Journal of
Mechanics of
Materials and Structures

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Volume 5, No. 6

June 2010

mathematical sciences publishers

JOURNAL OF MECHANICS OF MATERIALS AND STRUCTURES
Vol. 5, No. 6, 2010

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The uniform monolithic metallic plate is the standard for assessing structural performance against blasts. For example, this has been the reference in recent work assessing the blast resistance of sandwich plates. Here, the performance of monolithic plates made from a hardening metal with high ductility is investigated by systematic optimization. The thickness distribution of clamped monolithic wide plates is optimized with respect to minimizing the permanent deflection at midspan under impulsive loads. The optimized plates are compared to a uniform plate of the same mass. Two load cases are considered: one with impulses acting uniformly along the plate, and one with impulses acting over a central patch. In both cases the reduction of permanent deflection of the optimized plate relative to the uniform plate is pronounced for small impulses but progressively smaller for larger impulses. For large impulses the optimal thickness distribution differs only slightly from that of a uniform plate. The study confirms the effectiveness of uniform thickness plates against large impulses.

1. Introduction

Plates subjected to blast loads have been extensively studied; especially so, in recent years, in the context of all-metal sandwich plates [Fleck and Deshpande 2004; Xue and Hutchinson 2004; Qiu et al. 2005; Dharmasena et al. 2008]. In all of these studies the performance of the sandwich plates is compared to the performance of uniform monolithic plates of the same mass. The aim of the present study is to optimize the thickness distribution of monolithic plates for impulsive loads. This has direct applications in its own right, and the results can also be used to asses the effectiveness of the uniform plate as the standard reference for blast load resistance.

An infinite depth clamped plate with length $2L$ and thickness H in Figure 1 is considered. The plate is loaded with a pressure given as an exponentially decaying function typical of blast loads:

$$P(x,t) = f(x)\exp\left(\frac{-t}{\tau}\right), \quad -L \le x \le L, \quad t \ge 0, \tag{1}$$

where τ is the characteristic time of loading, t is time, x is the distance from midspan (see Figure 1), and $f(x)$ describes the spatial distribution of the pressure, which is detailed below. The characteristic time τ is chosen such that it is much shorter than the response time of the plate T (defined as the time to reach the maximum midspan deflection), hence the loading is effectively impulsive[1]. Two situations are considered: uniform loading with $f(x) = p_0$, and loading over a central patch with $f(x) = p_0\exp(-(x/a)^2)$, where

Keywords: impulsive loads, FE simulations, finite deflections, blast resistance, monolithic plates.

[1]Numerical studies show that for all practical purposes $T/\tau = 75$ is more than adequate for this purpose.

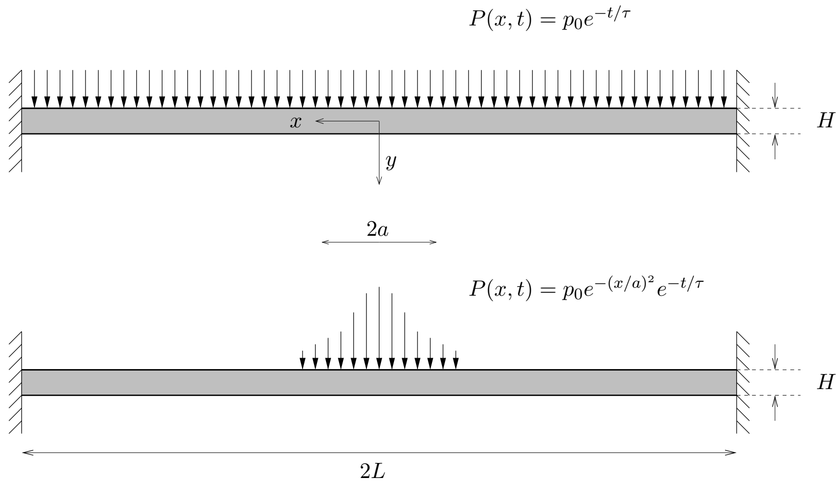

Figure 1. Clamped plate loaded with a pressure $P(x, t)$. Uniform loading (top) and loading over a central patch (bottom).

a is the characteristic spatial width of the pressure pulse. The impulse per unit depth of the plate is

$$I = 2\int_0^\infty\int_0^L P(x,t)\,dx\,dt = \begin{cases} 2Lp_0\tau & \text{for uniform loading,} \\ \sqrt{\pi}ap_0\tau & \text{for central patch loading } (a \ll L), \end{cases} \tag{2}$$

where the expression for central patch loading is an excellent approximation for localized pressure ($a \ll L$) as considered in this study, and the infinite upper limit on the integral over time is a good approximation when the simulation time is much larger than τ. The dimensionless impulse per unit depth of the plate $\hat{I} = I/(L^2\sqrt{\sigma_Y \rho})$ is used throughout this article, where σ_Y and ρ are defined below.

The plate material is assumed isotropic with power-law hardening stress-strain relationship

$$\sigma = \begin{cases} E\epsilon, & \epsilon \le \dfrac{\sigma_Y}{E}, \\ \sigma_Y\left(\dfrac{E\epsilon}{\sigma_Y}\right)^N, & \epsilon > \dfrac{\sigma_Y}{E}, \end{cases} \tag{3}$$

where σ is the true stress, ϵ the true strain, σ_Y the initial yield stress, E the Young's modulus, and N the hardening exponent.

The conventional J_2 flow rule is used. For a sufficiently ductile metal alloy such that necking occurs before any material failure, necking controls the maximum impulse the plate can withstand. This is due to the fact that if a plate is loaded with an impulse that is just large enough to produce necking at the boundary, then any larger impulse will almost certainly cause separation at the boundary because all the additional energy associated with the larger impulse will be absorbed only within the highly localized necking region. Necking and localization of plastic deformation is accurately captured by the finite strain finite element formulation used, which is detailed below. A material failure criterion that would predict details of the final fracture of the neck is not adopted. Failure by necking is evident in experiments on thin beams subject to a uniform impulsive load [Menkes and Opat 1973]. As detailed above, impulse levels above those required to produce the onset of necking caused the beams to fully separate at the clamped ends in a necking mode.

The material parameters used throughout the article are: $\sigma_Y = 260\,\text{MPa}$, $N = 0.2$, $E = 200\,\text{GPa}$, density $\rho = 7800\,\text{kg/m}^3$, and Poisson's ratio $\nu = 0.3$. Unless otherwise stated the following dimensions are used: $L = 100\,\text{mm}$, $H = 6\,\text{mm}$, and $a = 20\,\text{mm}$. For this choice of parameters numerical studies show that for $a/L < 0.15$ necking at midspan develops for the uniform plate at large impulses, while for $a/L > 0.15$ the plate fails by necking at the supports, which is also the case for uniform pressure. In this paper $a/L = 0.2$ is used, hence the failure modes for the central patch and uniform loading are identical. All computations are performed using the finite strain formulation available in the finite element code ABAQUS/Explicit version 6.6-1 [2006].

The clamped plate is modeled using four-node plane strain quadrilateral elements with reduced integration and hourglass control (CPE4R), with 5 elements through the thickness of the plate H and 125 elements along half of the plate length L. Symmetry is exploited; thus symmetry boundary conditions are used at the center and only half the plate is modeled. To evaluate the permanent deflection, an artificial viscous damping is introduced after a few elastic oscillations, which damps out the elastic vibrations (see Figure 2). This shows the time history of the vertical deflection (the y-axis of the coordinate system in Figure 1) of the midspan denoted δ. As only negligible plastic deformation takes place after the damping is activated it is assumed that this is a valid approach for finding the permanent deformation.

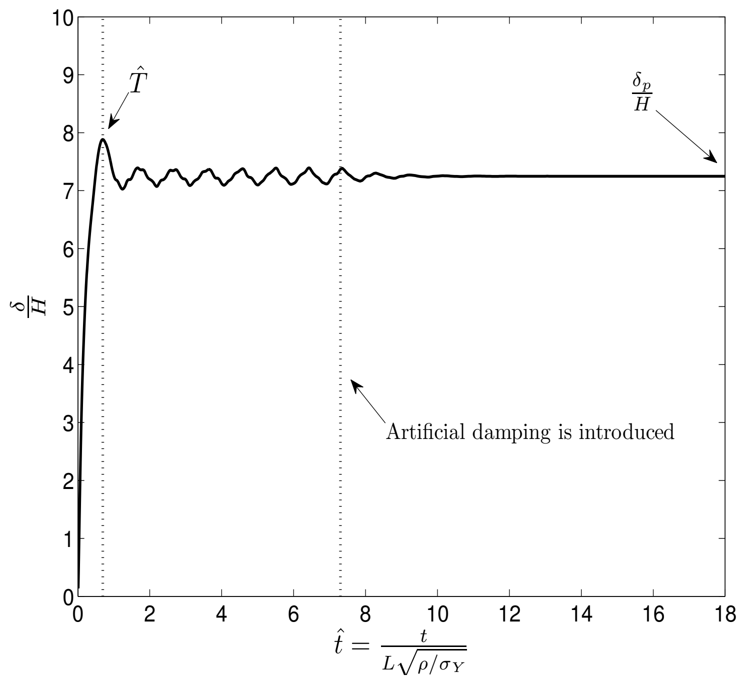

Figure 2. Normalized deflection at the midspan for central patch loading with $H/L = 6/100$, $\hat{I} = 0.0535$, and $a/L = 0.2$. The introduction of artificial viscous damping is indicated as is the dimensionless plate response time $\hat{T}$ and the normalized permanent deflection δ_p/H.

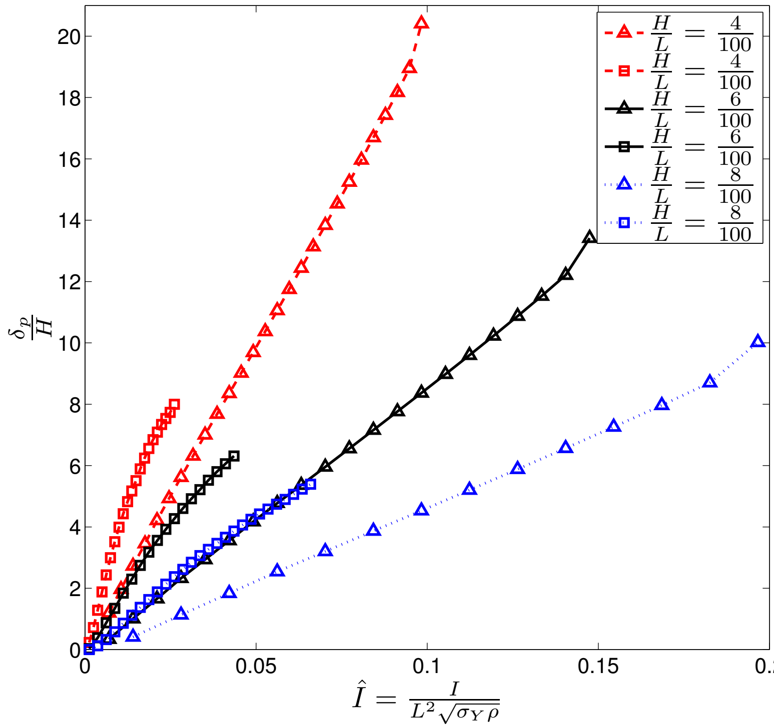

Figure 3. Permanent deflection at the midspan of uniform plates. Triangular markers indicate uniform pressure, and square markers indicate central patch pressure (with characteristic spatial width $a/L = 0.2$). The curves terminate at the impulse when substantial necking is observed.

Figure 3 gives an overview of the nondimensional permanent displacement of the midspan δ_{p}/H of uniform plates of different thicknesses. The curves are terminated at an impulse level within the small range of impulse levels where necking occurs. In Figure 3, a rapid upturn of the curves between the last two impulse levels is observed. For practical purposes this represents the limit of impulses that the plates can withstand.

2. Computational model and optimization scheme

In this study, the topology of the plate is not subject to change, that is, the top and bottom contours of the plate are parameterized and material is distributed throughout the thickness which these contours define, see Figure 4. Preliminary studies show that if both the top and bottom contours of the plate are allowed to change freely, the optimized structure will approach a V-shape such that the response is dominated by stretching from the initial application of the loading. In this study we are interested in plate-like structures, hence the contours must in some way be restrained. For example, restrictions could be put on the top or bottom contour, or on the midplane. We choose to restrict the top contour of the undeformed plate to be planar, as this ensures that the loading pressure does not change during optimization, see Figure 4. The gradient based code for constrained optimization SNOPT [Gill et al. 2005] is used to alter the design in an iterative fashion to minimize an object function, which is a measure of the performance of the

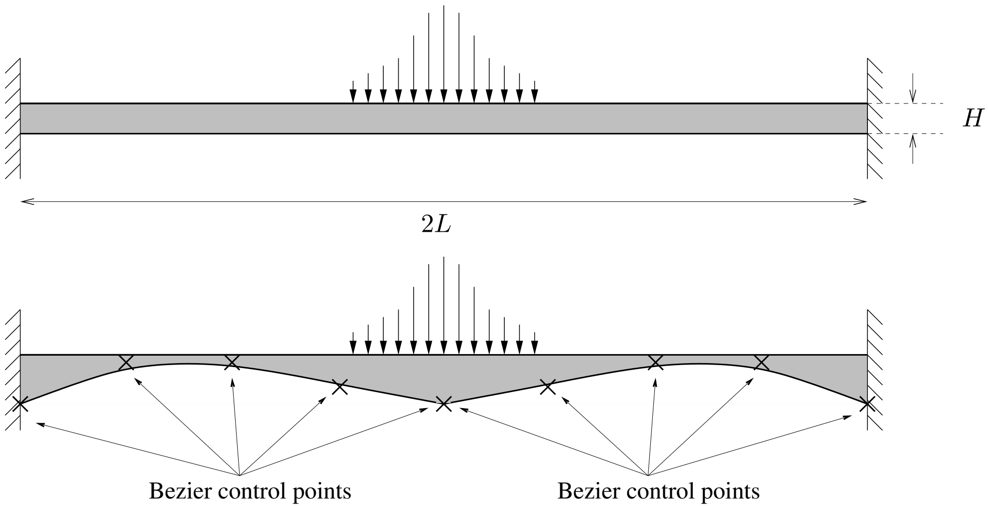

Figure 4. Plate loaded over a central patch. Uniform reference plate with thickness H (top) and optimized plate with average thickness H (bottom). The geometry of the lower contour of the plate is determined by equidistanced Bézier points, as indicated, which can move in the vertical direction. The top surface is restricted to be a plane. Symmetry is exploited, hence only the left half of the plate is modeled and parameterized.

structure. The gradients used in the optimization are obtained by finite differences with a perturbation of the control points equal to approximately $1/100$ of the element side length. The standard settings of SNOPT are used.

Object function. Several object functions could be used, such as minimization of average deflection, minimization of maximum deflection, minimization of maximum permanent deflection, et cetera. In this paper, minimization of the permanent deflection at midspan is used (specifically the bottom finite element node at the midspan symmetry line), which is denoted δ_{p}. The object function is normalized to cater for the performance of the optimization algorithm:

$$\psi = \frac{\delta_{\mathrm{p,ref}} - \delta_{\mathrm{p,opt}}}{\delta_{\mathrm{p,ref}}} \tag{4}$$

where $\delta_{\mathrm{p,ref}}$ is the permanent deflection of a uniform plate with the same mass acted upon by the same loading and $\delta_{\mathrm{p,opt}}$ is the permanent deflection of the optimized plate.

Parametrization and constraints. As mentioned earlier, to obtain plate-like geometries, the top surface of the plate is not allowed to change during the optimization process. The bottom surface of the plate is parameterized using a Bézier curve with control points at the left support and midspan and a number of equidistanced control points in between. This limits the design to relatively smooth designs, depending on the number of control points, and prevents infeasible designs with a large number of stiffers that depend on the spatial discretization, as observed for static elastic plate problems in [Cheng and Olhoff 1981]. The design variables of the problem are the vertical positions of the Bézier control points, see Figure 4. To establish an adequate number of control points, a plate has been optimized for a given load, with the number of Bézier control points ranging from 3 to 15. It was observed that the optimized designs were similar when using more than 4 control points, and that using a large number of control points made convergence to local minima more likely. Therefore, in this paper all optimizations are carried out using five Bézier control points, as this is the number of control points for which the object function seemed to plateau. The motion of the control points is constrained by enforcing a constant mass of the plate, that is, an average thickness equal to H. This is readily implemented using SNOPT, and the gradient of the mass constraint is evaluated in the same finite difference loop as the gradient of the object function. A feasibility constraint is employed to keep the optimization routine from requesting analysis of plates with very thin sections, which would cause numerical difficulties due to severely distorted elements during the finite element analysis. In this paper, the feasibility constraint demands that the thickness of the plate is everywhere more than $1/10$ of the uniform plate thickness. This limit is seldom reached during optimization, but it makes the optimization process more robust. Attention must be paid to two general problems arising in shape optimization: convergence to a local minimum, which is avoided in this paper by varying the initial guesses on the position of the control points, and the fact that the parametrization prevents the plate from reaching optimum, which is avoided by doing a convergence study in the number of parametrization variables, as mentioned above. For a general description of the shape optimization method the reader is referred to the review [Haftka and Grandhi 1986] and the more recent monograph [Haslinger and Mäkinen 2003].

3. Optimization results

The permanent midspan deflection of the optimized plates ($H/L = 0.06$) is compared to the permanent midspan deflection of a uniform plate with the same mass in Figure 5, left. To present the results more clearly, the relative improvement is plotted in Figure 5, right, against the dimensionless impulse, and in Figure 6 against the permanent deflection of the optimized plates (here several plate thicknesses are considered). As mentioned earlier, the curves have been terminated in the range where necking is observed. The figures show that the permanent deflection is significantly reduced in the range where the impulses are relatively small. Nevertheless the impulse levels in this range are still able to cause significant plastic deformation of the plate, giving rise to maximum permanent deflections less than about two times the plate thickness. The improvement relative to the uniform plate declines for larger impulses where the response transitions from being bending-dominated to stretching-dominated. The improvement over the uniform plate almost vanishes for impulses causing permanent midspan deflections greater than about 6 times the plate thickness. Nevertheless improvements of more than 10 percent can be achieved well into the stretching-dominated regime ($\delta_p/H > 1$). The plates optimized for this range of impulses do not, however, perform as well as uniform plates when subjected to larger impulses.

Reduction of imparted kinetic energy. In general, greater performance improvement is achieved in the optimized plates for central patch loading than for uniform loading; see Figures 5–6. This can be understood by studying the kinetic energy imparted to the plates. The kinetic energy imparted to the plates must be dissipated through plastic straining (neglecting elastic vibrations). Hence reducing the imparted kinetic energy is a potential means for reducing the permanent deflection. When considering a mass m initially at rest subjected to an impulse $I = mv$, where v is the velocity of the mass, the imparted kinetic

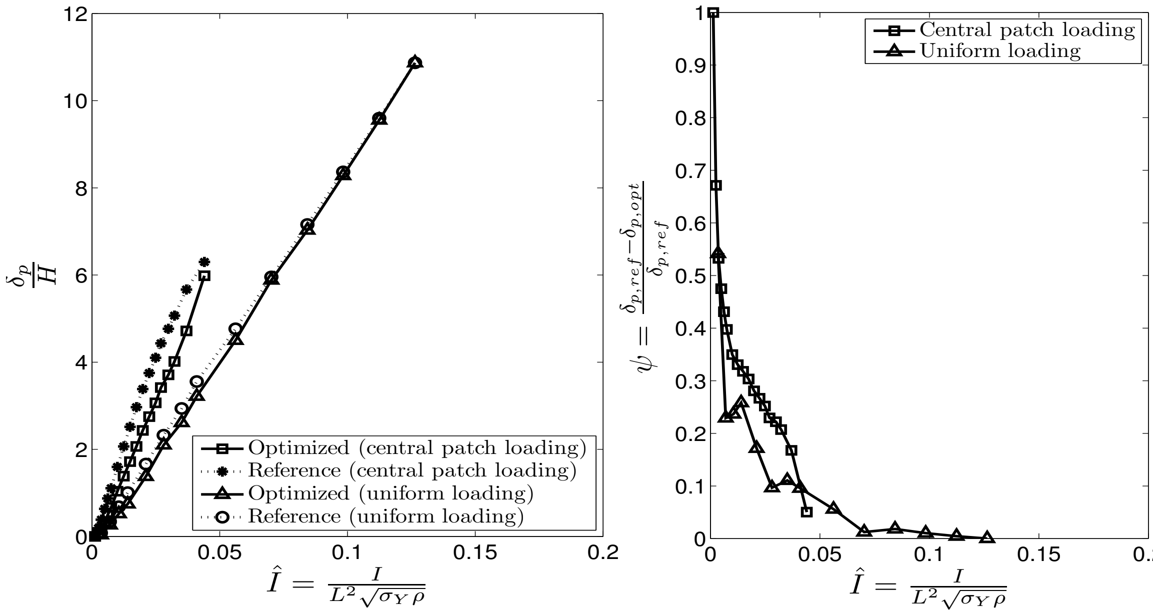

Figure 5. Left: permanent deflection at the midspan for a plate with dimensions $H/L = 0.06$. Right: relative improvement of the permanent deflection at the midspan for a plate with dimensions $H/L = 0.06$.

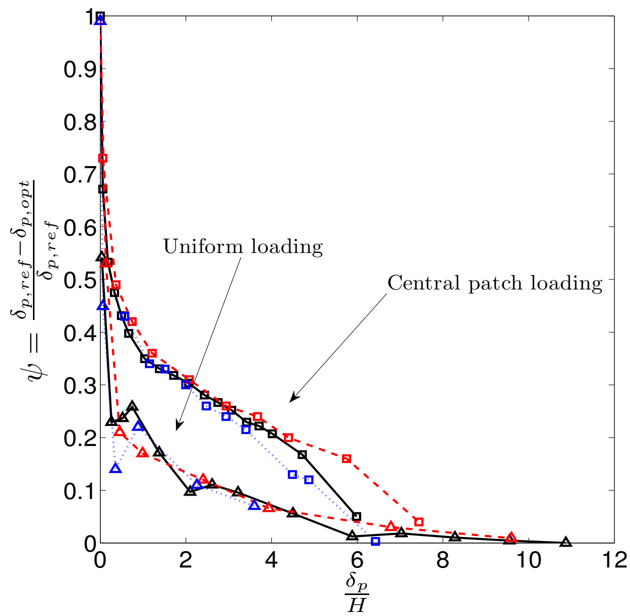

Figure 6. The relative reduction of permanent deflection plotted against the deflection of the optimized plates shows that the reduction depends but little on the height-length ratio of the plate, and that significant improvement can be obtained well into the stretching-dominated response range. The normalized plate thicknesses are $H/L = 0.04$ (dashed), 0.06 (continuous line), and 0.08 (dotted) for both uniform and central patch loading.

energy of this mass is

$$E_{\mathrm{kin}} = \frac{mv^2}{2} = \frac{I^2}{2m}. \tag{5}$$

Hence, increasing the mass on which a given impulse is acting will decrease the kinetic energy imparted to the mass. The same is true for the plates considered here. In the case of central patch loading, mass can be redistributed to the midspan where the loading acts, effectively decreasing the kinetic energy imparted to the plate, which in turn must be dissipated through plastic strain. This is seen in Figure 7, where the kinetic energy imparted to the optimized plates is plotted relative to the kinetic energy imparted to a uniform thickness plate. This effect is observed also in the case of uniform loading, as regions near the supports contribute only slightly to the kinetic energy. Thus it can be beneficial to move some mass to the midspan, but the effect is much smaller than for central patch loading.

Optimized thickness distributions. The inserts in Figure 7 show the optimized thickness distributions (for the left symmetric half) for a number of impulse levels. The optimized thickness distributions are similar for central patch and uniform loading. For small impulses the optimized plates resemble what is found for optimization of static elastic plates [Cheng and Olhoff 1981], that is, the thickness distribution

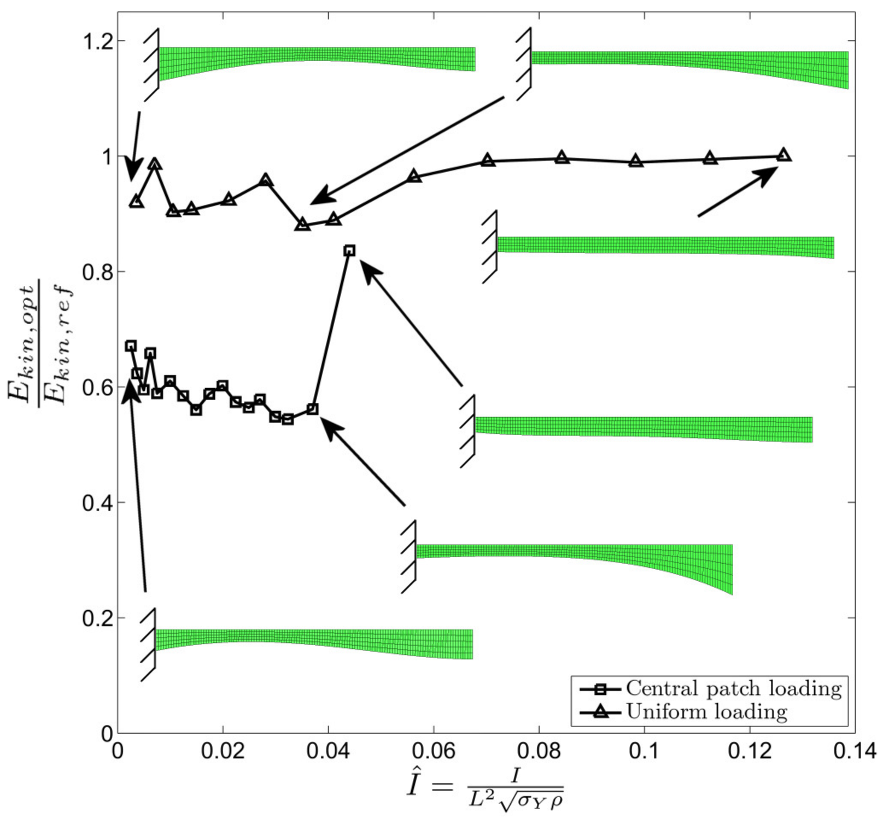

Figure 7. Kinetic energy imparted to the optimized plate relative to the kinetic energy imparted to the uniform plate. The inserts show the left symmetric half of the optimized plates.

varies with the bending moment distribution. For medium impulses, most of the redistributed mass is concentrated near the midspan of the plate which indicates that it is beneficial to reduce the kinetic energy imparted to the plate, compared to increasing the stiffness of the plate, as the discussion above suggests. For large impulses, causing permanent midspan deflections of more than about six times the plate thickness, the optimized plates are nearly uniform for both uniform and central patch loading. In this stretching-dominated range, uniform plates perform well, as changes in thickness variation promote necking in the thinnest regions.

4. Conclusions

In general, the reduction of permanent midspan deflection of the optimized plates compared to the uniform plates is significant for relatively small impulses, which give rise to permanent deflections that are less than two times the plate thickness. For larger impulses, which give rise to deflections greater than about six times the plate thickness, the payoff from optimization is less than five percent. Optimization of the thickness distribution shows more promise for plates subjected to central patch loading, as the kinetic energy imparted to the plates can be significantly reduced by relocating mass to areas below the loading. Three impulse domains have been identified:

- Small impulses (maximum permanent deflection less than the average plate thickness): The optimized plates resemble those found by shape optimization of linear elastic plates subjected to a static load at midspan [Cheng and Olhoff 1981]. This suggests that the response is primarily governed by the bending moment distribution. In this range of impulse levels the optimized plates perform significantly better than uniform plates.
- Medium impulses (maximum permanent deflection greater than the average plate thickness but less than about six times the average plate thickness): The optimized plates have a significant amount of material at the midspan, as to reduce the kinetic energy imparted to the plate. This effect is limited in the case of uniform loading, as pressure is applied everywhere on the top surface of the plate. Therefore, the impulse is transferred to the whole plate, hence moving mass from one region to another does not reduce the kinetic energy transferred to the plate, except near the supports. In this range of impulse levels the optimized plates perform 10–35% better than uniform plates in the case of central patch loading, and 5–15% better under uniform loading.
- Large impulses (maximum permanent deflection greater than about six times the average plate thickness): The plate response is dominated by stretching. Uniform plates perform very well, and only a slight reduction of permanent deflection can be obtained from optimization. In this range nonuniform thickness variations promote necking in regions where the thickness is minimum.

Finally, it can be emphasized again that the present study establishes that uniform plates perform well for large impulses, and that little is gained by varying the thickness distribution of the plate. This is an important observation for the monolithic plates studied here, and it establishes that the uniform monolithic plate is in fact a good reference for sandwich plates subjected to blast loads.

Acknowledgement

The author gratefully acknowledges discussions with John W. Hutchinson and Jens H. Andreasen.

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Received 4 May 2010. Revised 13 Oct 2010. Accepted 4 Nov 2010.

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JOURNAL OF MECHANICS OF MATERIALS AND STRUCTURES

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Founded by Charles R. Steele and Marie-Louise Steele

Cover design: Alex Scorpan Cover photo: Ev Shafrir

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JoMMS (ISSN 1559-3959) is published in 10 issues a year. The subscription price for 2010 is US $500/year for the electronic version, and $660/year (+$60 shipping outside the US) for print and electronic. Subscriptions, requests for back issues, and changes of address should be sent to Mathematical Sciences Publishers, Department of Mathematics, University of California, Berkeley, CA 94720–3840.

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Typeset in LaTeX

Journal of Mechanics of Materials and Structures

Volume 5, No. 6 June 2010

1559-3959(2010)5:6;1-C