Journal of
Mechanics of
Materials and Structures

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Volume 7, No. 4 April 2012

mathematical sciences publishers

JOURNAL OF MECHANICS OF MATERIALS AND STRUCTURES
Vol. 7, No. 4, 2012

ANALYTICAL STUDY OF PLASTIC DEFORMATION OF CLAMPED CIRCULAR PLATES SUBJECTED TO IMPULSIVE LOADING

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This paper presents an analytical method for determining deflection of fully clamped thin circular plates. The plates are made from a rigid perfectly plastic material and subjected to a transverse localized and uniform blast loading. The essence of the model is to describe the deformation profile with the aid of a zero-order Bessel function and to perform energy analysis. This provides a method for predicting the plastic deformation of circular plates under impulsive loading. It can be also regarded as an attempt to use the energy method for different impulsive loading conditions. Calculations of the cases indicate that the proposed analytical models are based on reasonable assumptions. The solutions obtained are in very good agreement with different sets of experimental results.

A list of symbols can be found on page 320.

1. Introduction

Circular plates are common structural elements, which may, in many applications, be subjected to impact and blast loads. Their large deflection and dynamic response under such conditions has received great attention [Jones 1989]. An understanding of the response of structures when subjected to dynamic loads which produce large plastic deformations and damage is important in solving a variety of engineering problems. Despite significant progresses which have been made in this field during the past decade, complete theoretical analysis of the dynamic structural response is still a formidable task, even for very simple structures such as beams and plates [Chen et al. 2005]. Recently, various simplified models have been developed for predicting the dynamic response of circular plate structures subjected to intense blast loads. Wen et al. [1995a] developed a quasistatic procedure to predict the deformation and failure of a clamped beam struck transversely at any point by a mass traveling at low velocities. A similar procedure has been proposed in [Wen et al. 1995b; 1995c] to construct failure maps for fully clamped metal beams and circular plates under impulsive loadings using a hybrid model (i.e., r.p.p. for the bending-membrane solution and a power law for the effects of local shear) [Wen 1998].

The present paper attempts to explain a theoretical formulation based on energy method. The major objective is to predict high rate plastic deformation of circular plate subjected to transverse impulsive loading and for clamped boundary condition. The shape of deformation profile was described using a Bessel function of zero order. The key assumption employed in the method is that effects of circumferential and radial strains are dominate during deformation process and thickness strain is negligible which in turn simplifies the formulation and reduces the mathematical complexity of the problem.

Keywords: circular plate, blast load, localized load, uniform load, deformation, Bessel function.

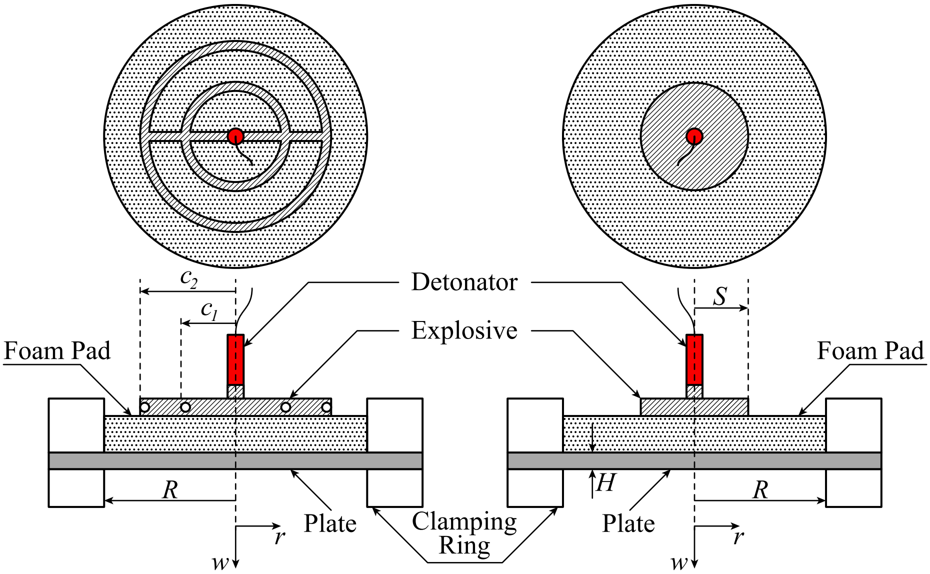

Figure 1. Schematic of two explosive conditions: uniform load (left) and localized load (right).

2. Analytical analysis

Uniform and localized loads on plates are two different explosive conditions which are applied in experimental work. For localized loading as in Figure 1 (left), the disc-shaped explosive is laid on a polystyrene pad. The explosive height and diameter is based on the loading required. In this case, the distribution of blast load is focused (localized).

For uniform loading as in Figure 1 (right), the explosive is laid out on concentric annuli. The explosive rings are connected by a cross leader of explosive. Concentric annuli are used instead of spreading the explosive over the entire area, because the explosive does not detonate if the charge thickness is less than two millimeters. According to the figure, the concentric annuli are set at radii $0.41R$ and $0.82R$. There is a polystyrene pad between shaped explosive and plate. In this case, the distribution of blast load on target plate is uniform.

Localized load. In the theoretical analysis, the large deflection problem of thin circular plate has been regarded as prototype on which various modeling concepts could be conveniently employed. One is wave form solution on the transient responses of circular membrane as proposed in [Symonds and Wierzbicki 1979; Wierzbicki and Nurick 1996]. The result obtained is the zero-order Bessel function of the first kind for the deflected shapes of membrane. Based on this deduction and similarity of zero-order Bessel function curve with deflection profile of circular plate, it is assumed that, a suitable mathematical function to describe the deflection profile of a circular plate is the zero-order Bessel function of the first kind as in [Gharababaei and Darvizeh 2010]

$$w(r) = W_\circ J_0\Big(\frac{a \cdot r}{R}\Big), \tag{1}$$

where $w(r)$ is the transverse displacement of the plate, $W_\circ$ is the transverse displacement at the center, r, R are the radial coordinate and outer radius of the plate, and a is first root of J_0, with approximate value 2.4048.

The strain energy per unit area in a circular plate is

$$dU = \sigma_r \, d\varepsilon_r + \sigma_\theta \, d\varepsilon_\theta. \tag{2}$$

It is assumed that radial and circumferential strains are significant and thickness strain ε_t is negligible.

The total strain energy during the deformation of plate is

$$U_T = \int_V U \cdot dV. \tag{3}$$

The total strain energy of a deformed circular plate consists of a bending strain energy U_b and a membrane strain energy U_m:

$$U_T = U_b + U_m. \tag{4}$$

For U_b, according to (1), the bending strains are

$$\varepsilon_{rb} = -z\frac{\partial^2 w}{\partial r^2} = W_\circ z\frac{a^2}{R^2}\left[J_0\left(\frac{ar}{R}\right) - \frac{R}{ra}J_1\left(\frac{ar}{R}\right)\right], \tag{5}$$

$$\varepsilon_{\theta b} = -z\frac{1}{r}\frac{\partial w}{\partial r} = W_\circ z\frac{a}{rR}J_1\left(\frac{ar}{R}\right), \tag{6}$$

where ε_{rb} and $\varepsilon_{\theta b}$ are the radial bending and circumferential bending strains, J_1 is first-order Bessel function of the first kind and z is the transverse coordinate.

$$U_b = \int_V (\sigma_r \varepsilon_{rb} + \sigma_\theta \varepsilon_{\theta b}) \, dV. \tag{7}$$

For materials insensitive to hydrostatic pressure, such as metals, the hydrostatic component of the stress tensor does not have a significant influence on yielding and plastic flow. Therefore these components can be subtracted from the stress tensor and it is common practice to employ the Tresca or von Mises yield criteria and the von Mises flow rules. Introducing the Tresca yield criterion and the von Mises flow relation for a rigid plastic material, one obtains for the circular plate the relation

$$\sigma_r = \sigma_\theta = \sigma_d, \tag{8}$$

where σ_d is the mean dynamic flow stress. Therefore, (7) can be converted to

$$U_b = \int_0^R \int_{-H/2}^{H/2} \sigma_d(\varepsilon_{rb} + \varepsilon_{\theta b}) 2\pi r \, dz \, dr = \tfrac{1}{2}\pi\sigma_d H^2 a J_1(a) W_\circ. \tag{9}$$

For U_m, the membrane strain is

$$\varepsilon_{rm} = \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 = 2\pi\left(\frac{a}{R}\right)^2 \frac{W_\circ^2}{2} J_1^2\left(\frac{ar}{R}\right). \tag{10}$$

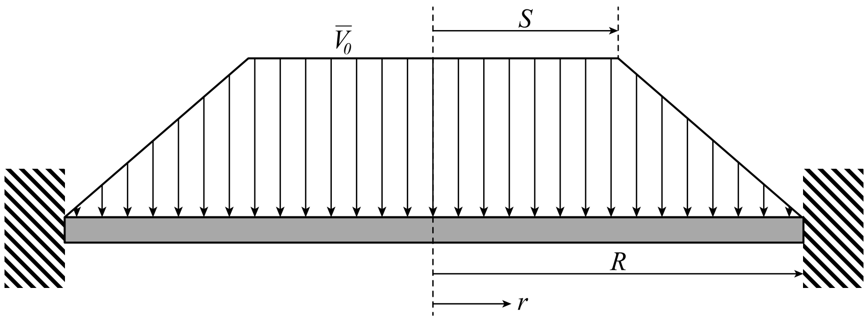

Figure 2. Initial velocity profile of circular plate subjected to localized load.

Then, the membrane energy can be obtained as

$$U_m = \int_V \sigma_r \varepsilon_{rm} dV = 2\pi\sigma_d \left(\frac{a}{R}\right)^2 \left[\int_0^R \int_{-H/2}^{H/2} \frac{W_\circ^2}{2} J_1^2\left(\frac{ar}{R}\right)\right] r\,dz\,dr = \tfrac{1}{2}\pi\sigma_d H (aJ_1(a)W_\circ)^2. \tag{11}$$

When the plate is under a large deformation, the bending strain energy is much lower than the membrane strain energy, because the ratio between the two is

$$\frac{U_m}{U_b} = \frac{aJ_1(a)W_\circ}{H}. \tag{12}$$

In large deformation, the mid point deflection of plate $W_\circ$ is approximately more than $5H$. Therefore,

$$U_m \approx 6U_b. \tag{13}$$

So, in the analysis of large deformation of circular plate, the bending strain energy can be neglected completely. Thus

$$U_T = U_m. \tag{14}$$

The total strain energy is equal to the initial kinetic energy of the plate. Initial velocity profile of circular plate subjected to localized load is not uniformly and it can be approximated as trapezoid distribution; see Figure 2. By conservation of momentum, the input impulse I to the plate can be calculated as

$$I = m_1 V_1(r) + m_2 V_2(r), \tag{15}$$

where m_1 and m_2 are the plate mass in the region under the explosive and in free region, respectively, and

$$V_1(r) = V_\circ \qquad \text{for } 0 \le r \le S, \tag{16}$$

$$V_2(r) = V_\circ \frac{R-r}{R-S} \qquad \text{for } S \le r \le R, \tag{17}$$

S being the radius of the explosive disc and $V_\circ$ the initial impulsive velocity in the central region of plate.

Substituting (16) and (17) into (15) yields

$$I = \rho H V_\circ \left[\int_0^S 2\pi r\,dr + \int_S^R 2\pi r\left(\frac{R-r}{R-S}\right) dr\right]. \tag{18}$$

Integrating (18) with respect to r gives

$$I = mV_\circ \psi, \tag{19}$$

where

$$m = \rho \pi R^2 H, \tag{20}$$

$$\psi = \left(\frac{S}{R}\right)^2 + \frac{1}{R-S}\left(\frac{2S^3}{3R^2} + \frac{R}{3} - \frac{S^2}{R}\right), \tag{21}$$

with ρ the density and m is the mass of the plate.

The kinetic energy imparted by an input impulse I to the plate can be calculated as

$$E_K = \frac{1}{2} m V_\circ^2 = \frac{I^2}{2m\psi^2}. \tag{22}$$

Equating (14) and (22) gives

$$[aJ_1(a)]^2 W_\circ^2 = \frac{I^2}{\rho \sigma_d (\pi R H)^2 \psi^2}. \tag{23}$$

The mean dynamic flow (yield) stress σ_d can be determined by the well known Cowper–Symonds empirical equation

$$\sigma_d = \sigma_y \left[1 + \left(\frac{\dot{\varepsilon}_m}{D}\right)^{1/q}\right], \tag{24}$$

where $\dot{\varepsilon}_m$ is the mean strain rate, D and q are material constants with typical values for mild steel of $D = 40.4\,\mathrm{s}^{-1}$ and $q = 5$[1], and σ_y is the quasistatic yield stress.

Equation (24) can be estimated by (see [Symonds 1973; Wojno and Wierzbicki 1979])

$$\sigma_d = \beta \sigma_y \left(\frac{\dot{\varepsilon}_m}{D}\right)^{1/\beta q}, \tag{25}$$

where β is a constant with value 2.5.

According to (10), the strain rate $\dot{\varepsilon}$ is obtained

$$\dot{\varepsilon} = \frac{d\varepsilon_{rm}}{dt} = W_\circ \dot{W}_\circ \left(\frac{a}{R} \cdot J_1\left(\frac{ar}{R}\right)\right)^2. \tag{26}$$

The mean strain rate at $r = R$ may be expressed as

$$\dot{\varepsilon}_m = W_\circ \dot{W}_\circ \left(\frac{a \cdot J_1(a)}{R}\right)^2, \tag{27}$$

where $\dot{W}_\circ$ is the mean velocity that is estimated as

$$\dot{W}_\circ = \frac{V_\circ}{2}. \tag{28}$$

Finally, (25) can be rewritten as

$$\sigma_d = \beta \sigma_y \zeta (W_\circ)^{1/\beta q}, \tag{29}$$

where

$$\zeta = \left(\frac{I\,(a \cdot J_1(a))^2}{2\pi \rho R^4 \psi D} \right)^{1/\beta q}. \tag{30}$$

Introducing ϕ as a dimensionless impulsive parameter

$$\phi = \frac{I}{\pi R H^2 \psi \sqrt{\beta \rho \sigma_y}}, \tag{31}$$

one can rewrite (30) as

$$\zeta = \left(\sqrt{\frac{\beta \sigma_y}{\rho}} \cdot \frac{(a \cdot J(a))^2}{2D} \cdot \frac{H^2 \phi}{R^3} \right)^{1/(\beta q)}. \tag{32}$$

By substituting (29) into (23), one reaches the expression for the relationship between the dimensionless displacement (ratio between midpoint deflection and thickness) and the dimensionless impulse parameter.

$$\frac{W_\circ}{H} = \left[\frac{1}{\zeta} \left(\frac{\phi}{a J_1(a)} \right)^2 \right]^{\beta q/(1+2\beta q)}. \tag{33}$$

Uniform load. In the case of a uniform load, the deflection profile is also described using the zero-order Bessel function of the first kind, but the initial velocity profile of circular plate is uniform; see Figure 3. So, by conservation of momentum, the initial velocity induced by the impulsive load is

$$\bar{V}_\circ = \frac{I}{m}, \tag{34}$$

$$\bar{E}_K = \frac{1}{2} m \bar{V}_\circ^2 = \frac{I^2}{2m}, \tag{35}$$

where $\bar{V}_\circ$ is the initial impulsive velocity and $\bar{E}_K$ is kinetic energy imparted by an input impulse I to the plate. Equating (35) and (14) gives

$$[a J_1(a)]^2 W_\circ^2 = \frac{I^2}{\rho \sigma_d (\pi R H)^2}. \tag{36}$$

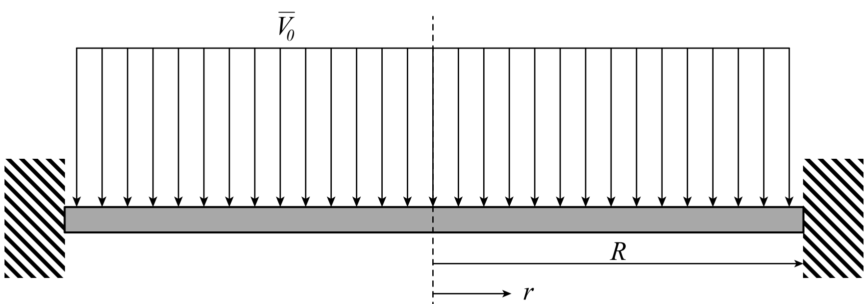

Figure 3. Initial velocity profile of circular plate subjected to uniform load.

Similarly to the preceding section, the mean dynamic flow (yield) stress σ_d can be determined as follows:

$$\sigma_d = \beta\sigma_y\bar{\xi}\left(\frac{W_\circ}{H}\right)^{1/\beta q}, \tag{37}$$

where

$$\bar{\xi} = \left(\frac{I(a \cdot J_1(a))^2}{2\pi\rho R^4 D}\right)^{1/\beta q}. \tag{38}$$

Introducing $\bar{\phi}$ as a dimensionless impulsive parameter

$$\bar{\phi} = \frac{I}{\pi R H^2\sqrt{\beta\rho\sigma_y}}. \tag{39}$$

Equation (38) also can be rewritten as

$$\bar{\xi} = \left(\sqrt{\frac{\beta\sigma_y}{\rho}} \cdot \frac{(a \cdot J(a))^2}{2D} \cdot \frac{H^2\bar{\phi}}{R^3}\right)^{1/\beta q}. \tag{40}$$

Similar to the procedure employed in previous section, by substituting (37) into (36), it leads to expression for the relationship between the midpoint-deflection-to-thickness ratio (dimensionless displacement) and dimensionless impulse parameter.

$$\frac{W_\circ}{H} = \left[\frac{1}{\bar{\xi}}\left(\frac{\bar{\phi}}{aJ_1(a)}\right)^2\right]^{\beta q/(1+2\beta q)}. \tag{41}$$

3. Results of experimental tests

The different experimental results used in this paper have been obtained through similar procedures. The experimental method consisted of creating an impulsive load with the aid of plastic explosive and measuring the impulse using a blast pendulum. The test plates are made from mild steel material with various thicknesses and radii. The test plates were sandwiched securely by support plates as shown in Figure 4 which were in turn fixed to ballistic pendulum [Jacob et al. 2007].

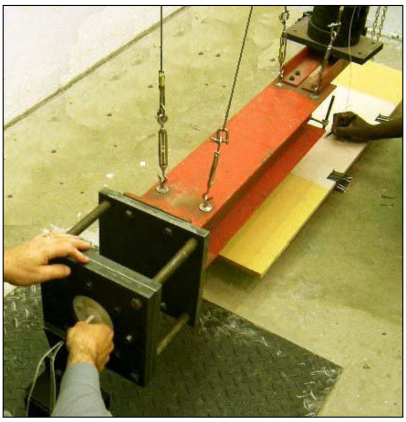

Figure 4. Experimental set-up [Jacob et al. 2007].

Experiment number and source	Data points	$2R$ (mm)	H (mm)	Load type	Load ratio S/R	σ_y (MPa)	$W_{\circ,\min}$ (mm)	$W_{\circ,\max}$ (mm)	$I_{\min}$ (N s)	$I_{\max}$ (N s)
1. [Chung Kim Yuen and Nurick 2000]	20	100	1.6, 3.6	loc.	0.25, 0.33 0.4	252	16	31.3	6.6	31.3
2. [Bodner and Symonds 1979]	21	64	1.9	loc.	0.33, 0.5	223	2.5	11.4	0.85	4.
3. [Nurick and Radford 1997]	42	100	1.6	loc.	0.18, 0.25 0.33, 0.4	194	5.4	29.9	2.7	12.4
4. [Nurick 1989]	21	100	1.6	unif.	—	282	6.14	19.8	5.6	15.6
5. [Nurick and Teeling-Smith 1994]	143	100	1.6	unif.	—	270	4.62	27.9	4.55	22.04
6. [Nurick and Lumpp 1996]	7	100	1.6	unif.	—	255	7.26	20.43	5.21	16.47
7. [Nurick et al. 1996]	113	60, 80 100, 120	1.6	unif.	—	290	2.7	34.3	1.4	30.9
8. [Bodner and Symonds 1979]	8	64	1.9	unif.	—	223	0.86	12.34	0.91	7.15
9. [Thomas and Nurick 1995]	20	100	3–3.9	unif.	—	262	5.6	20.9	4.6	16.3

Table 1. Summary of circular plate experimental conditions. Density $\rho = 7860\,\text{kg/m}^3$ in all cases. For the meaning of the variables see the text or the list on page 320.

The impulsive load is provided by a shaped plastic explosive charge placed over the test plate. The shape of charge adjusts localized or uniform according to Figure 1. The explosive charge is placed on a 12–16 mm thick polystyrene pad mounted directly onto the plate. The reason for using a polystyrene pad is to prevent spalling. A detonator taped with 1 gram of explosive was attached to the center of the explosive disk. Table 1 summarizes different load conditions and plate geometries for various experimental reports for clamped circular mild steel plates subjected to localized and uniform blast loads [Chung Kim Yuen and Nurick 2000; Bodner and Symonds 1979; Nurick and Radford 1997; Nurick 1989; Nurick and Teeling-Smith 1994; Nurick and Lumpp 1996; Nurick et al. 1996; Thomas and Nurick 1995].

4. Comparison of the experimental and analytical results

Comparison with experimental results. The results obtained from analytical models in the preceding section — (33) and (41) — are now compared with various sets of experimental results reported in the literature (Table 1).

Figure 5 compares the theoretical values of the midpoint-deflection-to-thickness ratio obtained using (33) with the experimental values for different series of experiments from the literature, performed using a localized load. The ratio is shown as a function of the dimensionless impulse (ϕ). The results of the present model are generally in good agreement with the experimental ones. In three cases — parts (a2), (b1), (c1) of the figure — the agreement is not so good, but even then the experimental values still fall within a range of ± 1 from the confidence line of (33) with confidence level of 83%, 100% and 52%, respectively.

Similarly, Figure 6 compares the theoretical values of the midpoint-deflection-to-thickness ratio obtained using (41) with the experimental values from the literature, in the case of a uniform load. Again we see generally good agreement, suggesting that the model can successfully predict central deflection.

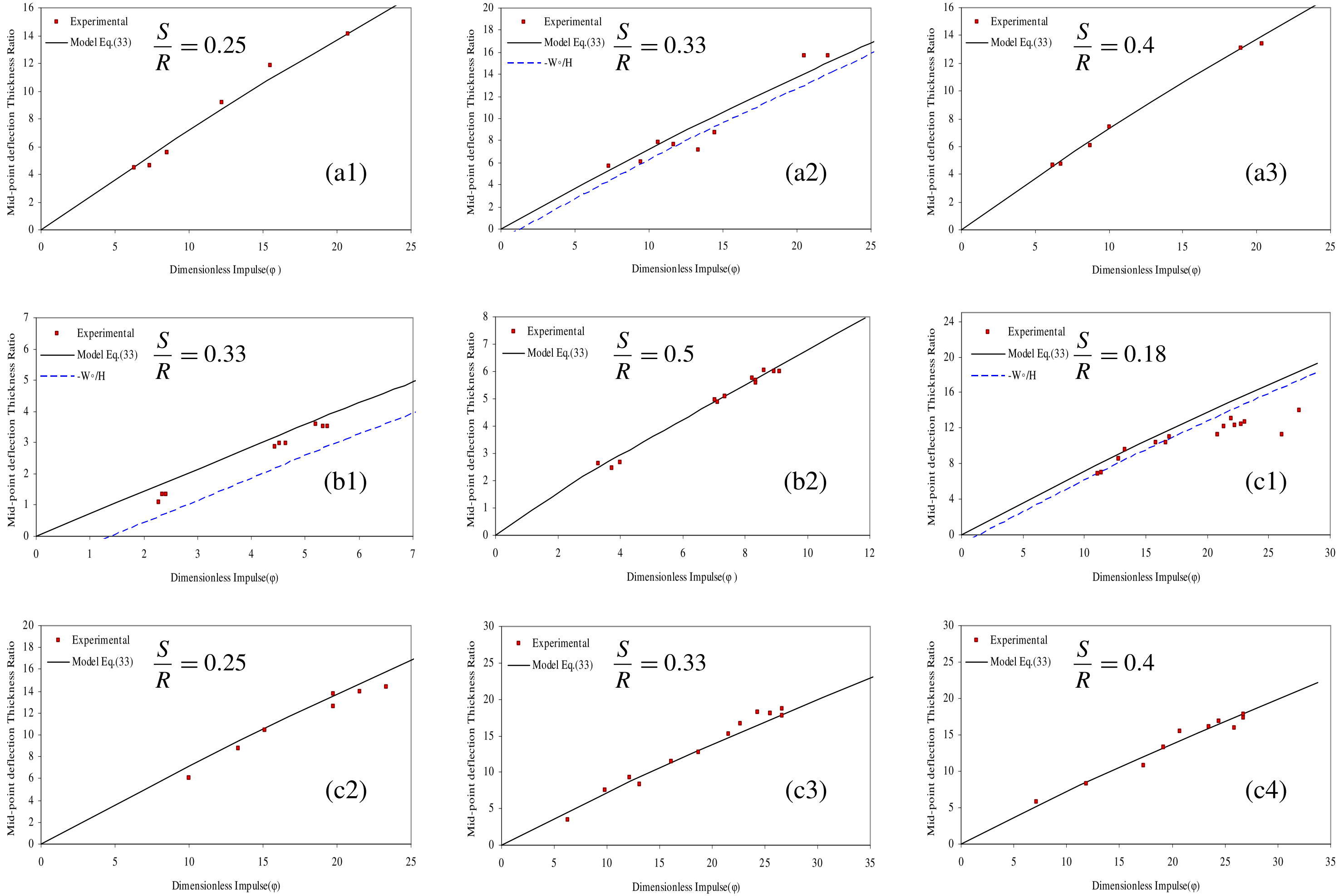

Figure 5. Comparison between experimental and model data: midpoint-deflection-to-thickness ratio versus dimensionless number (ϕ), under localized load. Comparison data for (a1)–(a3) from [Chung Kim Yuen and Nurick 2000]; for (b1)–(b2) from [Bodner and Symonds 1979]; for (c1)–(c4) from [Nurick and Radford 1997].

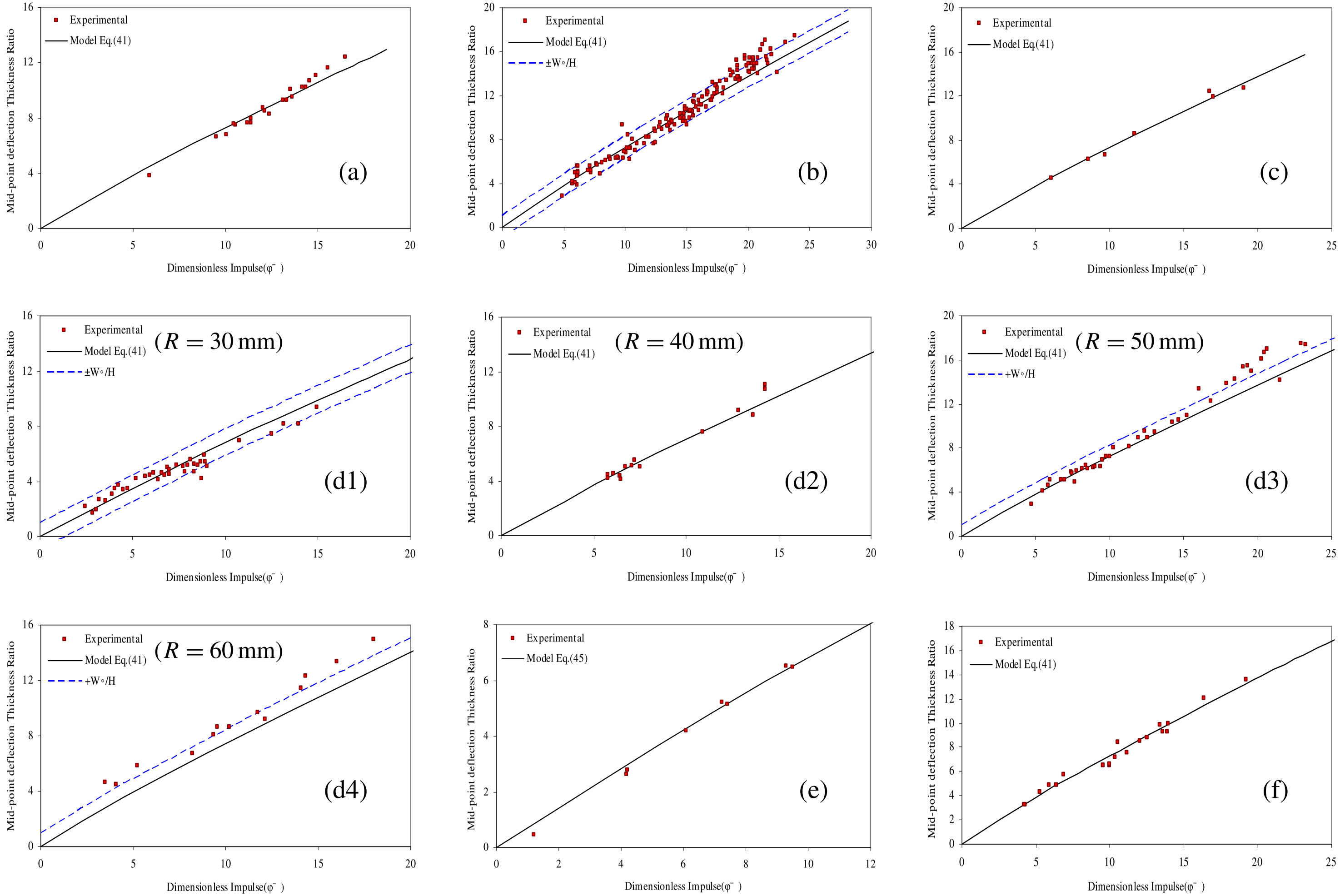

Figure 6. Comparison between experimental and model data: midpoint-deflection-to-thickness ratio versus dimensionless number (ϕ), under uniform load. Comparison data for (a) from [Nurick 1989]; for (b) from [Nurick and Teeling-Smith 1994]; for (c) from [Nurick and Lumpp 1996]; for (d1)–(d4) from [Nurick et al. 1996]; for (e) from [Bodner and Symonds 1979]; for (f) from [Thomas and Nurick 1995].

In parts (b), (d1), (d3), and (d4) some experimental points do not fit with values of model; these points fall within the ± 1 confidence line of (41) with confidence level of 94%, 99%, 78% and 42%, respectively.

Thus the proposed model can be used in most cases to predict midpoint deflection thickness ratio for different conditions of uniform and localized loads. Some experimental results obtained from literature as shown in Figures 5(c1) and 6(d4) do show a big difference with the results obtained from the present model. This could be due to incorrect impulse measurement or anisotropy in material properties.

Comparison with other formulas. There have been many research efforts for theoretical modeling the dynamic response and deformation of thin plates to predict the relationship of deflection-thickness ratio as a function of impulse, plate geometry and material properties. These models predict maximum midpoint deflection of circular plates for localized and uniform loads: we mention in particular [Nurick and Martin 1989], which supplies the equation

$$\frac{W_\circ}{H} = \frac{0.318 I}{H^2 R \sqrt{\rho \sigma_y}} \left(1 + \ln \frac{R}{S}\right) \tag{42}$$

for the ratio between midpoint deflection and thickness, and [Gharababaei et al. 2010], which supplies

$$\frac{W_\circ}{H} = \frac{0.12 I}{H^2 R \sqrt{\rho \sigma_y}} \frac{R}{S}. \tag{43}$$

Both of these apply to the case of localized loads. For case of uniform loads we have from [Nurick and Martin 1989]

$$\frac{W_\circ}{H} = \frac{f\, I}{H^2 R \sqrt{\rho \sigma_y}} \tag{44}$$

where the factor f is given by those authors as 0.135; values from other authors (cited in [Nurick and Martin 1989]) include 0.318 (Hudson), 0.212 (Symonds and Wierzbicki), 0.132 (Lipman), 0.260 (Jones), and 0.382 (Batra and Dubey).

The table below compares the RMSE obtained with the literature equations (42)–(44) and with the equations from our model, (33) and (41)). It can be observed that the models introduced in this paper have much less RMSE compared with those reported in (42)–(44).

model	RMSE	model	RMSE
(42) [Nurick and Martin 1989]	0.042	(44) [Nurick and Martin 1989]	1.055
(43) [Gharababaei et al. 2010]	0.045	(44) (Hudson)	12.914
(33) (present)	0.037	(44) (Symonds and Wierzbicki)	2.318
		(44) (Lipman)	1.136
		(44) (Jones)	8.439
		(44) (Batra and Dubey)	28.064
		(41) (present)	0.052

5. Conclusion

The models developed in the present work account for energy dissipation through plastic work. Two different analytical models were presented, for localized and uniform blast loads. The solution is determined completely through material and geometrical parameters. A zero Bessel function was used to determine the deflection profile for different types of impulsive loading (localized and uniform). Subsequent calculation indicated that the accuracy of quantities such as central deflection is sensitive to the function describing the exact shape of deflection profile. Analytical predictions of present models for central deflection of fully clamped impulsively loaded circular plates are in good agreement with numerous experimental data. This is despite of fact that calculation based on the analytical solution of present model involved only values of radial and circumferential strains, the values of thickness strain considered being relatively small and negligible.

Although the models in (33) and (41) indicate a nonlinear relationship between deflection and impulsive load, Figures 2 and 3 show an almost linear relationship between these parameters. In fact, the linearity or nonlinearity of the models depends on material properties (q) of the plates.

In summary, the models presented in this paper can predict central deflection of the plate subjected to impulsive loading accurately. The values obtained for central deflection from analytical models developed here show less error, in comparison with experimental results, than those obtained from other models already reported in literature. It is intended that this analytical approach will be complementary to the existing ones. However, the current establishment of equations in the present models is more straightforward.

Acknowledgements

The authors express their thanks to Prof. G. N. Nurick, Dr. G. S. Langdon and Mr. T. J. Cloete for valuable discussions. The authors also thank all the members of the BISRU group at the University of Cape Town.

List of symbols

w	Transverse displacement of plate
$W_\circ$	Transverse displacement of plate at center
r	Radial coordinate
z	Transverse coordinate
R	Radius of plate
H	Thickness of plate
J_0	Zero-order Bessel function
a	First root of J_0
J_1	First-order Bessel function
ε_r	Radial strain
ε_θ	Circumferential strain
ε_t	Thickness strain
σ_r	Radial stress
σ_θ	Circumferential stress
σ_d	Mean dynamic stress
σ_y	Quasistatic yield stress
$V_\circ, \bar{V}_\circ$	Initial impulsive velocity
m	Mass of plate
ρ	Density of plate
E_k	Kinetic energy for localized load
$\bar{E}_k$	Kinetic energy for uniform load
I	Input impulse
ψ	Coefficient of load (geometric)
ϕ	Dimensionless impulsive for localized load
$\bar{\phi}$	Dimensionless impulsive for uniform load
$\dot{\varepsilon}_m$	Mean strain rate
D	Material constant, defined in (24)
q	Material constant, defined in (24)
V	volume of plate
ζ	Dimensionless parameter for localized load
$\bar{\zeta}$	Dimensionless parameter for uniform load
U_T	Total strain energy
U_m	Membrane strain energy
U_b	Bending strain energy

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Received 1 Aug 2011. Revised 25 Mar 2012. Accepted 8 Apr 2012.

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JOURNAL OF MECHANICS OF MATERIALS AND STRUCTURES

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Cover design: Alex Scorpan Cover photo: Mando Gomez, www.mandolux.com

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JoMMS (ISSN 1559-3959) is published in 10 issues a year. The subscription price for 2012 is US $555/year for the electronic version, and $735/year (+$60 shipping outside the US) for print and electronic. Subscriptions, requests for back issues, and changes of address should be sent to Mathematical Sciences Publishers, Department of Mathematics, University of California, Berkeley, CA 94720–3840.

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Typeset in LaTeX

Journal of Mechanics of Materials and Structures

Volume 7, No. 4 April 2012