Journal of
Mechanics of Materials and Structures

RAYLEIGH-TYPE WAVE PROPAGATION IN AN AUXETIC DIELECTRIC

Andrzej Drzewiecki

Volume 7, No. 3 March 2012

msp

JOURNAL OF MECHANICS OF MATERIALS AND STRUCTURES
Vol. 7, No. 3, 2012
dx.doi.org/10.2140/jomms.2012.7.277

msp

RAYLEIGH-TYPE WAVE PROPAGATION IN AN AUXETIC DIELECTRIC

ANDRZEJ DRZEWIECKI

The propagation of Rayleigh-type waves in an elastic, isotropic, dielectric half-space for some orientation of the external electric field is considered, with Poisson's ratio in the range $-1 < \nu < 0.5$.

1. Introduction

Auxetics are materials that have a negative Poisson's ratio [Stręk et al. 2008; 2009; 2010; Poźniak et al. 2010]. The earliest publications providing information on the unusual auxetic properties of certain natural materials appeared almost a hundred years ago, but they did not arouse great interest because of the low reproducibility of the reported experimental results. Physicists and engineers were also convinced for a long time that there are no materials with negative Poisson's ratio in nature, despite the fact that their existence was known to be thermodynamically possible (see, for example, [Landau and Lifshits 1954]). The possible applications of materials having this property were also overlooked at the time.

Contemporary studies of auxetics were started after the publication of [Lakes 1987]. Soon, numerous possible applications of these materials were identified, such as body armor, packing material, knee and elbow pads, robust shock absorbing material, sponge mops and many others. This is the reason for the dynamic development of these materials research in recent years (see [Friis et al. 1988; Lakes and Wineman 2006], for example).

A few articles have considered the dynamic behavior of auxetics materials in the framework of coupled field theory. Here we discuss the problem of propagation of the Rayleigh-type waves in such media. Specifically, we consider elastic, isotropic, and dielectric half-space with Poisson's ratio in the range $(-1; 0.5)$, subject to an external electric field in some fixed orientation. We derive the propagation equation and solve it numerically in special cases.

We adopt a statistical model (according to the classification by K. Hutter and A. A. F. van de Ven) for the interaction between electromagnetic and mechanical fields. To render the basic equations amenable to direct analysis, they are be linearized with respect to some average state, referred to as the ξ state. The problem at hand suggests a natural definition of this state. It is assumed that ξ state is known or at least determinable. All physical fields may be decomposed into two parts: the tensor fields in the ξ state and tensor fields correction expressing the difference between the ξ state and the real state, in the framework of the model. In this paper they are denoted by ${}^0\varphi$ and ${}^*\varphi$ respectively, where φ is the tensor field in question. Because the perturbations ${}^*\varphi$ are assumed to be small, the governing equations can be linearized.

Partially supported by Poland's Ministry of Science and Higher Education (MNiSzW), grant 2363/B/TO2/2010/39.
Keywords: auxetics, Rayleigh wave, dielectric.

2. Setting up the equations

The specific version of the basic equations derived in [Hutter and van de Ven 1978] that is presented below corresponds with the problem under consideration. This specification is confined to the choice of free energy form and proper definition of some average state. It is assumed that the ξ state is a motionless, rigid body state and the electric field in this state is static and homogeneous. These assumptions are not contradictory, but in general a stress vector acting at the surface that bounds a dielectric is needed.

The free energy is postulated in the form (material description):

$$F = -\frac{1}{2\mu_0\rho_0}\frac{\kappa'}{1+\kappa'}B_\alpha B_\alpha + \frac{1}{2\rho_0\epsilon_0\kappa}P_\alpha P_\alpha - \tfrac{1}{2}n(\Theta-\Theta_0)^2 - \gamma(\Theta-\Theta_0)E_{\gamma\gamma} + \frac{1}{\rho_0}\mu' E_{\alpha\beta}E_{\alpha\beta} + \frac{1}{2\rho_0}\lambda' E_{\alpha\alpha}E_{\beta\beta}, \quad (1)$$

where κ' is the magnetic susceptibility, κ the electric susceptibility, λ', μ' are the elastic constants, ρ_0 the mass density, B_α the magnetic induction, P_α the magnetic induction, Θ the absolute temperature, γ, n the material constants, $C_{\alpha\beta}$ Green's deformation tensor, $E_{\alpha\beta} = \frac{1}{2}(C_{\alpha\beta} - \delta_{\alpha\beta})$ the Lagrangian strain tensor, ϵ_0 the electric permeability of vacuum and μ_0 the magnetic permeability of vacuum.

With this choice, the constitutive equations read:

$$S = -\frac{\partial F}{\partial\Theta} = n(\Theta-\Theta_0) + \delta E_{\alpha\alpha}, \quad E_\alpha = \frac{\partial F}{\partial(P_\alpha/\rho_0)} = \frac{1}{\epsilon_0\kappa}P_\alpha, \quad M_\alpha = -\rho_0\frac{\partial F}{\partial B_\alpha} = \frac{\kappa'}{\mu_0(1+\kappa')}B_\alpha,$$

$$\begin{aligned} T_{\alpha\beta} = T_{i\alpha}F^{-1}_{\beta i} &= 2\rho_0\frac{\partial F}{\partial C_{\alpha\beta}} - \big(P_\alpha E_\gamma - B_\alpha M_\gamma\big)C^{-1}_{\beta\gamma} - M_\gamma B_\gamma C^{-1}_{\alpha\beta} \\ &= 2\mu' E_{\alpha\beta} + \lambda'\delta_{\alpha\beta}E_{\gamma\gamma} - \rho_0\gamma\delta_{\alpha\beta}(\Theta-\Theta_0) \\ &\quad -\left(\frac{1}{\epsilon_0\kappa}P_\alpha P_\gamma - \frac{\kappa'}{\mu_0(1+\kappa')}B_\alpha B_\gamma\right)C^{-1}_{\beta\gamma} - \frac{\kappa'}{\mu_0(1+\kappa')}B_\gamma B_\gamma C^{-1}_{\alpha\beta}, \end{aligned} \quad (2)$$

where S is the entropy density, E_α the electric field intensity, M_α the magnetization, $T_{i\alpha}$ the Piola–Kirchhoff stress tensor, $F_{i\alpha}$ the material deformation gradient, $F^{-1}_{\alpha i}$ the spatial deformation gradient and $C^{-1}_{\alpha\beta}$ Cauchy's deformation tensor.

For an adiabatic process one obtains

$$\Theta - \Theta_0 = -\frac{\delta}{n}E_{\alpha\alpha} \quad (3)$$

and the constitutive relation (2) can be replaced by

$$T_{\alpha\beta} = 2\mu E_{\alpha\beta} + \lambda\delta_{\alpha\beta}E_{\gamma\gamma} - \left(\frac{1}{\epsilon_0\kappa}P_\alpha P_\gamma - \frac{\kappa'}{\mu_0(1+\kappa')}B_\alpha B_\gamma\right)C^{-1}_{\beta\gamma} - \frac{\kappa'}{\mu_0(1+\kappa')}B_\gamma B_\gamma C^{-1}_{\alpha\beta}, \quad (4)$$

where

$$\lambda = \lambda' + \rho_0\frac{\gamma^2}{n}, \qquad \mu = \mu' \quad (5)$$

are the adiabatic elastic constants.

As a result of the linearizing procedure one reaches the equations of motion

$$\rho_0\ddot{u}_\alpha = \mu u_{\alpha,\beta\beta} + (\lambda+\mu)u_{\beta,\beta\alpha} - \frac{1}{\epsilon_0\kappa}\,{}^0P_\alpha\,{}^*P_{\beta,\beta} \quad (6)$$

(u_α stands for the displacement), the Maxwell equations

$$
{}^*B_{\alpha,\alpha}=0,\quad {}^*\dot{B}_\alpha+\frac{1}{\epsilon_0\kappa}e_{\alpha\beta\gamma}\,{}^*P_{\gamma,\beta}=0,\quad \Big(\frac{1}{\kappa}+1\Big){}^*P_{\alpha,\alpha}-\frac{1}{\kappa}\,{}^0P_\beta u_{\beta,\alpha\alpha}=0,
$$

$$
-\Big(\frac{1}{\kappa}+1\Big){}^*\dot{P}_\alpha+\frac{1}{\kappa}\,{}^0P_\gamma\dot{u}_{\gamma,\alpha}+\frac{1}{\mu_0(1+\kappa')}e_{\alpha\beta\gamma}\,{}^*B_{\gamma,\beta}=0, \tag{7}
$$

$$
e_{\alpha\beta\gamma}\,{}^0P_{\gamma,\beta}=0,\quad {}^0P_{\alpha,\alpha}=0, \tag{8}
$$

and the jump conditions

$$
\Big[\epsilon_0\,{}^0E^+_\alpha-\Big(1+\frac{1}{\kappa}\Big){}^0P^-_\alpha\Big]N_\alpha=0,\quad e_{\alpha\beta\gamma}\Big({}^0E^+_\alpha-\frac{1}{\epsilon_0\kappa}\,{}^0P^-_\beta\Big)N_\gamma=0,\quad \big({}^*B^+_\alpha-{}^*B^-_\alpha\big)N_\alpha=0, \tag{9a}
$$

$$
T^+_i=\tfrac{1}{2}\delta_{i\beta}\,{}^0P^-_\alpha\,{}^0E^+_\beta N_\alpha-\frac{1}{2\epsilon_0\kappa}\delta_{i\beta}\,{}^0P^-_\alpha\,{}^0P^-_\beta N_\alpha,\quad e_{\alpha\beta\gamma}\Big({}^*E^+_\beta-\frac{1}{\epsilon_0\kappa}\,{}^*P^-_\beta\Big)N_\gamma=0, \tag{9b}
$$

$$
\Big[\epsilon_0\,{}^*E^+_\alpha-\big(1+\frac{1}{\kappa}\big){}^*P^-_\alpha\Big]N_\alpha+{}^0P^-_\alpha u_{\gamma,\gamma}N_\alpha-\big(\epsilon_0\,{}^0E^+_\gamma-\frac{1}{\kappa}\,{}^0P^-_\gamma\big)\big(u_{\gamma,\alpha}-u_{\alpha,\gamma}\big)N_\alpha=0, \tag{9c}
$$

$$
\frac{1}{\mu_0}e_{\alpha\beta\gamma}\Big({}^*B^+_\beta-\frac{1}{1+\kappa'}\,{}^*B^-_\beta\Big)N_\gamma+{}^0P^-_\gamma\dot{u}_\alpha N_\gamma-\big(\epsilon_0\,{}^0E^+_\alpha-\frac{1}{\kappa}\,{}^0P^-_\alpha\big)\dot{u}_\gamma N_\gamma=0, \tag{9d}
$$

$$
\big[\mu\big(u_{\omega,\beta}+u_{\beta,\omega}\big)-\delta_{\omega\beta}\lambda u_{\gamma,\gamma}\big]N_\beta=\tfrac{1}{2}u_{\alpha,\omega}\,{}^0P^-_\beta\Big({}^0E^+_\alpha+\frac{1}{\epsilon_0\kappa}\,{}^0P^-_\alpha\Big)N_\beta
$$

$$
-\tfrac{1}{2}\,{}^0P^-_\beta\Big({}^*E^+_\omega+\frac{1}{\epsilon_0\kappa}\,{}^*P^-_\omega\Big)N_\beta-\tfrac{1}{2}\,{}^*P^-_\beta\Big({}^0E^+_\omega+\frac{1}{\epsilon_0\kappa}\,{}^0P^-_\omega\Big)N_\beta. \tag{9e}
$$

The elastic dielectric is placed on the "−" side of the surface (see Figure 1). It is assumed that T^+_i (needed for the mechanical equilibrium of motionless rigid body placed in the electric field) is the only surface traction of the other than electromagnetic origin. The quantities ${}^0E^+_\alpha$, ${}^*E^+_\alpha$ and ${}^*B^+_\alpha$ are defined by

$$
{}^0E^+_\alpha=\delta_{i\alpha}\,{}^0e^+_i,\quad {}^*E^+_\alpha=\delta_{i\alpha}\,{}^*e^+_i+\delta_{i\beta}\,{}^0e^+_i u_{\beta,\alpha},\quad {}^*B^+_\alpha=\delta_{i\alpha}\,{}^*b^+_i, \tag{10}
$$

where ${}^0e^+_i$, ${}^*e^+_i$, ${}^*b^+_i$ are the limits to which tend the decomposed fields e_i, b_i on the surface. The latter vector fields are the electric field intensity and the magnetic induction in vacuum.

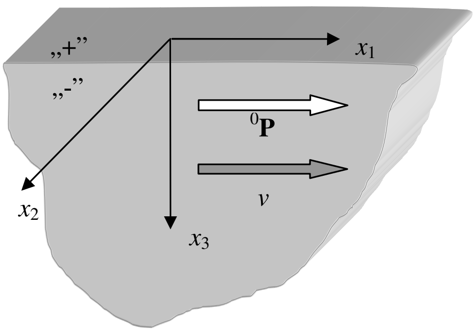

Figure 1. The geometry of the problem.

The vectors of electromagnetic field in vacuum satisfy the following equations:

$$e_{ijk}e_{k,j} = -\dot{b}_i, \quad e_{i,i} = 0, \quad \frac{1}{\mu_0}e_{ijk}b_{k,j} = \epsilon_0\dot{e}_i, \quad b_{i,i} = 0, \tag{11}$$

$$e_{ijk}\,{}^0e_{k,j} = 0, \quad {}^0e_{i,i} = 0, \quad e_{ijk}\,{}^*e_{k,j} = -{}^*\dot{b}_i, \quad {}^*e_{i,i} = 0, \quad \frac{1}{\mu_0}e_{ijk}\,{}^*b_{k,j} = \epsilon_0\,{}^*\dot{e}_i, \quad {}^*b_{i,i} = 0. \tag{12}$$

3. Rayleigh-type wave

The geometry of the problem is shown in Figure 1. The initial uniform electric polarization ${}^0\boldsymbol{P}$ has the direction of x_1 axis. Consider the propagation equations of a Rayleigh-type surface wave [Eringen and Suhubi 1975; Nowacki 1970; Miklowitz 1978] in the same direction:

$$\begin{aligned}\{u_1, u_3, P_1, P_3, B_2\} &= \left\{\tilde{u}_1(x_3), \tilde{u}_3(x_3), \tilde{P}_1(x_3), \tilde{P}_3(x_3), \tilde{B}_2(x_3)\right\}\exp[\mathrm{i}\gamma(\nu t - x_1)],\\ \{e_1, e_3, b_2\} &= \left\{\tilde{e}_1(x_3), \tilde{e}_3(x_3), \tilde{b}_2(x_3)\right\}\exp[\mathrm{i}\gamma(\nu t - x_1)].\end{aligned} \tag{13}$$

In the above equations and in the further considerations the following description (which should not lead to failures) is introduced:

$${}^*P_\alpha = P_\alpha, \quad {}^*B_\alpha = B_\alpha, \quad {}^*e_i = e_i, \quad {}^*b_i = b_i, \quad {}^0P_\alpha = \delta_{\alpha 1}P. \tag{14}$$

After simply calculations one obtains the following system of the ordinary differential equations valid in the half space $x_3 > 0$.

$$\begin{aligned} c_2^2\tilde{u}_1'' + \gamma^2(\nu^2 - c_1^2)\tilde{u}_1 - \mathrm{i}\gamma(c_1^2 - c_2^2)\tilde{u}_3' + \mathrm{i}\frac{\gamma P}{\rho_0\epsilon_0\kappa}\tilde{P}_1 - \frac{P}{\rho_0\epsilon_0\kappa}\tilde{P}_3' &= 0,\\ c_1^2\tilde{u}_3'' + \gamma^2(\nu^2 - c_2^2)\tilde{u}_3 - \mathrm{i}\gamma(c_1^2 - c_2^2)\tilde{u}_1' &= 0,\\ \tilde{P}_1'' - \gamma^2\alpha\tilde{P}_1 - \mathrm{i}\frac{\gamma^3\alpha}{1+\kappa}P\tilde{u}_1 + \mathrm{i}\frac{\gamma}{1+\kappa}P\tilde{u}_1'' &= 0,\\ \tilde{P}_3'' - \gamma^2\alpha\tilde{P}_3 + \frac{\gamma^2\alpha}{1+\kappa}P\tilde{u}_1' - \frac{1}{1+\kappa}P\tilde{u}_1''' &= 0.\end{aligned} \tag{15}$$

The amplitude $\tilde{B}_2$ satisfies the equation

$$\tilde{B}_2'' - \gamma^2\alpha\tilde{B}_2 = 0. \tag{16}$$

In (15) and (16) the following notation has been introduced:

$$\alpha = 1 - (1+\kappa)(1+\kappa')\frac{\nu^2}{c_0^2}, \quad c_1^2 = \frac{\lambda + 2\mu}{\rho_0}, \quad c_2^2 = \frac{\mu}{\rho_0}, \tag{17}$$

$$\frac{d\tilde{\phi}}{dx_3} = \tilde{\phi}', \quad \frac{d^2\tilde{\phi}}{dx_3^2} = \tilde{\phi}'', \quad \frac{d^3\tilde{\phi}}{dx_3^3} = \tilde{\phi}'''. \tag{18}$$

The characteristic equation of the system (15) is

$$\epsilon(\epsilon^2-\gamma^2\alpha)^2\big\{c_1^2(c_2^2-v^2)\epsilon^4+\big[\big(\nu^2+v^2\big)(c_1^2+c_2^2)-2c_1^2c_2^2-\nu^2v^2\big]\gamma^2\epsilon^2 \\ +\big[v^2(\nu^2-c_2^2)+\big(\nu^2-c_1^2\big)(\nu^2-c_2^2)\big]\gamma^4\big\}=0, \quad (19)$$

where

$$v^2=\frac{P^2}{\rho_0\epsilon_0\kappa(1+\kappa)}. \quad (20)$$

The solutions of the set of (15) have finally the form

$$\begin{aligned}
\tilde{u}_1&=S_1\exp(\epsilon_1x_3)+S_2\exp(\epsilon_2x_3),\\
\tilde{u}_3&=\frac{\mathrm{i}\gamma\epsilon_1(c_1^2-c_2^2)}{c_1^2\epsilon_1^2+\gamma^2(\nu^2-c_2^2)}S_1\exp(\epsilon_1x_3)+\frac{\mathrm{i}\gamma\epsilon_2(c_1^2-c_2^2)}{c_1^2\epsilon_2^2+\gamma^2(\nu^2-c_2^2)}S_2\exp(\epsilon_2x_3),\\
\tilde{P}_1&=S_3\exp(-\gamma\sqrt{\alpha}x_3)-\frac{\mathrm{i}\gamma P}{1+\kappa}S_1\exp(\epsilon_1x_3)-\frac{\mathrm{i}\gamma P}{1+\kappa}S_2\exp(\epsilon_2x_3),\\
\tilde{P}_3&=-\frac{\mathrm{i}}{\sqrt{\alpha}}S_3\exp(-\gamma\sqrt{\alpha}x_3)+\frac{\epsilon_1P}{1+\kappa}S_1\exp(\epsilon_1x_3)+\frac{\epsilon_2P}{1+\kappa}S_2\exp(\epsilon_2x_3),
\end{aligned} \quad (21)$$

where S_1, S_2 and S_3 are constants, and ϵ_1 and ϵ_2 are the roots of the expression in (outer) braces in (19), which satisfy the restriction Re $\epsilon<0$. There is no physical singularity for $\epsilon_1=\epsilon_2$ and $\epsilon_1=\epsilon_2=-\gamma\sqrt{\alpha}$. Similarly, the solution of the (16) is

$$\tilde{B}_2=\frac{\mathrm{i}}{\epsilon_0\kappa\nu}\frac{1-\alpha}{\sqrt{\alpha}}S_3\exp(-\gamma\sqrt{\alpha}x_3). \quad (22)$$

In the same way we obtain the following solution for the electromagnetic field in a vacuum:

$$\tilde{e}_1=G\exp(\gamma\sqrt{\beta}x_3),\quad \tilde{e}_3=\frac{\mathrm{i}}{\sqrt{\beta}}G\exp(\gamma\sqrt{\beta}x_3),\quad \tilde{b}_2=\frac{\mathrm{i}(\beta-1)}{\nu\sqrt{\beta}}G\exp(\gamma\sqrt{\beta}x_3), \quad (23)$$

where

$$\beta=1-\frac{\nu^2}{c_0^2}. \quad (24)$$

Successive relations between S_1, S_2, S_3 and G are the consequence of fact that jump conditions (9) hold true. In this case they become

$$e_1^++\frac{P}{\epsilon_0\kappa}u_{1,1}-\frac{1}{\epsilon_0\kappa}P_1^-=0,\quad e_3^++\frac{P}{\epsilon_0\kappa}u_{1,3}-\frac{\kappa+1}{\epsilon_0\kappa}P_3^-=0,\quad b_2^+-\frac{1}{1+\kappa'}B_2^-=0, \\ u_{1,3}+u_{3,1}=0,\quad (c_1^2-2c_2^2)u_{1,1}+c_1^2u_{3,3}=0. \quad (25)$$

After substitutions it is easy to obtain that

$$G=-\sqrt{\frac{\beta}{\alpha}}\frac{1+\kappa}{\epsilon_0\kappa}S_3. \quad (26)$$

This conclusion follows from the third equality in (25). The second equality in (25) is satisfied identically. Finally the following set of equations has to be considered:

$$R_1S_1+R_2S_2+R_3S_3=0,\quad Q_1S_1+Q_2S_2=0,\quad W_1S_1+W_2S_2=0, \quad (27)$$

where

$$R_1 = R_2 = \mathrm{i}\gamma\sqrt{\alpha}\frac{\kappa P}{1+\kappa}, \quad R_3 = \sqrt{\beta}(1+\kappa)+\sqrt{\alpha}, \quad k = 1, 2$$
$$Q_k = \left[1 + \frac{\gamma^2(c_1^2 - c_2^2)}{c_1^2\epsilon_k^2 + \gamma^2(\nu^2 - c_2^2)}\right]\epsilon_k, \quad W_k = \frac{\epsilon_k^2 c_1^2(c_1^2 - c_2^2)}{c_1^2\epsilon_k^2 + \gamma^2(\nu^2 - c_2^2)} - c_1^2 + 2c_2^2. \tag{28}$$

Then in the case under consideration, the propagation condition of the Rayleigh-type wave has the form

$$\begin{vmatrix} R_1 & R_2 & R_3 \\ Q_1 & Q_2 & 0 \\ W_1 & W_2 & 0 \end{vmatrix} = 0. \tag{29}$$

If the condition (29) holds true, it is possible to solve the system (27) with the respect of S_1 and S_2:

$$S_1 = -\frac{R_3 Q_2}{R_1 Q_2 - R_2 Q_1} S_3, \quad S_2 = \frac{R_3 Q_1}{R_1 Q_2 - R_2 Q_1} S_3. \tag{30}$$

For the analysis in the sequel it is convenient to use some dimensionless quantities

$$a = \frac{c_1^2}{c_2^2}, \quad g = \frac{\nu^2}{c_2^2}, \quad f = \frac{\nu^2}{c_2^2}, \quad \tilde{\epsilon} = \frac{\epsilon}{\gamma}. \tag{31}$$

The substitutions (31), applied to (29), yield

$$\left[a(\tilde{\epsilon}_1^2+1)+f-2\right]\left[a(\tilde{\epsilon}_2^2+1)+2f-2-fa\right]\tilde{\epsilon}_1 - \left[a(\tilde{\epsilon}_2^2+1)+f-2\right]\left[a(\tilde{\epsilon}_1^2+1)+2f-2-fa\right]\tilde{\epsilon}_2 = 0. \tag{32}$$

Similarly, writing explicitly the equation from which the roots ϵ_1 and ϵ_2 are determined (see (19)), the following expression has been obtained:

$$a(1-g)\tilde{\epsilon}^4 + \left[(f+g)(a+1) - 2a - fg\right]\tilde{\epsilon}^2 + \left[(f-1)(g+f-a)\right] = 0. \tag{33}$$

The root of (33) do not depend on γ. They are the functions of material constants, P and ν only. Then, from the propagation condition follows that ν does not depend on γ, too (it is convenient to consider this condition in the form (32)). There is no dispersion comparing to in the classical case [Stręk et al. 2010].

4. Some results for classical and auxetic materials

The propagation condition (32) can only be examined numerically. The only exception is the case $a = 2$ (Poisson ratio $\nu = 0$), for which analytical analysis is possible. The results obtained in this case are similar (though not identical) to results which are in force for auxetic materials. These results have not been presented in the paper.

In Figure 2 the dependence of dimensionless phase velocity f on dimensionless (low) external electric field strength g (see (20) and (31)) for conventional material is presented in two scales. In the area bounded by the curve in the shape of a loop, the roots of (33) are complex. There is an interval of values of parameter g for which the wave does not propagate ($g \in (g_B, g_C)$).

In Figure 3 the same dependence for strong external electric field is presented.

For the auxetic [Stręk et al. 2009] material and for the weak external electric field, character of examined dependence is similar to presented above (Figure 2). Significant differences become apparent in the case of strong external electric field ($g \geq g_C$). In this case two extra modes (rapidly vanishing

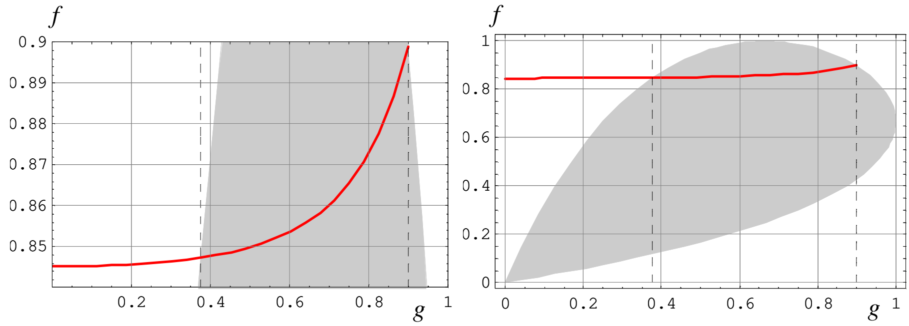

Figure 2. The case $a = 3$ (conventional material, $\nu = 0.25$); $g \leq g_B \approx 0.8990$.

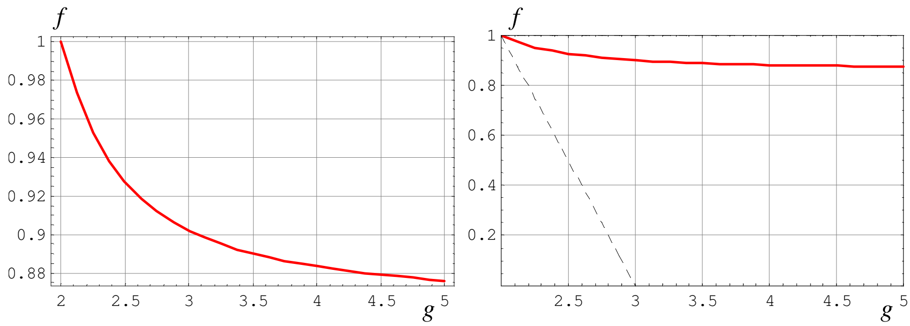

Figure 3. The case $a = 3$ (conventional material, $\nu = 0.25$); $g \geq g_C = 1$.

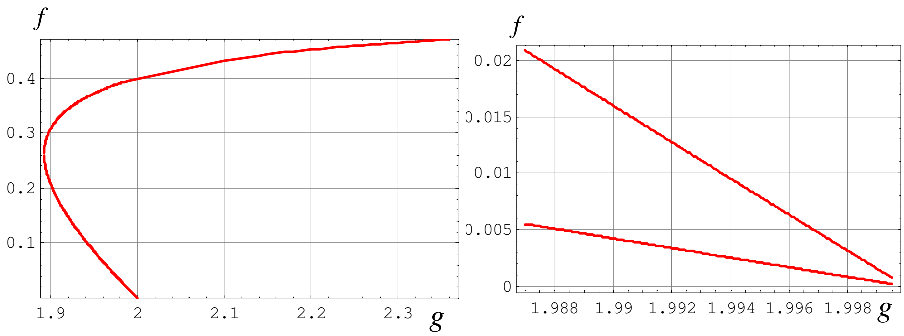

Figure 4. The case $a = 1.5$ (auxetic, $\nu = -0.25$), $g \geq g_C \approx 1.8925$.

with increasing of g) of the wave appear. In the previously reported case, f decreased with increasing g (Figure 3). The other situation is for the "fast", not disappearing mode of the wave in the auxetic. The increase in g results in an increase of f. Similar results were obtained in the previously mentioned case $\nu = 0$. In this case for strong external electric field only two modes of the wave appear.

In Figure 4 the dependence of dimensionless phase velocity f on dimensionless (strong) external electric field g for the auxetic material is presented in two scales. Lowest mode fades so quickly that it was necessary to present it choosing a different scale (Figure 4, right).

References

[Eringen and Suhubi 1975] A. C. Eringen and E. S. Suhubi, *Elastodynamics, II: Linear theory*, Academic Press, New York, 1975.

[Friis et al. 1988] E. A. Friis, R. S. Lakes, and J. B. Park, "Negative Poisson's ratio polymeric and metallic foams", *J. Mater. Sci.* **23**:12 (1988), 4406–4414.

[Hutter and van de Ven 1978] K. Hutter and A. A. F. van de Ven, *Field matter interactions in thermoelastic solids: unification of existing theories of electro-magneto-mechanical interactions*, Lecture Notes in Physics **88**, Springer, Berlin, 1978.

[Lakes 1987] R. S. Lakes, "Foam structures with a negative Poisson's ratio", *Science* **235**:4792 (1987), 1038–1040.

[Lakes and Wineman 2006] R. S. Lakes and A. Wineman, "On Poisson's ratio in linearly viscoelastic solids", *J. Elasticity* **85**:1 (2006), 45–63.

[Landau and Lifshits 1954] L. D. Landau and E. M. Lifshits, Механика сплошных сред, Gos. Izd. tekhn.-teoret. lit., Moscow, 1954. Translated as *Fluid mechanics*, Pergamon, London, 1959.

[Miklowitz 1978] J. Miklowitz, *The theory of elastic waves and waveguides*, Series in Applied Mathematics and Mechanics **22**, North-Holland, Amsterdam, 1978.

[Nowacki 1970] W. Nowacki, *Teoria sprężystości*, PWN, Warsaw, 1970.

[Poźniak et al. 2010] A. A. Poźniak, H. Kamiński, P. Kędziora, B. Maruszewski, T. Stręk, and K. W. Wojciechowski, "Anomalous deformation of constrained auxetic square", *Rev. Adv. Mater. Sci.* **23**:2 (2010), 169–174.

[Stręk et al. 2008] T. Stręk, B. Maruszewski, J. W. Narojczyk, and K. W. Wojciechowski, "Finite element analysis of auxetic plate deformation", *J. Non-Cryst. Solids* **354**:35–39 (2008), 4475–4480.

[Stręk et al. 2009] T. Stręk, P. Kędziora, B. Maruszewski, A. A. Poźniak, K. V. Tretiakov, and K. W. Wojciechowski, "Finite element analysis of auxetic obstacle deformation and fluid flow in a channel", *J. Non-Cryst. Solids* **355**:24–27 (2009), 1387–1392.

[Stręk et al. 2010] T. Stręk, B. Maruszewski, A. A. Poźniak, and K. W. Wojciechowski, "Computational modelling of auxetics", pp. 265–284 in *Finite element analysis*, edited by D. Moratal, Sciyo, Rijek, 2010.

Received 24 Jan 2011. Revised 12 Dec 2011. Accepted 29 Dec 2011.

ANDRZEJ DRZEWIECKI: andrzej.drzewiecki@put.poznan.pl
Poznań University of Technology, Institute of Applied Mechanics, ul. Jana Pawla II 24, 60-965 Poznań, Poland

JOURNAL OF MECHANICS OF MATERIALS AND STRUCTURES

jomms.net

Founded by Charles R. Steele and Marie-Louise Steele

Cover design: Alex Scorpan

See http://jomms.net for submission guidelines.

JoMMS (ISSN 1559-3959) is published in 10 issues a year. The subscription price for 2012 is US $555/year for the electronic version, and $735/year (+$60 shipping outside the US) for print and electronic. Subscriptions, requests for back issues, and changes of address should be sent to Mathematical Sciences Publishers, Department of Mathematics, University of California, Berkeley, CA 94720–3840.

JoMMS peer-review and production is managed by EditFlow® from Mathematical Sciences Publishers.

PUBLISHED BY

mathematical sciences publishers

http://msp.org/

A NON-PROFIT CORPORATION

Typeset in LaTeX

Journal of Mechanics of Materials and Structures

Volume 7, No. 3 March 2012

Special issue

Trends in Continuum Physics (TRECOP 2010)