a) \tag{17}$$

Let

$$\Pi_B = \int_0^l \rho h b\big((u_1|_{t=0} - u_1^0)\dot{u}_1 - \dot{u}_1^0 u_1 + (u_3|_{t=0} - u_3^0)\dot{u}_3 - \dot{u}_3^0 u_3\big)\, dx_1$$
$$+ \int_0^l \frac{\rho\chi}{I_y + I_z}\big((M_\varphi|_{t=0} - M_\varphi^0)\dot{M}_\varphi - \dot{M}_\varphi^0 M_\varphi\big)\, dx_1 + \int_0^l \frac{1}{I_y + I_z}\left(\int_0^{t_1} \frac{\rho c_e}{T_0}(M_\theta|_{t=0} - M_\theta^0) M_\theta\, dt\right) dx_1. \tag{18}$$

Generalized Hamilton principle for thermoelastic beams with voids. In all possible displacement fields $u_i(x_1, t)$, volume fraction field $\varphi(x_i, t)$and temperature field $\theta(x_i, t)$ satisfying the geometric constraint conditions and having the appointed motions at the initial and final time, the actual displacements

$u_i(x_1,t)$, volume fraction $\varphi(x_i,t)$ and temperature $\theta(x_i,t)$ make the following functional arrive at the stationary value

$$\Pi(u_i,\varphi,\theta)=\int_0^t (T+W+D-U)\,dt+\Pi_B, \tag{19}$$

where $H=T+W+D-U$ is a generalized Hamilton function.

Applying a variational calculation to (19) (whose detailed formulas are given in the Appendix) and substituting the results obtained into the variational equation of (19), that is, $\delta\Pi=0$, then integrating (19) with regard to time from 0 to the final time t, and observing the beam has the appointed motions at the initial and final time, as well as the arbitrariness of the variables δu_i, δM_φ, δM_θ on the interval $[0,l]$, we obtain the differential equations of motion in terms of u_i, M_φ, M_θ, in which the balance of entropy has been differentiated relative to t_1:

$$\begin{aligned}
&D_0hb\Big(u_{1,1}+\frac{(u_{2,1})^2+(u_{3,1})^2}{2}\Big)_{,1}+q_1=\rho hb\ddot{u}_1,\\
&D_0hb\Big(\Big(u_{1,1}+\frac{(u_{2,1})^2+(u_{3,1})^2}{2}\Big)u_{2,1}\Big)_{,1}-D_0I_zu_{2,1111}\\
&\qquad\qquad+\frac{I_z}{I_y+I_z}(b_vM_\varphi-\beta M_\theta)_{,11}+q_2-N_0u_{2,11}=\rho hb\ddot{u}_2-\rho I_z\ddot{u}_{2,11},\\
&D_0hb\Big(\Big(u_{1,1}+\frac{(u_{2,1})^2+(u_{3,1})^2}{2}\Big)u_{3,1}\Big)_{,1}-D_0I_yu_{3,1111}\\
&\qquad\qquad+\frac{I_y}{I_y+I_z}(b_vM_\varphi-\beta M_\theta)_{,11}+q_3-N_0u_{3,11}=\rho hb\ddot{u}_3-\rho I_y\ddot{u}_{3,11},\\
&\alpha_vM_{\varphi,11}+b_v(I_zu_{2,11}+I_yu_{3,11})-\xi_vM_\varphi+q_\varphi+m_vM_\theta=\rho\chi\ddot{M}_\varphi,\\
&KM_{\theta,11}+\beta T_0(I_z\dot{u}_{2,11}+I_y\dot{u}_{3,11})-m_vT_0\dot{M}_\varphi+q_\theta=\rho c_e\dot{M}_\theta.
\end{aligned} \tag{20}$$

This is a set of coupled nonlinear equations for u_i, M_φ and M_θ, in which the effects of the axial forces N_0, the neutral layer inertia $\rho hb\ddot{u}_1$, and the rotation inertias $\rho I_z\ddot{u}_{2,11}$ and $\rho I_y\ddot{u}_{3,11}$ are included.

It can be also seen that the boundary conditions at the end designated forces may be derived from the boundary virtual work equation in the variational equation $\delta\Pi=0$. If we only consider a clamped-beam without axial forces, the boundary conditions at the ends ($x_1=0,l$) are

$$u_i=0,\ u_{2,1}=0,\quad u_{3,1}=0,\quad M_{\varphi,1}=0,\quad M_\theta=0. \tag{21}$$

Observing that formula (18) is included in (19), the initial conditions at the initial time are given as

$$u_i=u_i^0,\quad M_\varphi=M_\varphi^0,\quad M_\theta=M_\theta^0,\quad \dot{u}_i=\dot{u}_i^0,\quad \dot{M}_\varphi=\dot{M}_\varphi^0, \tag{22}$$

in which u_i^0, M_φ^0, M_θ^0, $\dot{u}_i^0$, $\dot{M}_\varphi^0$ are the known functions of x_1. Especially, if the beam is at rest at the initial time, these functions are equal to zeros.

4. Solution method

As application of the mathematical model above, the nonlinear mechanical characteristics of a two end fixed beam without the axial force are investigated, and the influences of parameters are considered. For convenience, we here study the plane bending of the beam only, that is, $u_2(x_1, t) \equiv 0$. As it is difficult to obtain the solution of the problem directly, we will apply the differential quadrature method (DQM) to discretize the nonlinear system on the spatial domain.

The DQM is a numerical technique for solving boundary-valued problems. It was developed in [Bellman and Casti 1971], and since then it has been successfully employed to solve all kinds of problems in engineering and science due to the DQM owns the advantages of little amount of nodes and computation, high precision and good convergence and so on.

The DQM approximates the derivative of a function, with respect to the independent variable at a given discrete point, as a weighted linear sum of the values of the function at all the discrete points chosen in the solution domain of the independent variable, in which the weighting coefficients are only associated with the given discrete points in the solution domain and independent of a certain problem. Therefore, any differential equations can be transformed into a set of the corresponding algebraic equations.

Introduce the nondimensional variables and parameters

$$X = \frac{x_1}{h}, \quad U = \frac{u_1}{h}, \quad W = \frac{u_3}{h}, \quad \beta_1 = \frac{h}{l}, \quad \tau = t\frac{V_1}{l}, \quad \psi = 12\frac{M_\varphi}{h^2}, \quad \Theta = 12\frac{M_\theta}{T_0 h^2}, \tag{23}$$

$$a_1 = \frac{b_v}{D_0}, \quad a_2 = \frac{\beta T_0}{D_0}, \quad a_3 = \frac{b_v h^2}{\alpha_v}, \quad a_4 = \frac{\xi_v h^2}{\alpha_v}, \quad a_5 = \frac{m_v T_0 h^2}{\alpha_v}, \quad a_6 = \left(\frac{V_1}{V_3}\right)^2, \quad a_8 = \frac{\beta}{\rho c_e},$$
$$a_9 = \frac{m_v}{\rho c_e}, \quad a_{10} = \frac{\hbar}{2\rho c_e V_1}, \quad a_{11} = \frac{\rho c_e V_1 h}{K}, \quad \bar{p} = \frac{q_3}{D_0}, \quad V_1 = \sqrt{\frac{D_0}{\rho}}, \quad V_3 = \sqrt{\frac{\alpha_v}{\rho \chi}}. \tag{24}$$

From (24), one sees that the coefficients a_1 and a_3 are the coupling deformation-void parameters, which represent the coupling degree of the deformation and volume fraction field. The coefficient a_6 is the ratio of the longitudinal wave velocity to the volume fraction wave velocity, which represents the ratio of the elastic constant to the void constant. The coefficient a_4 is a void parameter. The coefficients a_2 and a_8 are the coupling deformation-heat parameters, which represent the coupling degree of the deformation and temperature field. The coefficient a_{10} is a convection heat transfer parameter, and a_{11} is the ratio of dilational wave velocity to thermal conductive coefficient. The coefficients a_5 and a_9 are the coupling void-heat parameters, which represent the coupling degree of the volume fraction and temperature field.

The differential quadrature discretization forms of the nondimensional differential equations are

$$\sum_{k=1}^{N} A_{ik}^{(2)} U_k + \beta_1 \sum_{k=1}^{N} A_{ik}^{(1)} W_k \cdot \sum_{l=1}^{N} A_{il}^{(2)} W_l = \ddot{U}_i,$$

$$\beta_1 \left(\sum_{k=1}^{N} A_{ik}^{(2)} U_k \cdot \sum_{l=1}^{N} A_{il}^{(1)} W_l + \sum_{k=1}^{N} A_{ik}^{(1)} U_k \cdot \sum_{l=1}^{N} A_{il}^{(2)} W_l \right) + \frac{3}{2}\beta_1^2 \left(\sum_{k=1}^{N} A_{ik}^{(1)} W_k \right)^2 \sum_{l=1}^{N} A_{il}^{(2)} W_l$$
$$- \frac{\beta_1^2}{12} \sum_{k=1}^{N} A_{ik}^{(4)} W_k + \frac{a_1}{12} \sum_{k=1}^{N} A_{ik}^{(2)} \psi_k - \frac{a_2}{12} \sum_{k=1}^{N} A_{ik}^{(2)} \Theta_k + \frac{\bar{p}}{\beta_1^2} = \ddot{W}_i - \frac{\beta_1^2}{12} \sum_{k=1}^{N} A_{ik}^{(2)} \ddot{W}_k, \tag{25}$$

$$\sum_{k=1}^{N} A_{ik}^{(2)} \psi_k + a_3 \sum_{k=1}^{N} A_{ik}^{(2)} W_k - \frac{a_4 + 12}{\beta_1^2} \psi_i + \frac{a_5}{\beta_1^2} \Theta_i = a_6 \ddot{\psi}_i,$$

$$\frac{\beta_1}{a_{11}} \sum_{k=1}^{N} A_{ik}^{(2)} \Theta_k + a_8 \beta_1^2 \sum_{k=1}^{N} A_{ik}^{(2)} \dot{W}_k - a_9 \dot{\psi}_i - \left(a_{10} + \frac{1}{a_{11}} \right) \frac{12}{\beta_1} \frac{\theta_+ - \theta_-}{T_0} = \dot{\Theta}_i,$$

where i ranges from 1 to N, the number of discrete points, and $A_{ik}^{(j)}$ is the weighting coefficient of the j-th partial derivative of the function with respect to the independent variable X. In this paper, the polynomial function is adopted as the test function to obtain the weighting coefficient, and the zeros of the Chebyshev–Lobatto polynomial are adopted as the coordinates of the grid points [Bellman and Casti 1971].

The DQ discretization forms of the nondimensional boundary conditions can be expressed as

$$\begin{gathered} U_1 = U_N = 0, \quad W_1 = W_N = 0, \quad \Theta_1 = \Theta_N = 0, \\ \sum_{k=1}^{N} A_{1k}^{(1)} \psi_k = \sum_{k=1}^{N} A_{Nk}^{(1)} \psi_k = 0, \quad \sum_{k=1}^{N} A_{1k}^{(1)} W_k = \sum_{k=1}^{N} A_{Nk}^{(1)} W_k = 0. \end{gathered} \tag{26}$$

If the initial values of variables are all zero, we have the initial conditions of nondimensional forms

$$U(0) = \dot{U}(0) = W(0) = \dot{W}(0) = \psi(0) = \dot{\psi}(0) = \Theta(0) = 0 \tag{27}$$

Assuming that the temperature distribution is given by $\theta(x_1, x_3) = \theta_0(1/2 + x_3/h)$, we have $\theta_+ = \theta_0$, $\theta_- = 0$.

Hence, the static problem is converted to solving the nonlinear algebraic equations (25) under the boundary conditions (26); while the dynamical problem is converted to solving the nonlinear ordinary differential equations (25) under the boundary conditions (26) and initial condition (27). The Newton–Raphson and Runge–Kutta methods are adopted in the calculation of the static and dynamical problems, respectively.

To illustrate the correctness of the theory and method in this paper, a simply supported elastic beam subjected to a uniformly distributed load $\bar{p}$ is considered. The analytic solution is available in the case of small deformation. Figure 2a shows the comparison between the analytic solution and the numerical result obtained from the theory and method of this paper when $\bar{p} = 10^{-5}$ and 7 nodes are collocated. It can be seen that the results are accordant. Figure 2b shows the comparison between the analytic solution and the numerical results when $\bar{p} = 10^{-4}$ and 7 and 9 nodes are collocated, respectively. It is seen that the solutions of linear elastic beam under small deformation and the nonlinear theory are no longer consistent. The DQ solution is slightly less than the analytical solution due to the nonlinear effect. From Figure 2b, one can see that the DQ solution has good convergence also.

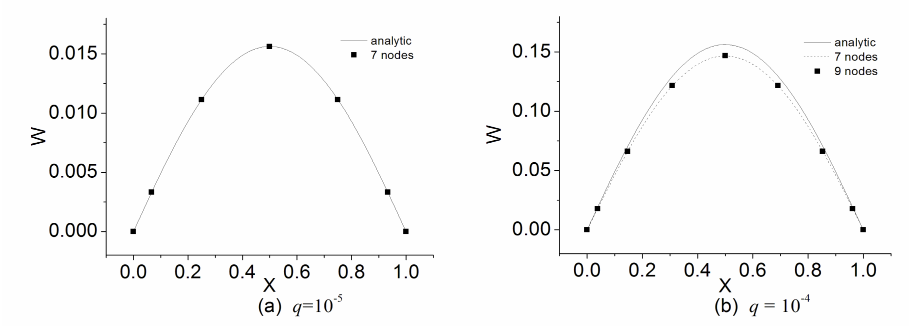

Figure 2. Comparison of the DQ solution with the analytic solution.

5. Nonlinear mechanical characteristics of thermoelastic beam with voids

Nonlinear static behavior of thermoelastic beams with voids. The Newton–Raphson method is used to solve the static system (25)–(26) numerically. In this situation, the thermal moment, which is independent of the deflection and void moment but just dependent of the lower and upper surface temperature, can be solved directly from $(25)_4$. In Table 1 we give the effects of the node distribution on the DQ solution of the dimensionless deflection and void moment at the middle point of the beam for four different materials. The parameters are chosen as follows (see [Puri and Cowin 1985; Kumar and Rani 2005]):

$$\beta_1 = 0.1, \quad a_1 = 0.33, \quad a_2 = 0.027, \quad a_3 = 5, \quad a_4 = 6, \quad a_{11} = 4.56 \times 10^6, \quad \theta_0 = 10, \quad \bar{p} = 0.0001.$$

One can see that satisfactory results can be obtained when 9 nodes are collocated on the physical interval [0, 1] for elastic beams (EB) and thermoelastic beams (TEB), while 15 nodes are needed for elastic beams with voids (EVB) and thermoelastic beams with voids (TEVB). Hence, we set $N = 15$ in the calculation. When $\theta_0 > 0$, this means that the beam is subjected to a thermal moment, which is just opposite to the bending moment of external force. From the comparison of the deflections of EB and EVB, we see that the deflections of EVB are larger than the ones of EB. We can conclude that the presence of voids enlarges the deflection.

Effect of a_1. It can be observed from Figure 3 that the deflection and void moment of EVB increase with a_1. From Figure 4 it is seen that the deflection of TEVB decreases as a_1 increases, while the corresponding void moment increases.

Effect of a_3. As for EVB, the results are similar to Figure 3, that is to say, the deflection and void moment of EVB increases with a_3. From Figure 5, one can see that the deflection of TEVB increases with an increase in a_3, while the void moment decreases.

Effect of a_4. Figure 6 shows that the deflection and void moment of EVB decrease with an increase in a_4. Figure 7 suggests that the deflection of TEVB increases with an increase in a_4, while the corresponding void moment decreases.

Effect of a_2. As for TEB and TEVB, the temperature can be solved directly because the energy equation is an ordinary differential equation about X. So, the parameter a_2 has no effect on the temperature. The

	EB	TEB	EVB		TEVB	
nodes	W	W	W	ψ	W	ψ
7	0.031232	−0.25341	0.034650	−0.0015148	−0.20074	−0.095229
9	0.031228	−0.25123	0.034422	−0.0015084	−0.20265	−0.095241
11	0.031228	−0.25123	0.034351	−0.0015074	−0.20373	−0.095229
13	0.031228	−0.25123	0.034330	−0.0015066	−0.20406	−0.095216
15	0.031228	−0.25123	0.034323	−0.0015065	−0.20416	−0.095216
17	0.031228	−0.25123	0.034322	−0.0015065	−0.20419	−0.095215
19	0.031228	−0.25123	0.034322	−0.0015065	−0.20420	−0.095215
21	0.031228	−0.25123	0.034322	−0.0015065	−0.20420	−0.095215

Table 1. Effect of the node distribution on the DQ solutions for static system.

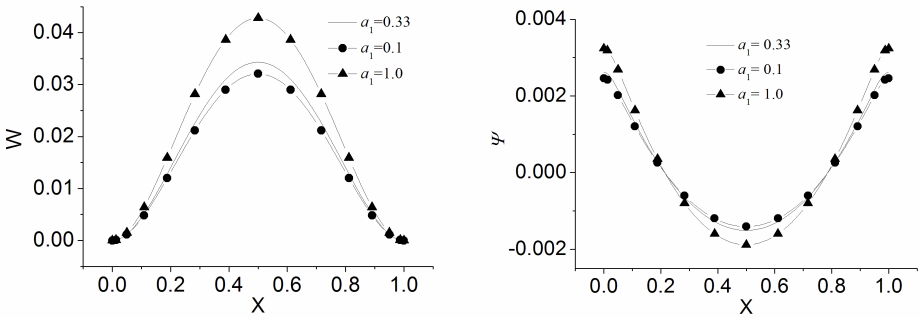

Figure 3. Variation of deflection W and void moment ψ of EVB with X, for various a_1.

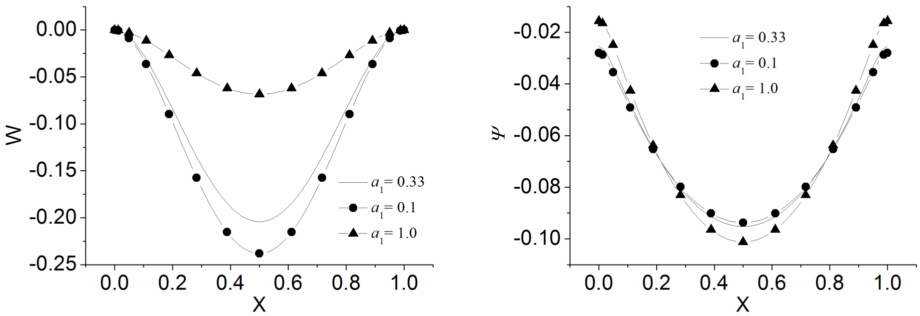

Figure 4. Variation of deflection W and void moment ψ of TEVB with X, for various a_1.

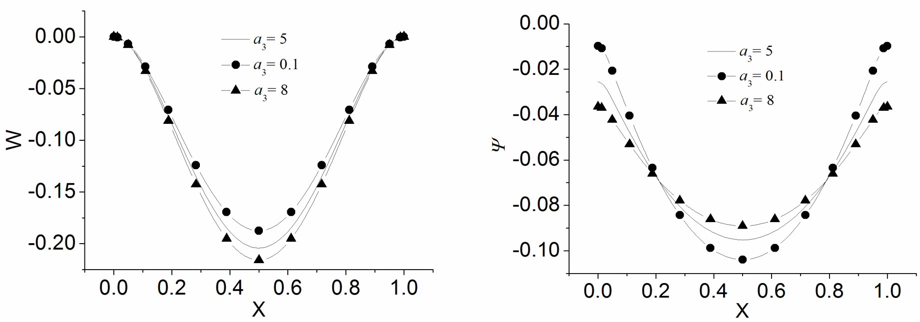

Figure 5. Variation of deflection W and void moment ψ of TEVB with X, for various a_3.

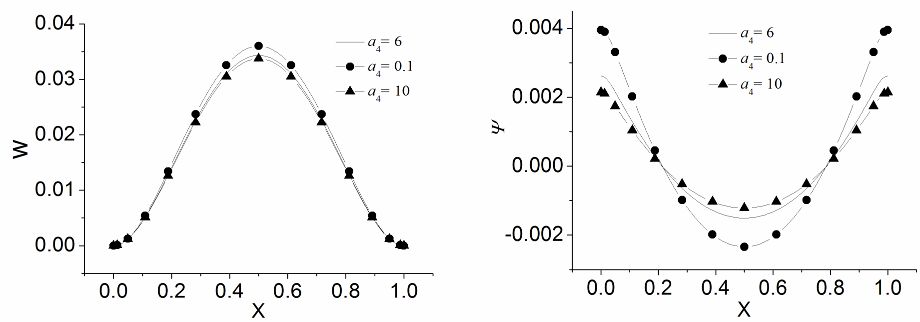

Figure 6. Variation of deflection W and void moment ψ of EVB with X, for various a_4.

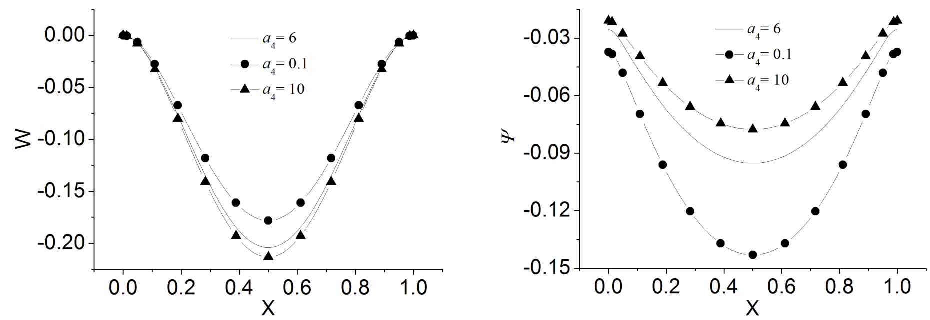

Figure 7. Variation of deflection W and void moment ψ of TEVB with X, for various a_4.

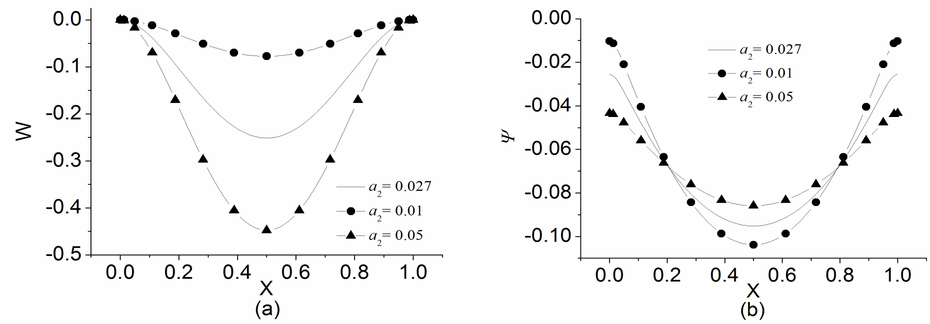

Figure 8. Variation of deflection W of EVB and void moment ψ of TEVB with X, for various a_2.

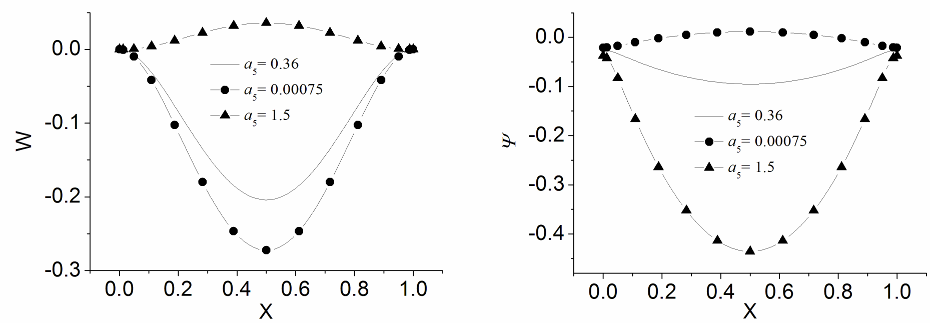

Figure 9. Variation of deflection W and void moment ψ of TEVB with X, for various a_5.

deflection of TEB increases with an increase in a_2, as seen in Figure 8a, and so does the deflection of TEVB. In contrast, the void moment of TEVB decreases with an increase in a_2, as seen in Figure 8b.

Effect of a_5. The parameter a_5 is a coupling one of volume fraction and temperature field. Figure 9 shows that the deflection of TEVB varies from negative to positive with an increase in a_5, while the void moment varies from positive to negative.

Effect of a_{10} *and* a_{11}. When $a_{10} = 0$, the boundary of the beam is adiabatic, meanwhile, the temperature is independent of a_{11} but just dependent of the thickness-length ratio of the beam. When $a_{10} \to \infty$, the

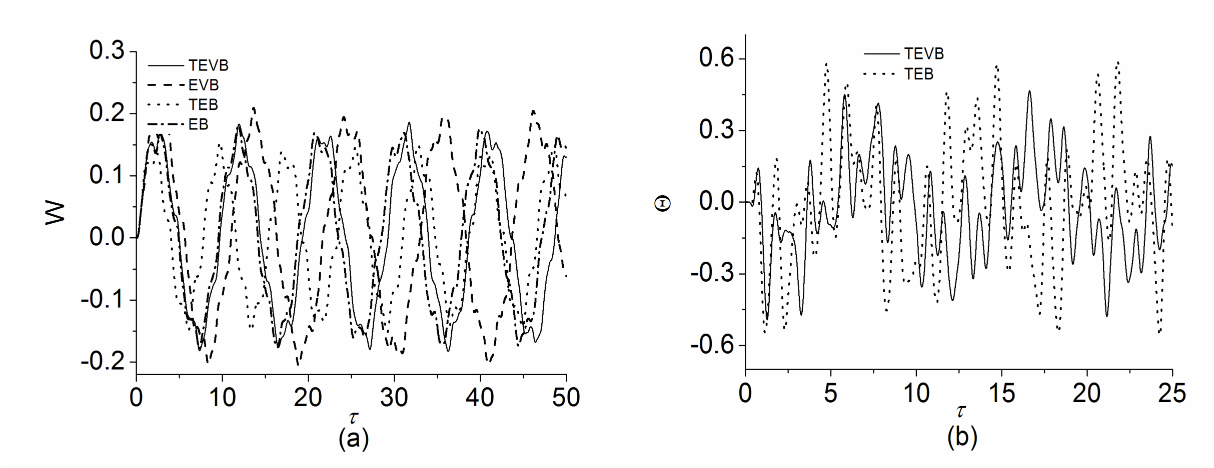

Figure 10. Time-history curves of the beams deflection W and thermal moment Θ.

boundary of beam is isothermal. The numerical calculation shows that the deflection, void moment and thermal moment all increase with an increase in $a_{10}$and a_{11}, respectively.

To summarize, we can see that the effect of the parameters a_1, a_2, a_3, a_4, a_5 on the deflection and the void moment is significant for the static problem.

Nonlinear dynamical behavior of thermoelastic beams with voids. The Runge–Kutta method of fourth order is used to solve the nonlinear dynamical system (25)–(27) numerically, to obtain time-history curves of the corresponding variables at the middle point of the beam. In computation, setting $N = 15$, the parameters are given as follows (see [Puri and Cowin 1985; Kumar and Rani 2005]):

$$\beta_1 = 0.1, \quad \bar{p} = 0.005\sin(2\pi\tau), \quad h = 0.1, \quad \theta_0 = 100, \quad a_1 = 1.0, \quad a_2 = 0.05, \quad a_3 = 5.0,$$
$$a_4 = 6.0, \quad a_5 = 0.36, \quad a_6 = 0.6, \quad a_8 = 10.0, \quad a_9 = 8.0, \quad a_{10} = 1.0\times 10^{-8}, \quad a_{11} = 4.56\times 10^6.$$

For comparison, the time-history curves of the deflection for four beams (EB, TEB, EVB, TEVB) are presented in Figure 10a, and those of the thermal moment for TEB and TEVB are shown in Figure 10b. It can be observed that the presence of voids enlarges beam deflection, while the thermal effect is the opposite: maybe the external work is partially transformed to thermal energy and dissipated. It is also seen that the thermal moment increases due to the presence of voids. Comparing the time-history curves of the void moment of TEVB and EVB, one can see that void moment of TEVB is slightly larger.

Next, the effect of parameters will be investigated. For the static problem, the parameters a_6, a_8, a_9 are absent. For the dynamical problem, the parameter a_6 has no influence on the dynamical behavior; one only has to study the effects of a_8 and a_9.

Effect of a_8. Figure 11 depicts the time-history curves of thermal moment of TEVB and TEB. It can be observed that thermal moment decreases sharply then increases with the decrease of a_8 for TEVB, from Figure 11a; while the thermal moment just decreases sharply with the decrease of a_8 for TEB, from Figure 11b. Meanwhile, the deflection of these beams increases with the decrease of a_8, and the void moment has barely changes.

Effect of a_9. The effect of the parameter a_9 on the deflection and void moment of TEVB is negligible, but Figure 12a shows that the thermal moment of TEVB increases with the decrease of a_9.

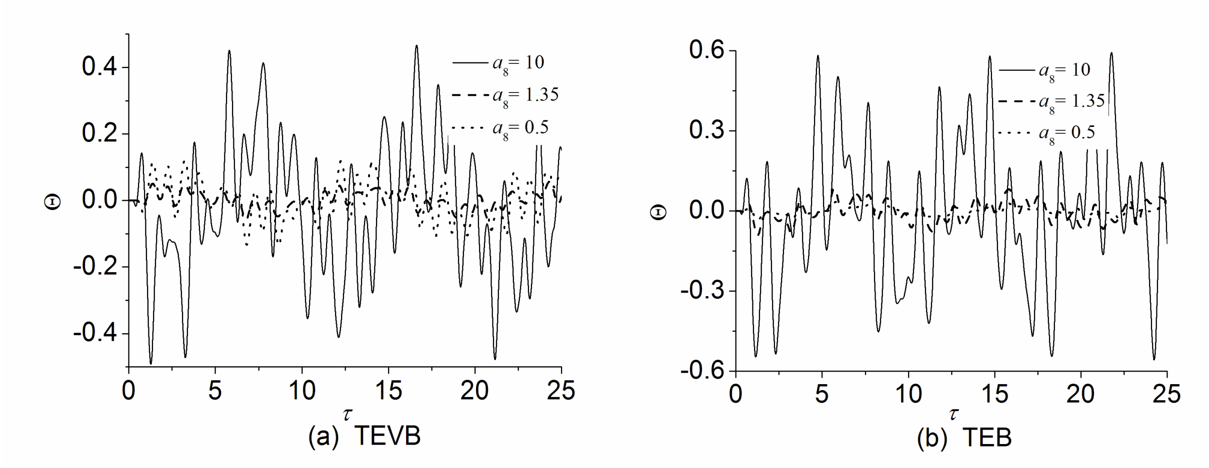

Figure 11. Time-history curves of the thermal moment of TEVB and TEB, for various values of a_8.

Effect of other parameters. For EVB and TEVB, the deflection and void moment all decrease with the decrease of the parameters a_1, a_3, a_5, respectively, while increase with the decrease of a_4. Similarly to the static results, the effect of the parameters a_3 and a_4 on the void moment of EVB is prominent, at the same time, the effect of the parameters a_3, a_4 and a_5 on the void moment of TEVB is significant, but on the corresponding thermal moment may be negligible.

For TEVB, the deflection, void moment and thermal moment all increase with the decrease of a_2, and the latter two vary more obviously. As for TEB, the deflection and thermal moment also increase with the decrease of a_2.

For TEVB and TEB, the effect of the parameter a_{10} on the deflection is negligible. The thermal moment no longer vibrates with the X-axis but gradually deviated from the X-axis downward with time with an increase in a_{10}, this means that the boundary of beam varies from adiabatic to isothermal. Figure 12b shows the time-history curves of the thermal moment of TEVB for different a_{10}. Besides, the curves of the void moment are similar to the thermal moment, but the value changes slightly.

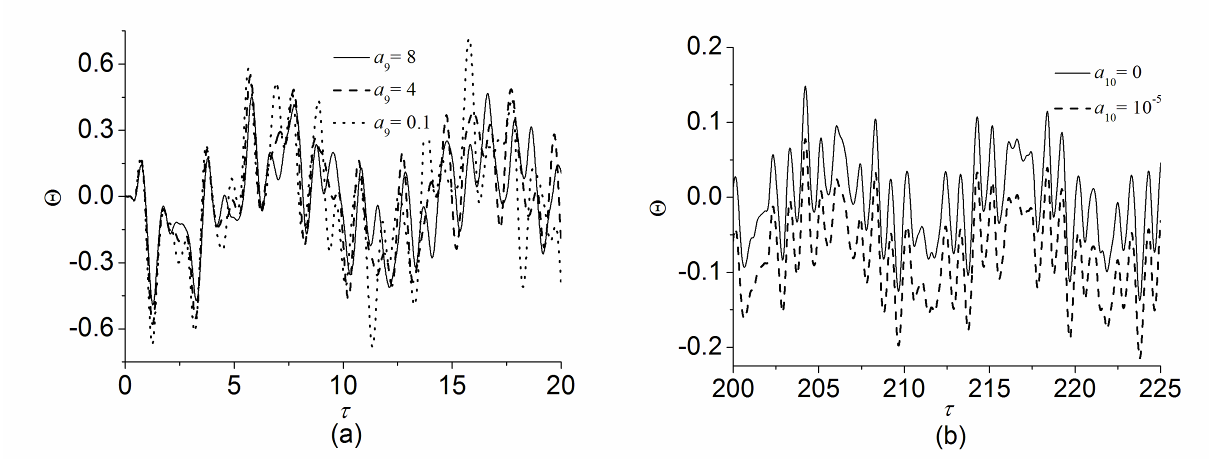

Figure 12. Time-history curves of thermal moment of TEVB, for various values of a_9 (left) and a_{10} (right).

For TEVB and TEB, the amplitudes of deflection all decrease slightly with time as the parameter a_{11} increases. Besides, the variations of the void moment and thermal moment with a_{11} are similar to a_{10}.

6. Conclusion

In this paper, the equation for the balance of entropy is firstly converted to an equivalent form without the first-order time-derivative by integral, and introducing the moments caused by the change of the void volume fraction and temperature, the Hamilton variational principle is extended to the three-dimensional thermoelastic beams with voids under finite deformations. A nonlinear theory of thermoelastic beams with voids is established based on the Kirchhoff–Love hypothesis, in which the influences of the axial forces, neutral layer inertia and rotation inertia are considered. The theory presents a set of coupling nonlinear differential equations of three displacements, one void moment and one thermal moment, in which not only the accelerations of the deflection and void moment are included, but also the effects of the neutral layer inertia and rotation inertia. So the theory is a complete nonlinear mathematical model of thermoelastic beams with voids under the Kirchhoff–Love hypothesis.

To illustrate the correctness of the theory and method of this paper, the plane bending problem of a simply supported elastic beam with small deformations is first studied, and the results are compared with the analytic solution. One can see that the numerical solution obtained from the DQM is accordant with the analytic solution, and the DQ solution has good convergence also.

As application, the static and dynamic responses of the plane bending of a fixed-fixed beam are studied. It is difficult to obtain the analytic solution of the nonlinear problem, so one kind of numerical methods is adopted to solve the problem. Firstly, the DQM is used to discrete the nonlinear system on the spatial domain, a set of nonlinear algebraic equations or ordinary differential equations are obtained for the static or dynamic problem, respectively. Then, the Newton–Raphson method and Runge–Kutta method are adopted to calculate the static and dynamical system, respectively. Mechanical characteristics of the system are investigated for four beams, that is, TEVB, TEB, EVB and EB, and the influences of parameters are all investigated. The general conclusion is that the presence of voids enlarges beam deflection. The deformation-void coupling parameters mainly bring changes in the deflection and void moment. The effect of the deformation-heat coupling parameters is significant. The void-heat coupling parameters change the void moment and thermal moment greatly, but the effect on the deflection is little. Beam deflection increases with an increase in a_1, a_3, a_5 but decreases with an increase in a_2, a_4, a_8. The effect of other parameters on the deflection is not obvious. For the dynamical problem, the effects of the temperature on the deflection and void moment are less obvious than the static case, which maybe the external work is transformed to the thermal energy partially and to be dissipated.

Appendix: Derivation of Hamilton variational principle of thermoelastic beam with voids

Letting $u_2(x_1, t) \equiv 0$ in the equations above corresponds to considering only plane bending of the beam. Applying a variational calculation to (19), we then obtain the variational formulas

$$\delta T_1 = -\iint_\Omega \{\rho h(\ddot{u}_1 \delta u_1 + \ddot{u}_3 \delta u_3) + \rho\chi \ddot{M}_\varphi \delta M_\varphi\}\, d\Omega + \iint_\Omega \frac{\partial}{\partial t}\{\rho h(\dot{u}_1 \delta u_1 + \dot{u}_3 \delta u_3) + \rho\chi \dot{M}_\varphi \delta M_\varphi\}\, d\Omega,$$

$$\delta T_2 = I\rho \oint_{\partial\Omega}\Big(\frac{\partial \dot{u}_3}{\partial x_1} l\Big)\delta\dot{u}_3\,ds - I\rho\iint_{\Omega}\Big(\frac{\partial}{\partial t}\Big(\frac{\partial^2 \dot{u}_3}{\partial x_1^2}\delta u_3\Big) - \frac{\partial^2 \ddot{u}_3}{\partial x_1^2}\delta u_3\Big)\,d\Omega,$$

$$\delta W = \iint_{\Omega}(q_1\delta u_1 + q_3\delta u_3)\,d\Omega + N_{x0}\delta u_1(0) - N_{xa}\delta u_1(a),$$

$$\delta D(M_\theta) = \iint_{\Omega}\int_0^{t_1}\Big(-\frac{K}{T_0}M_{\theta,11}\delta M_\theta + \Big(\frac{h\hbar}{2T_0}+\frac{K}{T_0}\Big)\frac{12}{h^2}M_\theta\delta M_\theta\Big)\,dt_1\,d\Omega + \oint_{\partial\Omega}\int_0^{t_1}\frac{K}{T_0}(lM_{\theta,1}\delta M_\theta)\,dt_1\,ds,$$

$$\begin{aligned}\delta U = {}& -(\lambda+2\mu)h\iint_{\Omega}\Big(\frac{\partial \varepsilon_{11}^0}{\partial x_1}\delta u_1 + \frac{\partial}{\partial x_1}\Big(\varepsilon_{11}^0\frac{\partial u_3}{\partial x_1}\Big)\delta u_3\Big)\,d\Omega + \frac{(\lambda+2\mu)h^3}{12}\iint_{\Omega}\frac{\partial^4 u_3}{\partial x_1^4}\delta u_3\,d\Omega\\ & + \iint_{\Omega}\Big(-\alpha_v M_{\varphi,11}\delta M_\varphi + \xi_v M_\varphi\delta M_\varphi + \frac{12\alpha_v}{h^2}M_\varphi\delta M_\varphi\Big)\,d\Omega - \iint_{\Omega} m_v(M_\varphi\delta M_\theta + M_\theta\delta M_\varphi)\,d\Omega\\ & - \frac{h^3}{12}\iint_{\Omega}\Big((b_v\delta M_\varphi - \beta\delta M_\theta)\frac{\partial^2 u_3}{\partial x_1^2} + \Big(b_v\frac{\partial^2 M_\varphi}{\partial x_1^2} - \beta\frac{\partial^2 M_\theta}{\partial x_1^2}\Big)\delta u_3\Big)\,d\Omega - \iint_{\Omega}\frac{\rho c_e}{T_0}M_\theta\delta M_\theta\,d\Omega + \delta U_B,\end{aligned}$$

in which

$$\begin{aligned}\delta U_B = {}& (\lambda+2\mu)h\oint_{\partial\Omega}\Big(l\varepsilon_{11}^0\delta u_1 + l\varepsilon_{11}^0\frac{\partial u_3}{\partial x_1}\delta u_3\Big)\,ds + \frac{(\lambda+2\mu)h^3}{12}\oint_{\partial\Omega}\Big(l\frac{\partial^2 u_3}{\partial x_1^2}\delta\frac{\partial u_3}{\partial x_1} - l\frac{\partial^3 u_3}{\partial x_1^3}\delta u_3\Big)\,ds\\ & + \oint_{\partial\Omega}\Big(\alpha_v\Big(l\frac{\partial M_\varphi}{\partial x_1}\delta M_\varphi\Big) - \frac{h^3}{12}\Big(l(b_v M_\varphi - \beta M_\theta)\delta\frac{\partial u_3}{\partial x_1}\Big) + \frac{h^3}{12}\big(l(b_v M_\varphi - \beta M_\theta)_{,1}\big)\delta u_3\Big)\,ds,\end{aligned}$$

$$\begin{aligned}\delta\Pi_B = {}& \int_0^t\Big(\int_{\partial V_\sigma}\overline{T}_\alpha\delta u_\alpha\,ds + \int_{\partial V_u}(u_\alpha - \bar{u}_\alpha)\delta T_\alpha\,ds + \int_{\partial V_h}\overline{M}_h\delta M_\varphi\,ds + \int_{\partial V_\varphi}(M_\varphi - \overline{M}_\varphi)\delta M_h\,ds\\ & + \int_{\partial V_Q}\frac{1}{T_0}\overline{M}_Q\delta M_\theta\,ds + \int_{\partial V_\theta}\frac{1}{T_0}(M_\theta - \overline{M}_\theta)\delta M_Q\,ds\Big)\,dt + \iint_{\Omega}\rho h\big((u_\alpha|_{t=0} - u_\alpha^0)\delta\dot{u}_\alpha - \dot{u}_\alpha^0\delta u_\alpha\big)\,d\Omega\\ & + \iint_{\Omega}\rho\chi h\big((M_\varphi|_{t=0} - M_\varphi^0)\delta\dot{M}_\varphi - \dot{M}_\varphi^0\delta M_\varphi\big)\,d\Omega + \iint_{\Omega}h\Big(\int_0^{t_1}\frac{\rho c_e}{T_0}(M_\theta|_{t=0} - M_\theta^0)\delta M_\theta\,dt\Big)\,d\Omega.\end{aligned}$$

Substituting these variational formulas into $\delta\Pi = 0$, we can obtain the differential equations of motion in terms of u_1, u_3, M_φ, M_θ, that is, Equations (20).

Boundary conditions can be derived from the following boundary virtual work equation in variational equation:

$$\begin{aligned}(\lambda+2\mu)h & \oint_{\partial\Omega}\Big(l\varepsilon_{11}^0\delta u_1 + l\varepsilon_{11}^0\frac{\partial u_3}{\partial x_1}\delta u_3\Big)\,ds + \frac{(\lambda+2\mu)h^3}{12}\oint_{\partial\Omega}\Big(l\frac{\partial^2 u_3}{\partial x_1^2}\delta\frac{\partial u_3}{\partial x_1} - l\frac{\partial^3 u_3}{\partial x_1^3}\delta u_3\Big)\,ds\\ & + \oint_{\partial\Omega}\Big(\alpha_v\Big(l\frac{\partial M_\varphi}{\partial x_1}\delta M_\varphi\Big) - \frac{h^3 l}{12}(b_v M_\varphi - \beta M_\theta)\delta\frac{\partial u_3}{\partial x_1} + \frac{h^3 l}{12}(b_v M_\varphi - \beta M_\theta)_{,1}\delta u_3\Big)\,ds\\ & + \oint_{\partial\Omega}\int_0^{t_1}\frac{K}{T_0}(lM_{\theta,1}\delta M_\theta)\,dt_1\,ds - N_{xa}\delta u_1(a) + N_{x0}\delta u_1(0) = 0.\end{aligned}$$

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