ELECTROMAGNETOELASTIC WAVES IN A VORTEX LAYER OF A SUPERCONDUCTOR

Bogdan T. Maruszewski, Andrzej Drzewiecki and Roman Starosta
Magnetic field enters the type-II superconducting body along a discrete arrangement of magnetic vortex lines. The paper aims at investigating the dispersion and amplitude distributions of magnetoelastic waves propagating solely in the vortex field of the superconducting layer. Our attention has been focused on the dispersion features and amplitudes for various wave velocities. The vortex field consists only of soft vortices (the superconducting crystal is free of lattice defects).

1. Introduction

Magnetic flux can penetrate the type-II superconductor in the form of Abrikosov vortices (also called flux lines, flux tubes or fluxons), each carrying a quantum of magnetic flux. Since the vortices are formed by the applied magnetic field, the supercurrent flows around of each of them. There exist also Lorentz force interactions among them, which give rise to an additional thermomechanical (stress) field in type-II superconductors. Near the lower critical magnetic intensity limit $H_{c1}$, this field has elastic character. However, if the density of the supercurrent is above its critical value and/or the temperature is sufficiently high, there occurs a flow (creep or diffusion) of vortex lines in the superconducting body. The fluidity of the vortex array has been also observed when the applied magnetic field approaches its upper critical limit $H_{c2}$ [Blatter et al. 1994; Brandt 1995; Cyrot and Pavuna 1992]. Following a magnetothermomechanical model of interactions in such II-type superconductors both for the lattice-like and fluid-like states of the vortex field, where the specific definition of the vortex field stress tensor has been introduced [Maruszewski 1998; 2007], this paper aims at determining the dispersion and amplitude distributions of magnetoelastic waves propagating along soft vortices in the superconducting layer.

If the crystal lattice of the layer is free of lattice defects, there is a parallel arrangement of soft vortex cores (Figure 1, left). However, in real crystals, imperfections such as vacancies, dislocations, disclinations, grain boundaries, and interstitial atoms cause that the vortex lines can be curved or even tangled because of pinning (Figure 1, right). We have confined in the paper only to the lattice-like state. The choice of the geometry of the vortex field (layer) was motivated by a need to check if those magnetoelastic waves and electromagnetoelastic waves in such peculiar environment are possible to be guided.

The natural conditions where devices may use such a technological solution exist in space where temperature is about 4K and majority of elements and materials behave as superconductors. In the paper the particular analysis of the SH magnetoelastic and compressional and flexural electromagnetoelastic modes has been presented.

Partially supported by Poland’s Ministry of Science and Higher Education (MNiSzW) grant 1101/T02/2006/30 and DS grant 21-320/2009.

Keywords: magnetoelastic waves, dispersion, superconductor.
2. General wave equations

Let us now consider a problem which deals with the dynamics of the previously defined vortex field. Following the thermodynamical model of the electromagnetomechanical interactions in continuous media, i.e., the momentum balance with Hooke’s law and the Maxwell equations in the moving frames with the first London equation we are able to formulate proper equations describing dynamics of the vortex field in the elastic superconductor. For the sake of simplicity thermal influences have been omitted. The governing equations of the electromagnetoelastic vortex waves read as follows [Maruszewski et al. 2007]:

\begin{align}
\mu u_{i,ij} + \eta \dot{u}_{i,ij} + (\lambda + \mu)u_{j,ij} + \frac{1}{3} \eta \ddot{u}_{j,ij} - \mu_0(h_{r,i} - h_{i,r})H_{r}^0 - \rho \dddot{u}_i = 0, \\
\lambda_0^2 h_{i,kk} - h_i + u_{i,k}H_k^0 - u_{k,k}H_i^0 = 0, \\
\lambda_0^2 e_{i,jj} - e_i + \mu_0 e_{i,jk} \dddot{u}_j H_k^0 = 0,
\end{align}

(1)

where $u_k$ is the displacement vector of the vortex field point, $H_k^0$ is the applied magnetic field normal to the limiting surfaces of the layer (Figure 1), $h_r$ is the small contribution to the total magnetic field in the layer comparing to $H_r^0$ and describes its perturbations, because the linear form of equations (1) results from

\begin{equation}
H_k = H_k^0 + h_k, \quad |h_k| \ll |H_k^0|.
\end{equation}

(2)

Here $e_k$ is the electric field intensity understood in the sense of (2), $\lambda$ and $\mu$ are Lamé’s constants, $\mu_0$ is the permeability of vacuum, $\eta$ is the viscosity coefficient (inside each vortex core flows a normal current, so the Ohmic resistivity occurs there [Cyrot and Pavuna 1992]), $\rho$ is the vortex density [Maruszewski 2007; Maruszewski et al. 2007], and $\lambda_0$ is the London penetration depth. Any thermal influences on the considered waves have been omitted. The solutions of equations (1) are looked for in the form

\begin{equation}
f(x_1, x_3, t) = \tilde{f}(x_1) \exp[i(\omega t - kx_3)],
\end{equation}

(3)

where $\tilde{f}(x_1)$ are the amplitudes of signals $u_1, u_2, u_3, h_1, h_2, h_3, e_1, e_2, e_3$ propagating in $x_1$ direction with the velocity $v$:

\begin{equation}
\tilde{f}(x_1) = \{u_1, u_2, u_3, h_1, h_2, h_3, e_1, e_2, e_3\}
\end{equation}

(4)

if the geometry of the problem is as shown in Figure 2.
Figure 2. Geometry of the problem.

To facilitate the analysis of the waves, all the relations will be presented in the dimensionless form, with the help of the following substitutions, where $\tilde{c}$ is the speed of light:

$$x_1 = hx, \quad x_2 = hy, \quad x_3 = hz, \quad t = T\tau, \quad T = \frac{h}{v_T}, \quad v_T = \sqrt{\frac{\mu}{\rho}}, \quad \Omega = \omega T,$$

$$c = \frac{kvt}{\omega} = \frac{v_T}{v}, \quad \tilde{\lambda}_0 = \frac{\lambda_0}{h}, \quad \tilde{\mu} = \frac{K\mu}{\tilde{\lambda}},$$

$$K = \lambda + \frac{\gamma}{3}\mu, \quad \tilde{\lambda} = \frac{\lambda}{\mu}, \quad \tilde{\rho} = \frac{\rho v_T^2}{\mu} = 1, \quad H^0 = H_c H_0, \quad E = \tilde{c}\mu_0 H_c,$$

$$c_1 = \frac{v_T}{\tilde{c}}, \quad u_1 = h u_x, \quad u_2 = h u_y, \quad u_3 = h u_z,$$

$$\tilde{\eta} = \eta \frac{h}{v_T}, \quad e_1 = E e_x, \quad e_2 = E e_y, \quad e_3 = E e_z, \quad \tilde{\mu} = \frac{\mu_0 H_c^2}{\mu},$$

$$h_1 = H_c h_x, \quad h_2 = H_c h_y, \quad h_3 = H_c h_z.$$

(5)

3. SH magnetoelastic waves

Let us consider at the beginning the SH magnetoelastic wave propagating along the vortex layer shown in Figure 2. The general equations (1) in this case reduce to

$$\mu u_{i,j} + \eta u_{i,j} + (\lambda + \mu)u_{j,i} + \frac{1}{3}\eta u_{j,ij} - \mu_0(h_{r,i} - h_{i,r})H^0_r - \rho \tilde{u}_i = 0,$$

$$\lambda_0^2 h_{i,kk} - h_i + u_{i,k} H^0_k - u_{k,k} H^0_i = 0,$$

(6)

(7)

since the SH wave amplitudes in the layer are now looked for in the form (3) with

$$\tilde{f}(x_1) = \{u_2, h_2\},$$

(8)

because

$$u = [0, u_2, 0], \quad h = [0, h_2, 0].$$

(9)

Taking (5) into account, the SH wave equations (6) and (7) become in dimensionless form

$$\tilde{\mu} + i\tilde{\eta} \frac{d^2 u_y}{dx^2} + \frac{\Omega^2}{c^2}(v^2 \tilde{\rho} - \tilde{\mu} - i\Omega \tilde{\eta})u_y + \tilde{\mu} H_0 \frac{dh_y}{dx} = 0,$$

$$\tilde{\lambda}_0^2 \frac{d^2 h_y}{dx^2} - (\tilde{\lambda}_0^2 \frac{\Omega^2}{c^2} + 1)h_y + H_0 \frac{du_y}{dx} = 0.$$
On using relations (5) in (8), the solutions of (10) determine the following amplitudes of the SH magnetoelastic waves:

\[ u_x = S_1 e^{p_1 x} + S_2 e^{-p_1 x} + S_3 e^{p_2 x} + S_4 e^{-p_2 x}, \]
\[ h_y = -M(p_1, \Omega, c)S_1 e^{p_1 x} + M(p_1, \Omega, c)S_2 e^{-p_1 x} - M(p_2, \Omega, c)S_3 e^{p_2 x} + M(p_2, \Omega, c)S_4 e^{-p_2 x}, \]

where

\[ M(p_k, \Omega, c) = \frac{p_k}{\mu H_0} + \frac{\Omega^2 (1 - 1/c^2)}{\mu H_0 p_k}, \quad k = 1, 2. \]

The parameters \( p_1 \) and \( p_2 \) were determined from the characteristic equation

\[ \tilde{\lambda}_0^2 A(\Omega) p^4 + \left[ \tilde{\lambda}_0^2 B(\Omega, c) - F(\Omega, c) A(\Omega) - \mu_0 H_0^2 \right] p^2 - F(\Omega, c) B(\Omega, c) = 0 \]

where

\[ A(\Omega) = 1 + i\Omega\tilde{\eta}, \quad B(\Omega, c) = \frac{\Omega^2}{c^2} (c^2 - 1 - i\Omega\tilde{\eta}), \quad F(\Omega, c) = \tilde{\lambda}_0^2 \frac{\Omega^2}{c^2} + 1. \]

A detailed analysis of the characteristic equation shows that wave propagation is possible only if

\[ B(\Omega, c) > 0. \]

From (13) we get that if SH magnetoelastic wave amplitude is a function of \( x \), then

\[ c = \frac{v}{v_r} < 1 \]

We neglect, from now on, viscous features of the vortex field, in view of their very weak influence on the propagation process; see [Cyrot and Pavuna 1992; Maruszewski et al. 2008].

Since the waves propagate along the layer, we should now take into account the proper boundary conditions for solving the set of equations (10). They follow from Figure 2, and read

\[ h_y = 0 \quad \text{and} \quad \frac{du_y}{dx} = 0 \quad \text{for} \quad x = -1 \quad \text{and} \quad x = 0. \]

The first of these conditions expresses the continuity of the tangent component of the magnetic field intensity, while the second follows from the equality \( \sigma_{yz} = 0 \) (the surfaces are free of loadings).

Substituting (11) into (15) we obtain the set of algebraic equations

\[ -M(p_1, \Omega, c)S_1 e^{-p_1} + M(p_1, \Omega, c)S_2 e^{p_1} - M(p_2, \Omega, c)S_3 e^{-p_2} + M(p_2, \Omega, c)S_4 e^{p_2} = 0, \]
\[ S_1 p_1 e^{-p_1} - S_2 p_1 e^{p_1} + S_3 p_2 e^{-p_2} + S_4 p_2 e^{p_2} = 0, \]
\[ -M(p_1, \Omega, c)S_1 + M(p_1, \Omega, c)S_2 - M(p_2, \Omega, c)S_3 + M(p_2, \Omega, c)S_4 = 0, \]
\[ S_1 p_1 - S_2 p_1 + S_3 p_2 + S_4 p_2 = 0, \]

or

\[ W \cdot S = 0, \]

where

\[ S = \{S_1, S_2, S_3, S_4\}^T. \]

Hence we arrive at the dispersion relation for SH magnetoelastic wave in the form

\[ \det W = 0. \]
A unique solution for (19) exists if
\[ c = 1. \] (20)

Thus the SH wave is nondispersive and its amplitude is independent of \( x \). (See [Oliner 1978, Chapter 5].) So, the supposition (14) is no longer valid. But the final conclusion is such that we are dealing only with the plane SH wave mode in the vortex layer, which propagates with constant velocity — (20).

4. Compressional (C) and flexural (F) electromagnetoelastic waves

Let us now focus on electromagnetoelastic waves propagating along the magnetic vortex field layer. Our aim is to answer the question of whether they can be guided by such a layer. It is well known that, besides the SH magnetoelastic waves considered in the previous section, compressional (symmetric) and flexural (antisymmetric) waves may also propagate along waveguides in the form of a rod, plate or ribbon [Achenbach 1976, Chapter 8; Oliner 1978, Chapter 5; Eringen and Suhubi 1975, Chapter 7] (Figure 3).

The solutions of the \( C \) and \( F \) wave equations (1) are looked this time for in the form (3) where \( \tilde{f}(x_1) \) now stands for \( \tilde{f}(x_1) = \{u_1, u_3, h_1, h_3, e_2\} \) (21) since (see Figure 4)
\[ u = [u_1, 0, u_3], \quad h = [h_1, 0, h_3], \quad e = [0, e_2, 0]. \] (22)

On using now (21) and (5) in (1) the dimensionless \( F \) and \( C \) wave equations read
\[ \left( \frac{4}{3} + \frac{1}{3} \tilde{G} \right) u_{x,x} + \Omega^2 (1 - c^2) u_x - \frac{1}{2} i \Omega c (\tilde{G} + 1) u_{z,x} = 0, \]
\[ u_{z,x} + \Omega^2 \left[ 1 - c^2 (\frac{4}{3} + \frac{1}{3} \tilde{G}) \right] u_z - \frac{1}{2} i \Omega c (\tilde{G} + 1) u_{x,x} + i \tilde{\mu} H_0 \Omega c h_x + \tilde{\mu} H_0 h_{z,x} = 0, \]
\[ \tilde{\lambda}_0^2 h_{x,xx} - (1 + \tilde{\lambda}_0^2 \Omega^2 c^2) h_x + i \Omega c H_0 u_z = 0, \]
\[ \tilde{\lambda}_0^2 h_{z,zz} - (1 + \tilde{\lambda}_0^2 \Omega^2 c^2) h_z + H_0 u_{z,x} = 0, \]
\[ \tilde{\lambda}_0^2 e_{y,xx} - (1 + \tilde{\lambda}_0^2 \Omega^2 c^2) e_y - i \Omega c_1 H_0 u_z = 0. \] (23)

**Figure 3.** Symmetry of the vortex field displacements [Eringen and Suhubi 1975]: symmetric-compressional wave (left pair) and antisymmetric-flexural wave (right pair).

**Figure 4.** Geometry of the problem for the \( C \) and \( F \) waves in the vortex layer, following (5).
These equations should be completed by the electromagnetic field equations in vacuum:

\[ h_{x,xx}^p - \Omega^2(c^2 - c_1^2)h_x^p = 0, \quad h_{z,xx}^p - \Omega^2(c^2 - c_1^2)h_z^p = 0, \quad e_{y,xx}^p - \Omega^2(c^2 - c_1^2)e_y^p = 0, \tag{24} \]

where superscript \( p \) denotes fields in a vacuum.

The jump and boundary conditions for (23) and (24) at the upper and lower planes of the layer read (see Figure 4)

\[
\begin{align*}
\sigma_{xx}|_{x=-1} = 0 & \Rightarrow \frac{du_x}{dx}|_{x=-1} = 0; \quad \sigma_{zx}|_{x=-1} = 0 \Rightarrow \frac{du_z}{dx}|_{x=-1} = 0; \\
(h_x = h_x^p)|_{x=0} \text{ or } (e_y = e_y^p)|_{x=0}; \quad (h_z = h_z^p)|_{x=0}; \\
\sigma_{xx}|_{x=1} = 0 & \Rightarrow \frac{du_x}{dx}|_{x=1} = 0; \quad \sigma_{zx}|_{x=1} = 0 \Rightarrow \frac{du_z}{dx}|_{x=1} = 0; \\
(h_x = h_x^p)|_{x=1} \text{ or } (e_y = e_y^p)|_{x=1}; \quad (h_z = h_z^p)|_{x=1}.
\end{align*}
\tag{25} \]

The desired \( C \) and \( F \) wave amplitudes are (using \( \text{sh} \) and \( \text{ch} \) to denote the hyperbolic sine and cosine)

\[
\begin{align*}
 u_x &= P(\xi_2)S_4 \text{ch}(\xi_2x) + P(\xi_2)S_3 \text{sh}(\xi_2x) + P(\xi_3)S_6 \text{ch}(\xi_3x) + P(\xi_3)S_5 \text{sh}(\xi_3x) + P(\xi_4)S_8 \text{ch}(\xi_4x) \\
&\quad + P(\xi_4)S_7 \exp(\xi_4x), \\
 u_z &= Q(\xi_2)S_3 \text{ch}(\xi_2x) + Q(\xi_2)S_4 \text{sh}(\xi_2x) + Q(\xi_3)S_5 \text{ch}(\xi_3x) + Q(\xi_3)S_6 \text{sh}(\xi_3x) + Q(\xi_4)S_7 \text{ch}(\xi_4x) \\
&\quad + Q(\xi_4)S_8 \text{sh}(\xi_4x), \\
 h_x &= S_3 \text{ch}(\xi_2x) + S_4 \text{sh}(\xi_2x) + S_5 \text{ch}(\xi_3x) + S_6 \text{sh}(\xi_3x) + S_7 \text{ch}(\xi_4x) + S_8 \text{sh}(\xi_4x), \\
 h_z &= -\frac{i}{\Omega c}[\xi_2S_4 \text{ch}(\xi_2x) + \xi_2S_3 \text{sh}(\xi_2x) + \xi_3S_6 \text{ch}(\xi_3x) + \xi_3S_5 \text{sh}(\xi_3x) + \xi_4S_8 \text{ch}(\xi_4x) + \xi_4S_7 \text{sh}(\xi_4x)], \\
 e_y &= -\frac{c_1}{c}[S_3 \text{ch}(\xi_2x) + S_4 \text{sh}(\xi_2x) + S_5 \text{ch}(\xi_3x) + S_6 \text{sh}(\xi_3x) + S_7 \text{ch}(\xi_4x) + S_8 \text{sh}(\xi_4x)],
\end{align*}
\tag{26} \]

where

\[
\begin{align*}
P(\xi) &= -\frac{\xi(\tilde{G} + 1)(\tilde{\lambda}_0^2 - 1 - \tilde{\lambda}_0^2\Omega^2c^2)}{H_0[(4 + \tilde{G})\xi^2 + 3\Omega^2(1 - c^2)]}, \\
Q(\xi) &= \frac{\tilde{\lambda}_0^2\xi^2 - 1 - \tilde{\lambda}_0^2\Omega^2c^2}{\Omega c H_0}.
\end{align*}
\]

The parameters \( \xi_m \) are the solutions of the characteristic equation

\[
(A\xi^6 + B\xi^4 + C\xi^2 + D)(\Omega^2c^2\tilde{\lambda}_0^2 - \tilde{\lambda}_0^2\xi^2 + 1) = 0,
\tag{27} \]

whose coefficients \( A, B, C \) and \( D \) are given in the Appendix.

The amplitudes of the electromagnetic waves outside the layer read as follows:

– above the upper plane:

\[
\begin{align*}
 h_x^p &= S_1[\text{ch}(\tilde{\xi}_1x) + \text{sh}(\tilde{\xi}_1x)], \\
 h_z^p &= -\frac{i\tilde{\xi}_1}{\Omega c}S_1[\text{ch}(\tilde{\xi}_1x) + \text{sh}(\tilde{\xi}_1x)], \\
 e_y^p &= -\frac{c_1}{c}S_1[\text{ch}(\tilde{\xi}_1x) + \text{sh}(\tilde{\xi}_1x)],
\end{align*}
\tag{28} \]

– below the lower plane:

\[
\begin{align*}
 h_x^p &= S_2[\text{ch}(\tilde{\xi}_1x) - \text{sh}(\tilde{\xi}_1x)], \\
 h_z^p &= \frac{i\tilde{\xi}_1}{\Omega c}S_2[\text{ch}(\tilde{\xi}_1x) - \text{sh}(\tilde{\xi}_1x)], \\
 e_y^p &= -\frac{c_1}{c}S_2[\text{ch}(\tilde{\xi}_1x) + \text{sh}(\tilde{\xi}_1x)].
\end{align*}
\tag{29} \]
Now using solutions (26), (28) and (29) in the boundary and jump conditions (25) we get a set of eight algebraic equations for $S_1$ through $S_8$. They are given in the Appendix. After laborious calculations this set of eight equation splits into two uncoupled sets:

$$W_C \cdot S_C = 0,$$

$$W_F \cdot S_F = 0,$$

where

$$S_C = \{\frac{1}{2}(S_1 - S_2), S_4, S_6, S_8\}^T, \quad S_F = \{\frac{1}{2}(S_1 + S_2), S_3, S_5, S_7\}^T.$$

Hence the dispersion relations for the considered waves are

- for compressional modes: $\det W_C = 0,$
- for flexural modes: $\det W_F = 0.$

5. Numerical results

In this section we limit ourselves to the numerical analysis of the compressional and flexural electromagnetic wave propagation conditions since the SH magnetic wave is nondispersive propagating with the constant velocity and is amplitude independent of the thickness of the layer. All the calculations have been made for YBa$_2$Cu$_3$O$_{6+x}$, or YBCO, ceramics. We have restricted ourselves to the lattice-like state of the vortex field, so the external magnetic $H_0$ is taken slightly higher than the limiting lower magnetic field value $H_{c1}$.

Firstly, let us take care of the existence of the $C$ and $F$ waves in the layer. The conditions dealing with their existence result from characteristic equation (27). That situation is illustrated in Figure 5.

Before we start analysis of the amplitude distribution of the $C$ and $F$ waves in the layer we should consider their dispersion. Figure 6 shows the dispersion of the $C$ waves calculated from (32), and the $F$ waves, from (33).

Remark that both in the $C$ and $F$ wave cases only one single modes propagate. That feature is anomalous and differs from that for the classical elastic body where infinite number of modes propagate [Eringen and Suhubi 1975, Chapter 7]. From Figure 6 it is also visible that waves $C$ and $F$ characterize anomalous dispersion: they propagate with acoustic velocity having optical frequencies ($C$ wave has

![Figure 5. Area of existence of $C$ and $F$ waves.](image-url)
higher frequency comparable to the frequency spectrum of the visible light then $F$ wave has lower frequency comparable to the frequency spectrum of the infrared rays). Moreover, for both cases the dispersion decreases if the external magnetic field $H_0$ increases. Now let us take care of the amplitudes.

**Compressional waves.** The compressional ($C$) wave amplitudes for $h_x, h_z, e_y, u_x, u_z, \sigma_{xx}$ and $\sigma_{xz}$ are presented in Figure 7. Shaded areas indicate the layer region.

**Figure 6.** Dispersion of $C$ waves (left) and of $F$ waves (right) for $h = 10^{-7}$ m. Black: $H_0 = 1$; red: $H_0 = 10$.

**Figure 7.** Compressional wave amplitudes for $H_0 = 1$ and $h = 10^{-7}$ m. Top row: $h_x, h_z, e_y$. Middle row: $u_x, u_z$. Bottom row: $\sigma_{xx}, \sigma_{xz}$. Green: $\Omega = 0.2$. Blue: $\Omega = 0.6$. Red: $\Omega = 0.9$. 
The simple look at the drawings in Figure 7 shows that in the case of the electromagnetic part of the C waves (top row) their amplitudes reach extremal values at the lateral planes of the layer and are continuous across them. But the mechanical part of the C waves behave differently (middle and bottom rows). They naturally exist only between the lateral planes of the layer. Then the longitudinal \(u_x\) and shear \(\sigma_{xz}\) modes are symmetric with respect to the middle surface of the layer. But the general conclusion for all modes is such that all their amplitudes decreases if the frequency \(\Omega\) increases.

**Flexural waves.** The flexural (F) wave amplitudes for \(h_x, h_z, e_y, u_x, u_z, \sigma_{xx}\) and \(\sigma_{xz}\) are presented in Figure 8.

The properties of the F wave amplitudes differ from their C counterpart. For example, \(h_x\) and \(e_z\) amplitudes are extremal at the middle surface of the layer and do not “feel” its lateral planes disappearing smoothly in infinity. Then antisymmetric \(h_z\) is extremal at the above planes. The mechanical mode amplitudes behave as follows:

- \(u_x\) mode is linear and antisymmetric vanishing at the middle surface; it exist solely inside the layer.
- \(u_z\) is symmetric with respect to the middle surface reaching there extremum.

![Figure 8](image.png)

**Figure 8.** Flexural wave amplitudes for \(H_0 = 1\) and \(h = 10^{-7}\) m. Green: \(\Omega = 0.2\). Blue: \(\Omega = 0.6\). Red: \(\Omega = 0.9\). Top row: \(h_x, h_z, e_y\). Middle row: \(u_x, u_z\). Bottom row: \(\sigma_{xx}, \sigma_{xz}\). The last panel, (h), shows the same graphs as (g) but at different scales.
The coefficients of (27) are given by

\[ \sigma_{xx} \text{ (the trace part of the stress associated with the wave propagation direction) behaves like } u_z; \text{ however, it vanish at the lateral planes.} \]

\[ \sigma_{xz} \text{ is antisymmetric with respect to the middle plane vanishing at the lateral surfaces. However it behaves anomalously; from the last two panels in the bottom row of Figure 8 we see that there exists a frequency } \Omega_{cr} \text{ for which } \sigma_{xz} \text{ values are equal to zero across the layer, so the } \sigma_{xz} \text{ stress values change sign if they are observed for } \Omega < \Omega_{cr} \text{ or for } \Omega > \Omega_{cr}. \]

According to the dependence of } \Omega \text{ the electromagnetic mode amplitudes decrease if the frequency } \Omega \text{ increases contrary to the mechanical modes behavior (Figure 8).}

6. Conclusions

The considerations made in the paper show that two general types of waves can propagate in the vortex array existing solely in the superconducting layer. The first one deals with the SH magnetoelastic modes and the second type concerns electromagnetoelastic compressionals and flexural modes. It is known that layers can be used in some situations as waveguides. Investigations made in the paper confirmed that guided signal transmission along the magnetic vortex field is possible. Natural conditions for such an application exist in free space, so such a technology, which is not energy consuming, might be used in space ships or other similar equipment.

Except for classical properties, electromagnetoelastic } C \text{ and } F \text{ waves involve the same anomalous features, as seen in Figure 8(c), (g), (h). The } u_x \text{ mode is linearly distributed across the layer, which differs from the classical (mechanical non-vortex) situation. But a much more interesting result is presented in Figure 8 (g), (h). We see that there exists a definite critical frequency } \Omega_{cr} \text{ for which the amplitude of the } F \sigma_{xz} \text{ mode vanishes across the entire layer. That means that for such critical frequency } \Omega_{cr} \sigma_{xz} \text{ mode does not propagate.}

Appendix: The coefficients of (27) and the algebraic equations encoded in (30)–(31)

The coefficients of (27) are given by

\[
A = \tilde{\lambda}_0^2(4 + \tilde{G}),
\]

\[
B = -4(3\Omega^2c^2\tilde{\lambda}_0^2 + 1) - \tilde{G}(3\Omega^2c^2\tilde{\lambda}_0^2 + 1) - 4H_0^2\tilde{\mu} + 7\Omega^2\tilde{\lambda}_0^2 + \tilde{G}(\Omega^2\tilde{\lambda}_0^2 - H_0^2\tilde{\mu}),
\]

\[
C = 4\Omega^2c^2(3\Omega^2c^2\tilde{\lambda}_0^2 + 2) + \Omega^2[\tilde{G}c^2(3\Omega^2c^2\tilde{\lambda}_0^2 + 2) + 7(H_0^2c^2\tilde{\mu} - 2\Omega^2c^2\tilde{\lambda}_0^2 - 1)]
+ \Omega^2[\tilde{G}(H_0^2c^2\tilde{\mu} - 2\Omega^2c^2\tilde{\lambda}_0^2 - 1) - 3(H_0^2\tilde{\mu} - \Omega^2\tilde{\lambda}_0^2)],
\]

\[
D = -4\Omega^2c^4(\Omega^2c^2\tilde{\lambda}_0^2 + 1) - \Omega^4c^2[\tilde{G}c^2(\Omega^2c^2\tilde{\lambda}_0^2 + 1) + 3H_0^2c^2\tilde{\mu} - 7(\Omega^2c^2\tilde{\lambda}_0^2 + 1)]
+ \Omega^4[\tilde{G}c^2(\Omega^2c^2\tilde{\lambda}_0^2 + 1) + 3(H_0^2c^2\tilde{\mu} - \Omega^2c^2\tilde{\lambda}_0^2 - 1)].
\]

We next give the algebraic equations satisfied for } S_1, S_2, \ldots, S_8. \text{ Let}

\[ Y(\xi) = \frac{1}{3}(4 + \tilde{G})P(\xi)\xi - \frac{1}{3}i\Omega c(\tilde{G} - 2)Q(\xi) \quad \text{and} \quad Z(\xi) = [-Q(\xi)\xi + i\Omega cP(\xi)], \]
where \( \text{sh} \) and \( \text{ch} \) denote the hyperbolic sine and cosine. Then

\[
Y(\xi_2) \text{ch} \xi_2 S_3 - Y(\xi_2) \text{ch} \xi_2 S_4 + Y(\xi_3) \text{sh} \xi_3 S_5 - Y(\xi_3) \text{sh} \xi_3 S_6 + Y(\xi_4) \text{ch} \xi_4 S_7 - Y(\xi_4) \text{sh} \xi_4 S_8 = 0,
\]

\[
Z(\xi_2) \text{ch} \xi_2 S_3 - Z(\xi_2) \text{sh} \xi_2 S_4 + Z(\xi_3) \text{ch} \xi_3 S_5 - Z(\xi_3) \text{sh} \xi_3 S_6 + Z(\xi_4) \text{ch} \xi_4 S_7 - Z(\xi_4) \text{sh} \xi_4 S_8 = 0,
\]

\[
\bar{\xi}_1 (\text{ch} \bar{\xi}_1 - \text{sh} \bar{\xi}_1) S_1 + \bar{\xi}_2 \text{sh} \xi_2 S_3 - \bar{\xi}_2 \text{ch} \xi_2 S_4 - \bar{\xi}_3 \text{sh} \xi_3 S_5 - \bar{\xi}_3 \text{ch} \xi_3 S_6 + \bar{\xi}_4 \text{sh} \xi_4 S_7 - \bar{\xi}_4 \text{ch} \xi_4 S_8 = 0,
\]

\[
(\text{ch} \bar{\xi}_1 - \text{sh} \bar{\xi}_1) S_1 - \text{ch} \xi_2 S_3 + \text{sh} \xi_2 S_4 - \text{ch} \xi_3 S_5 + \text{sh} \xi_3 S_6 - \text{ch} \xi_4 S_7 + \text{sh} \xi_4 S_8 = 0,
\]

\[
Y(\xi_2) \text{ch} \xi_2 S_3 + Y(\xi_2) \text{sh} \xi_2 S_4 + Y(\xi_3) \text{sh} \xi_3 S_5 + Y(\xi_3) \text{ch} \xi_3 S_6 + Y(\xi_4) \text{ch} \xi_4 S_7 + Y(\xi_4) \text{sh} \xi_4 S_8 = 0,
\]

\[
Z(\xi_2) \text{ch} \xi_2 S_3 + Z(\xi_2) \text{sh} \xi_2 S_4 + Z(\xi_3) \text{ch} \xi_3 S_5 + Z(\xi_3) \text{sh} \xi_3 S_6 + Z(\xi_4) \text{ch} \xi_4 S_7 + Z(\xi_4) \text{sh} \xi_4 S_8 = 0,
\]

\[
\bar{\xi}_1 (\text{ch} \bar{\xi}_1 - \text{sh} \bar{\xi}_1) S_2 + \bar{\xi}_2 \text{sh} \xi_2 S_3 + \bar{\xi}_2 \text{ch} \xi_2 S_4 + \bar{\xi}_3 \text{sh} \xi_3 S_5 + \bar{\xi}_3 \text{ch} \xi_3 S_6 + \bar{\xi}_4 \text{sh} \xi_4 S_7 + \bar{\xi}_4 \text{ch} \xi_4 S_8 = 0,
\]

\[
(\text{ch} \bar{\xi}_1 - \text{sh} \bar{\xi}_1) S_2 - \text{ch} \xi_2 S_3 - \text{sh} \xi_2 S_4 - \text{ch} \xi_3 S_5 - \text{sh} \xi_3 S_6 - \text{ch} \xi_4 S_7 - \text{sh} \xi_4 S_8 = 0.
\]
Journal of Mechanics of Materials and Structures
Volume 7, No. 3 March 2012

Special issue
Trends in Continuum Physics (TRECOP 2010)

Preface  Bogdan T. Maruszewski, Wolfgang Muschik, Joseph N. Grima and Krzysztof W. Wojciechowski 225

The inverse determination of the volume fraction of fibers in a unidirectionally reinforced composite for a given effective thermal conductivity  Magdalena Mierzwiczak and Jan Adam Kołodziej 229

Analytical-numerical solution of the inverse problem for the heat conduction equation  Michał Ciałkowski, Andrzej Maćkiewicz, Jan Adam Kołodziej, Uwe Gampe and Andrzej Frąckowiak 239

Analysis of stress-strain distribution within a spinal segment  Zdenka Sant, Marija Cauchi and Michelle Spiteri 255

A mesh-free numerical method for the estimation of the torsional stiffness of long bones  Anita Usciloska and Agnieszka Fraska 265

Rayleigh-type wave propagation in an auxetic dielectric  Andrzej Drzewiecki 277

Local gradient theory of dielectrics with polarization inertia and irreversibility of local mass displacement  Vasyl Kondrat and Olha Hrytsyna 285

Electromagnetoelastic waves in a vortex layer of a superconductor  Bogdan T. Maruszewski, Andrzej Drzewiecki and Roman Starosta 297