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REFLECTION OF P AND SV WAVES FROM THE FREE SURFACE OF A TWO-TEMPERATURE THERMOELASTIC SOLID HALF-SPACE

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The present paper is concerned with the propagation of plane waves in an isotropic generalized thermoelastic solid half-space with two temperatures. The governing equations are modified in the context of the Lord–Shulman theory of generalized thermoelasticity and are solved to show the existence of three plane waves, namely, P, thermal, and SV waves in the x-z plane. The reflection of the P and SV waves from a thermally insulated free surface is studied to obtain the reflection coefficients in closed form. For numerical computations of the speeds and reflection coefficients, a particular material is chosen. The speeds of the plane waves are shown graphically against the two-temperature parameter. The reflection coefficients are also shown graphically against the angle of incidence for different values of the two-temperature parameter.

1. Introduction

Lord and Shulman [1967] and Green and Lindsay [1972] extended the classical dynamical coupled theory of thermoelasticity to generalized thermoelasticity theories. Their theories treat heat propagation as a wave phenomenon rather than a diffusion phenomenon and predict a finite speed of heat propagation. Ignaczak and Ostoja-Starzewski [2010] explained these theories in detail. The representative theories in the range of generalized thermoelasticity are reviewed in [Hetnarski and Ignaczak 1999]. Wave propagation in thermoelasticity has many applications in various engineering fields. Several problems in wave propagation in coupled or generalized thermoelasticity have been studied by various researchers [Deresiewicz 1960; Sinha and Sinha 1974; Sinha and Elsibai 1996; 1997; Sharma et al. 2003; Othman and Song 2007; Singh 2008; 2010].

Gurtin and Williams [1966; 1967] suggested a second law of thermodynamics for continuous bodies in which the entropy due to heat conduction was governed by one temperature, that of the heat supply by another. Based on this suggestion, Chen and Gurtin [1968] and Chen et al. [1968; 1969] formulated a theory of thermoelasticity which depends on two distinct temperatures, the conductive temperature Φ and the thermodynamic temperature T. Two-temperature theory involves a material parameter $a^* > 0$. The limit $a^* \to 0$ implies that $\Phi \to T$ and hence classical theory can be recovered from two-temperature theory. The two-temperature model has been widely used to predict electron and phonon temperature distributions in ultrashort laser processing of metals.

Warren and Chen [1973] stated that these two temperatures can be equal in time-dependent problems under certain conditions, whereas Φ and T are generally different in particular problems involving wave propagation. Following [Boley and Tolins 1962], they studied wave propagation in the two-temperature theory of coupled thermoelasticity. They showed that the two temperatures T and Φ and the strain are

Keywords: two-temperature parameter, generalized thermoelasticity, plane waves, reflection coefficients.

represented in the form of a traveling wave plus a response, which occurs instantaneously throughout the body. Puri and Jordan [2006] discussed the propagation of harmonic plane waves in two-temperature theory. Quintanilla and Jordan [2009] presented exact solutions of two initial-boundary value problems in the two-temperature theory with dual-phase-lag delay.

Youssef [2006] formulated a theory of two-temperature generalized thermoelasticity. Kumar and Mukhopadhyay [2010] extended the work of Puri and Jordan [2006] in the context of the linear theory of two-temperature generalized thermoelasticity formulated in [Youssef 2006]. Magaña and Quintanilla [2009] studied the uniqueness and growth of solutions in two-temperature generalized thermoelastic theories. Youssef [2011] also presented a theory of two-temperature thermoelasticity without energy dissipation. Ezzat and El-Karamany [2011] developed a two-temperature theory in generalized magnetothermoelasticity with two relaxation times.

In the present paper, we have applied the theory of [Youssef 2006] to the study of wave propagation in an isotropic two-temperature thermoelastic solid. The expressions for the speeds of plane waves are obtained. The required boundary conditions at a thermally insulated stress-free surface are satisfied by the appropriate solutions in an isotropic thermoelastic solid half-space to obtain the reflection coefficients in closed form for a particular incident wave. The speeds and reflection coefficients of plane waves are computed numerically for a particular model of the half-space to observe the effect of the two-temperature parameter.

2. Basic equations

Following [Youssef 2006], the governing equations for two-temperature anisotropic generalized thermoelasticity with one relaxation parameter are:

- The stress-strain-temperature relations:

$$\sigma_{ij} = c_{ijkl}e_{kl} - \gamma_{ij}(T - \Phi_0), \tag{1}$$

- The displacement-strain relation:

$$e_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}), \tag{2}$$

- The equation of motion:

$$\rho\ddot{u} = \sigma_{ji,j} + \rho F_i, \tag{3}$$

- The energy equation:

$$-q_{i,i} = \rho T_0 \dot{S}, \tag{4}$$

- The modified Fourier's law:

$$-K_{ij}\phi_{,j} = q_i + \tau_0 \dot{q}_i, \tag{5}$$

- The entropy-strain-temperature relation:

$$\rho S = \frac{\rho c_E}{T_0}\theta + \gamma_{ij}e_{ij}. \tag{6}$$

Here, γ_{ij} are the coupling parameters, T is the mechanical temperature, $\Phi_0 = T_0$ is the reference temperature, $\theta = T - T_0$ with $|\theta/T_0| \ll 1$, σ_{ij} is the stress tensor, e_{kl} is the strain tensor, c_{ijkl} is the tensor of elastic constants, ρ is the mass density, q_i is the heat conduction vector, K_{ij} is the thermal conductivity tensor, c_E is the specific heat at constant strain, u_i are the components of the displacement vector, S is the entropy per unit mass, τ_0 is the thermal relaxation time (which will ensure that the heat conduction equation will predict finite speeds of heat propagation), and ϕ is the conductive temperature satisfying the relation

$$\Phi - T = a^* \Phi_{,ii}, \tag{7}$$

where $a^* > 0$ is the two-temperature parameter.

3. Formulation and solution of the problem

We consider a homogeneous and isotropic thermoelastic medium of infinite extent, with a Cartesian coordinate system (x, y, z), which is previously at a uniform temperature. The origin is taken on the plane surface and the z-axis is taken normally into the medium ($z \geq 0$). The surface $z = 0$ is assumed stress-free and thermally insulated. The present study is restricted to the plane strain parallel to the x-z plane, with the displacement vector $\mathbf{u} = (u_1, 0, u_3)$. With the help of (1)–(3), we obtain the following two components of the equation of motion:

$$(\lambda + 2\mu) u_{1,11} + (\lambda + \mu) u_{3,13} + \mu u_{1,33} - \gamma \theta_{,1} = \rho \ddot{u}_1, \tag{8}$$

$$(\lambda + 2\mu) u_{3,33} + (\lambda + \mu) u_{1,13} + \mu u_{3,11} - \gamma \theta_{,3} = \rho \ddot{u}_3. \tag{9}$$

Equations (4)–(6) lead to the following heat conduction equation:

$$K(\Phi_{,11} + \Phi_{,33}) = \rho c_E (\dot{\theta} + \tau_0 \ddot{\theta}) + \gamma T_0 (\dot{u}_{1,1} + \tau_0 \ddot{u}_{1,1}) + \gamma T_0 (\dot{u}_{3,3} + \tau_0 \ddot{u}_{3,3}), \tag{10}$$

and (7) becomes

$$\Phi - T = a^* (\Phi_{,11} + \Phi_{,33}). \tag{11}$$

The displacement components u_1 and u_3 are written in terms of potentials q and ψ as

$$u_1 = \frac{\partial q}{\partial x} - \frac{\partial \psi}{\partial z}, \quad u_3 = \frac{\partial q}{\partial z} + \frac{\partial \psi}{\partial x}. \tag{12}$$

Using (12) in (8)–(11), we obtain

$$(\lambda + 2\mu)\left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2}\right) - \gamma \left[\Phi - a^* \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2}\right)\right] = \rho \frac{\partial^2 q}{\partial t^2}, \tag{13}$$

$$\begin{aligned} K(\Phi_{,11} + \Phi_{,33}) = \rho c_E \left(\frac{\partial \Phi}{\partial t} + \tau_0 \frac{\partial^2 \Phi}{\partial t^2}\right) - a^* \rho c_E \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial}{\partial t} \left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2}\right) \\ + \gamma T_0 \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \frac{\partial}{\partial t} \left(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2}\right), \end{aligned} \tag{14}$$

$$\mu \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2}\right) = \rho \frac{\partial^2 \psi}{\partial t^2}. \tag{15}$$

Here (15) is uncoupled, whereas (13) and (14) are coupled in q and Φ. Solutions (13)–(15) are now sought in the form of a harmonic traveling wave:

$$(q, \Phi, \psi) = (A, B, C)\exp(ik(x\sin\theta + z\cos\theta - vt)), \tag{16}$$

in which v is the phase speed, k is the wave number, and $(\sin\theta, \cos\theta)$ denotes the projection of the wave normal onto the x-z plane. Inserting (16) into (13)–(15), we obtain the following formulae for the speeds of the plane waves:

$$v_1 = \sqrt{\frac{1}{2\rho}\Big[\{(K_a+\epsilon)+(\lambda+2\mu)\} + \sqrt{(K_a+\epsilon)^2+(\lambda+2\mu)^2-2(K_a-\epsilon)(\lambda+2\mu)}\,\Big]}, \tag{17}$$

$$v_2 = \sqrt{\frac{1}{2\rho}\Big[\{(K_a+\epsilon)+(\lambda+2\mu)\} - \sqrt{(K_a+\epsilon)^2+(\lambda+2\mu)^2-2(K_a-\epsilon)(\lambda+2\mu)}\,\Big]}, \tag{18}$$

$$v_3 = \sqrt{\frac{\mu}{\rho}}, \tag{19}$$

where $\epsilon = (\gamma^2 T_0)/(\rho c_E)$ is the thermocoupling coefficient and $K_a = K/(c_E\tau^*(1+a^*k^2))$, with $\tau^* = \tau_0 + i/\omega$, $\omega = kv$. The speeds v_1, v_2, and v_3 correspond to the P, thermal, and SV waves, respectively. From (17)–(19), it is clear that the speeds of the modified P and thermal waves are functions of the two-temperature parameter a^*. The speed of the SV wave is not affected by a^*.

If we neglect the thermal parameters (that is, $K_a = 0$, $\epsilon = 0$), the speed v_1 reduces to $\sqrt{(\lambda+2\mu)/\rho}$, the speed of a P wave in an elastic solid. The thermal wave will disappear.

4. Boundary conditions

Let us now consider an incident P or SV wave. The boundary conditions at the stress-free thermally insulated surface $z = 0$ are satisfied if the incident P or SV wave gives rise to a reflected shear (SV) and two reflected longitudinal waves (P and thermal). The required boundary conditions at the free surface $z = 0$ are:

- Vanishing of the normal stress component:

$$\sigma_{zz} = 0, \tag{20}$$

- Vanishing of the tangential stress component:

$$\sigma_{zx} = 0, \tag{21}$$

- Vanishing of the normal heat flux component:

$$\frac{\partial\Phi}{\partial z} = 0, \tag{22}$$

where

$$\sigma_{zz} = \lambda\Big(\frac{\partial^2 q}{\partial x^2} + \frac{\partial^2 q}{\partial z^2}\Big) + 2\mu\Big(\frac{\partial^2\psi}{\partial x\partial z}\Big) + 2\mu\frac{\partial^2 q}{\partial z^2} - \gamma\Big[\Phi - a^*\Big(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial z^2}\Big)\Big], \tag{23}$$

$$\sigma_{zx} = \mu\Big[2\frac{\partial^2 q}{\partial x\partial z} - \frac{\partial^2\psi}{\partial z^2} + \frac{\partial^2\psi}{\partial x^2}\Big]. \tag{24}$$

The appropriate displacement potentials ψ, q, and Φ are taken in the form

$$\psi = C_1 \exp(ik_3(x \sin\theta_0 + z\cos\theta_0 - v_3 t)) + C_2 \exp(ik_3(x \sin\theta_3 - z\cos\theta_3 - v_3 t)), \tag{25}$$

$$q = A_1 \exp(ik_1(x \sin\theta_0 + z\cos\theta_0 - v_1 t)) + A_2 \exp(ik_1(x \sin\theta_1 - z\cos\theta_1 - v_1 t)) + A_3 \exp(ik_2(x \sin\theta_2 - z\cos\theta_2 - v_2 t)), \tag{26}$$

$$\Phi = \eta_1 A_1 \exp(ik_1(x \sin\theta_0 + z\cos\theta_0 - v_1 t)) + \eta_1 A_2 \exp(ik_1(x \sin\theta_1 - z\cos\theta_1 - v_1 t)) + \eta_2 A_3 \exp(ik_2(x \sin\theta_2 - z\cos\theta_2 - v_2 t)), \tag{27}$$

where the wave normal to the incident P or SV wave makes an angle θ_0 with the positive direction of the z-axis and those of the reflected P, thermal, and SV waves make angles θ_1, θ_2, and θ_3, respectively, with the same direction, and

$$\frac{\eta_1}{{k_1}^2} = \frac{\rho {v_1}^2 - (\lambda + 2\mu)}{\gamma(1 + a^* {k_1}^2)}, \qquad \frac{\eta_2}{{k_2}^2} = \frac{\rho {v_2}^2 - (\lambda + 2\mu)}{\gamma(1 + a^* {k_2}^2)}. \tag{28}$$

5. Reflection coefficients

The ratios of the amplitudes of the reflected waves to the amplitude of incident P wave, namely C_2/A_1, A_2/A_1, and A_3/A_1, are the reflection coefficients (amplitude ratios) of the reflected SV, reflected P, and reflected thermal waves, respectively. Similarly, for the incident SV wave, C_2/C_1, A_2/C_1, and A_3/C_1 are the reflection coefficients of the reflected SV, reflected P, and reflected thermal waves, respectively. The wave numbers k_1, k_2, and k_3 and the angles θ_0, θ_1, θ_2, and θ_3 are connected by the relation

$$k_1 \sin\theta_0 = k_3 \sin\theta_0 = k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3, \tag{29}$$

at the surface $z = 0$. In order to satisfy the boundary conditions (20)–(22), we write (29) as

$$\frac{\sin\theta_0}{v_1 \text{ or } v_3} = \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} = \frac{\sin\theta_3}{v_3}. \tag{30}$$

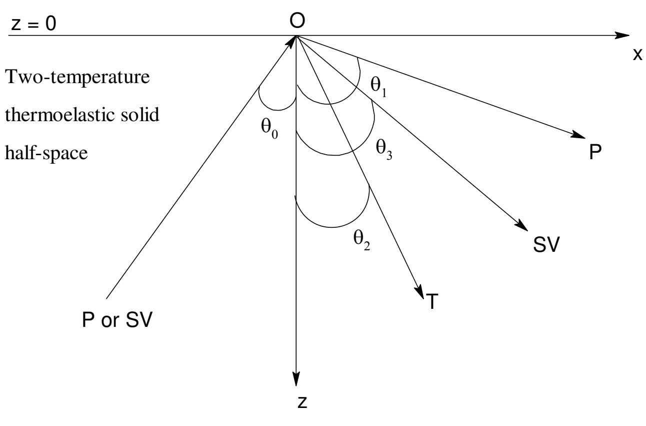

Figure 1. Geometry of the problem.

5.1. ***Incident P wave.*** Making use of the potentials given by (25)–(27) in the boundary conditions (20)–(22), we obtain a system of three nonhomogeneous equations which results in the following expressions for the reflection coefficients of the SV, P, and thermal waves:

$$\frac{C_2}{A_1}=\frac{D_1}{D},\qquad \frac{A_2}{A_1}=\frac{D_2}{D},\qquad \frac{A_3}{A_1}=\frac{D_3}{D},\tag{31}$$

where

$$D=\frac{{v_1}^2}{v_2v_3}\bigg[2\mu\sin\theta_0\sin 2\theta_0(\eta_2-\eta_1)\sqrt{1-\frac{{v_3}^2}{{v_1}^2}\sin^2\theta_0}\sqrt{1-\frac{{v_2}^2}{{v_1}^2}\sin^2\theta_0}\\ -\frac{v_1}{v_3}\Big(1-2\frac{{v_3}^2}{{v_1}^2}\sin^2\theta_0\Big)(\rho{v_1}^2-2\mu\sin^2\theta_0)\Big(-\eta_2\sqrt{1-\frac{{v_2}^2}{{v_1}^2}\sin^2\theta_0}+\eta_1\frac{v_2}{v_1}\cos\theta_0\Big)\bigg],\tag{32}$$

$$D_1=\frac{v_1}{v_2}\sin 2\theta_0(\rho{v_1}^2-2\mu\sin^2\theta_0)(\eta_2-\eta_1)\sqrt{1-\frac{{v_2}^2}{{v_1}^2}\sin^2\theta_0}\Big(1+\sqrt{1-\frac{{v_2}^2}{{v_1}^2}\sin^2\theta_0}\Big),\tag{33}$$

$$D_2=\frac{{v_1}^2}{v_2v_3}\bigg[2\mu\sin\theta_0\sin 2\theta_0(\eta_2-\eta_1)\sqrt{1-\frac{{v_3}^2}{{v_1}^2}\sin^2\theta_0}\sqrt{1-\frac{{v_2}^2}{{v_1}^2}\sin^2\theta_0}\\ -\frac{v_1}{v_3}\Big(1-2\frac{{v_3}^2}{{v_1}^2}\sin^2\theta_0\Big)(\rho{v_1}^2-2\mu\sin^2\theta_0)\Big(\eta_2\sqrt{1-\frac{{v_2}^2}{{v_1}^2}\sin^2\theta_0}+\eta_1\frac{v_2}{v_1}\cos\theta_0\Big)\bigg],\tag{34}$$

$$D_3=2\eta_1\cos\theta_0\frac{{v_1}^2}{{v_3}^2}\Big(1-2\frac{{v_3}^2}{{v_1}^2}\sin^2\theta_0\Big)(\rho{v_1}^2-2\mu\sin^2\theta_0).\tag{35}$$

5.2. ***Incident SV wave.*** Similarly, making use of the potentials given by (25)–(27) in the boundary conditions (20)–(22), we obtain the following expressions for the reflection coefficients of the SV, P, and thermal waves:

$$\frac{C_2}{C_1}=\frac{D_1'}{D'},\qquad \frac{A_2}{C_1}=\frac{D_2'}{D'},\qquad \frac{A_3}{C_1}=\frac{D_3'}{D'}.\tag{36}$$

Here,

$$D'=-\frac{{v_3}^3}{v_1v_2}\bigg[\frac{2\mu}{{v_1}^2}\sin\theta_0\sin 2\theta_0(\eta_1-\eta_2)\sqrt{1-\frac{{v_3}^2}{{v_1}^2}\sin^2\theta_0}\sqrt{1-\frac{{v_2}^2}{{v_1}^2}\sin^2\theta_0}\\ +\Big(1-2\frac{{v_3}^2}{{v_1}^2}\sin^2\theta_0\Big)(\rho{v_1}^2-2\mu\sin^2\theta_0)\Big(\frac{\eta_2}{v_1}\sqrt{1-\frac{{v_2}^2}{{v_1}^2}\sin^2\theta_0}-\eta_1\frac{v_2}{{v_1}^2}\cos\theta_0\Big)\bigg],\tag{37}$$

$$D_1'=-\frac{{v_3}^3}{v_1v_2}\bigg[\frac{\mu}{v_1}\sin^2 2\theta_0\sqrt{1-\frac{{v_2}^2}{{v_1}^2}\sin^2\theta_0}(\eta_2-\eta_1)\\ -\frac{(\rho{v_1}^2-2\mu\sin^2\theta_0)}{v_1}\Big(\eta_2\cos 2\theta_0\sqrt{1-\frac{{v_2}^2}{{v_1}^2}\sin^2\theta_0}-\frac{\eta_1}{v_1}v_2\cos\theta_0\Big)\bigg],\tag{38}$$

$$D_2' = 2\mu\eta_2 \frac{v_3}{v_2} \sin\theta_0 \sqrt{1 - \frac{{v_2}^2}{{v_1}^2} \sin^2\theta_0} [\frac{v_3}{v_1} \cos 2\theta_0 \sqrt{1 - \frac{{v_3}^2}{{v_1}^2} \sin^2\theta_0} + \cos\theta_0 (1 - 2\frac{{v_3}^2}{{v_1}^2} \sin^2\theta_0)], \tag{39}$$

$$D_3' = -2\mu\eta_1 \frac{v_3}{v_1} \sin 2\theta_0 [\frac{v_3}{v_1} \cos 2\theta_0 \sqrt{1 - \frac{{v_3}^2}{{v_1}^2} \sin^2\theta_0} + \cos\theta_0 (1 - 2\frac{{v_3}^2}{{v_1}^2} \sin^2\theta_0)]. \tag{40}$$

6. Numerical results and discussion

To study numerically the effects of the two-temperature parameter on the speeds of propagation and reflection coefficients, we consider the following physical constants of aluminum as an isotropic thermoelastic solid half-space:

$$\lambda = 7.59 \times 10^{10}\,\mathrm{Nm}^{-2}, \quad \mu = 1.89 \times 10^{10}\,\mathrm{Nm}^{-2}, \quad K = 237\,\mathrm{Wm}^{-1}\mathrm{deg}^{-1},$$
$$C_e = 24.2\,\mathrm{Jkg}^{-1}\mathrm{deg}^{-1}, \quad \rho = 2.7 \times 10^3\,\mathrm{kgm}^{-3}, \quad T_0 = 296\,\mathrm{K}, \quad \tau_0 = 0.05\,\mathrm{s}, \quad \omega = 20\,\mathrm{s}^{-1}.$$

Using the relation $V_j^{-1} = v_j^{-1} - \iota\omega^{-1} q_j$ $(j = 1, \ldots, 3)$, the real values of the propagation speeds of the P, SV, and thermal waves are computed for the range $0 \le a^* \le 1$ of the two-temperature parameter. The speeds of the P, SV, and thermal waves are shown graphically against the two-temperature parameter a^* in Figure 2. The speed of the P wave decreases with an increase in the two-temperature parameter, whereas the speed of the thermal wave increases. The speed of the SV wave is not affected by change in the two-temperature parameter.

With the help of (31), the reflection coefficients of the reflected P, SV, and thermal waves are computed for the incidence of a P wave. For the range $0° < \theta_0 \le 90°$ of the angle of incidence of the P wave, the reflection coefficients of the P, thermal, and SV waves are shown graphically in Figure 3, when $a^* = 0$, 0.5, and 1. For $a^* = 1$, the reflection coefficient of the P wave decreases from its maximum value of 1.211 at $\theta_0 = 1°$ to its minimum value of 0.88 at $\theta_0 = 69°$. Thereafter, it increases up to the grazing incidence. For $a^* = 1$, the reflection coefficient of the thermal wave decreases from its maximum

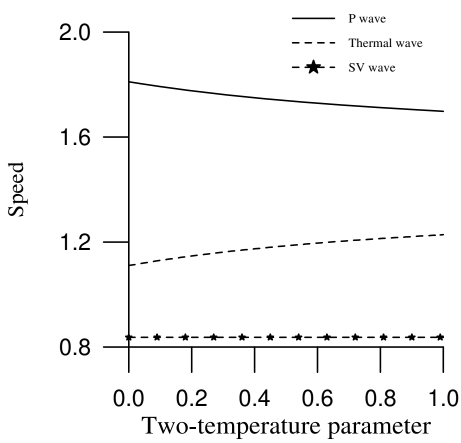

Figure 2. Variations of speed of reflected P wave (solid line), thermal wave (dashed line, and SV wave (dashed line with stars) against the two-temperature parameter.

value of 0.7132 at $\theta = 1°$ to its minimum value of zero at $\theta_0 = 90°$. For $a^* = 1$, the reflection coefficient of SV wave increases from its minimum value of zero at normal incidence to its maximum value of 0.5062 at $\theta_0 = 48°$. Beyond $\theta_0 = 48°$, it decreases to its minimum value of zero at grazing incidence. From Figure 3, it is observed that the effect of a^* on the reflection coefficients of the P and thermal waves is maximal at normal incidence. The effect of the two-temperature parameter on these waves decreases with increase in the angle of incidence. For grazing incidence, there is no effect of the two-temperature parameter on these reflected waves. The reflection coefficient of SV is also affected by two-temperature parameter. For normal and grazing incidences, there is no effect of two-temperature parameter on the reflected SV wave. The maximal effect of the two-temperature parameter on the reflected SV wave is observed at $\theta_0 = 45°$.

With the help of (36), the reflection coefficients of the reflected P, SV, and thermal waves are computed for the incidence of a SV wave. For the range $0° < \theta_0 \leq 27°$ of the angle of incidence of the SV wave, the reflection coefficients of the P, thermal, and SV waves are shown graphically in Figure 4, when $a^* = 0$, 0.5 and 1. For $a^* = 1$, the reflection coefficient of the P wave increases from its minimum

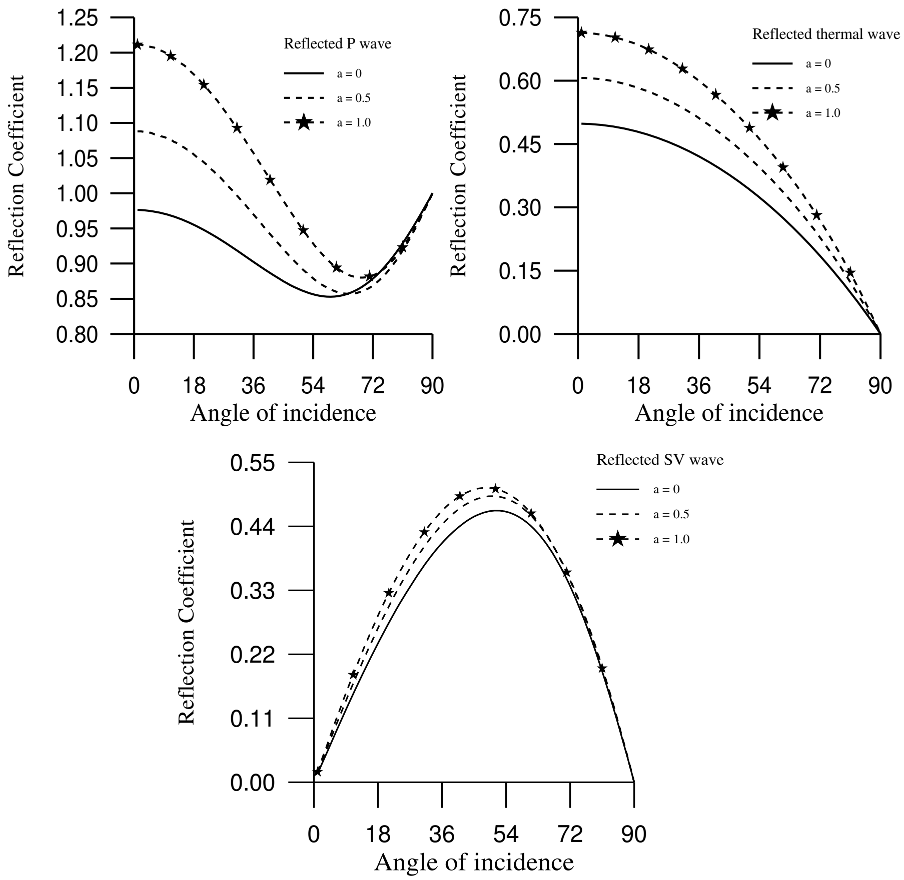

Figure 3. Variation in the reflection coefficients of the reflected P (top left), thermal (top right), and SV (bottom) waves against the angle of incidence of the incident P wave.

value of 0.07 at $\theta_0 = 1°$ to its maximum value of 1.926 at $\theta_0 = 27°$. For $a^* = 1$, the reflection coefficient of the thermal wave increases from its minimum value of 0.02 at $\theta = 1°$ to its maximum value of 0.3887 at $\theta_0 = 23°$. Thereafter, it decreases up to the angle of incidence $\theta = 27°$. For $a^* = 1$, the reflection coefficient of the SV wave decreases from its maximum value of one at normal incidence to its minimum value of 0.5992 at $\theta_0 = 26°$. Thereafter, it increases to a value of 0.6058 at $\theta_0 = 27°$. From Figure 4, it is observed that the effect of a^* on the reflection coefficients of the P wave is maximal at angles near $\theta_0 = 20°$. There is no effect of the two-temperature parameter on this wave at normal incidence. At normal incidence, the reflected thermal and SV waves are also not affected by a^*. The effect of a^* on these reflected waves increases with the increase in the angle of incidence and it becomes maximal at angles near $\theta_0 = 25°$.

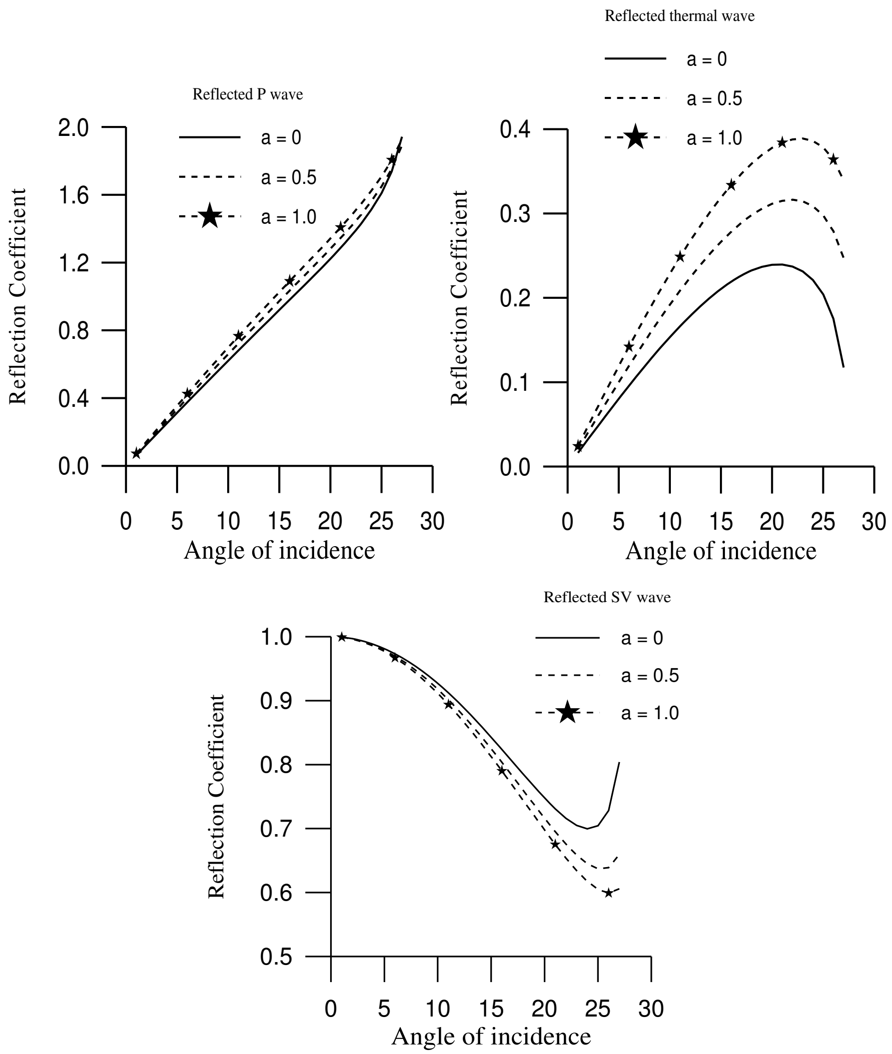

Figure 4. Variation in the reflection coefficients of the reflected P (top left), thermal (top right), and SV (bottom) waves against the angle of incidence of the incident SV wave.

7. Conclusion

The two-dimensional solution of the governing equations of an isotropic two-temperature thermoelastic medium indicates the existence of three plane waves, namely the P, thermal, and SV waves. The expressions for the speeds of the P, thermal, and SV waves are obtained explicitly. The reflection coefficients of the reflected P, thermal, and SV waves are also obtained in closed form for the incidence of P and SV waves. The speeds and reflection coefficients of plane waves are computed for a particular material representing the model. From the theory and numerical results, it is observed that the speeds and reflection coefficients of the plane waves are significantly affected by the two-temperature parameter.

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