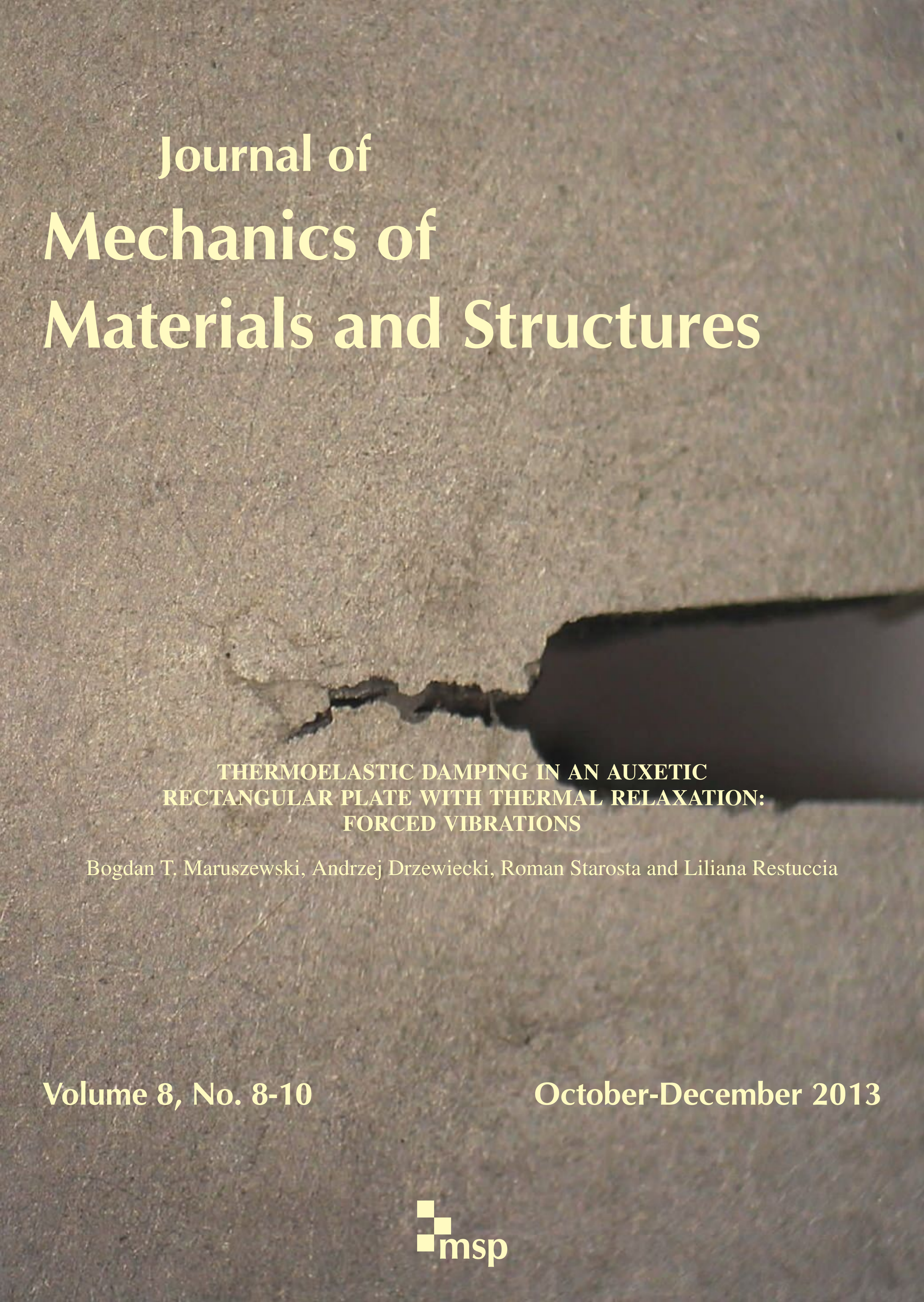

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THERMOELASTIC DAMPING IN AN AUXETIC RECTANGULAR PLATE WITH THERMAL RELAXATION: FORCED VIBRATIONS

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ROMAN STAROSTA AND LILIANA RESTUCCIA

We analyze the forced vibrations of an auxetic rectangular thermoelastic plate. In contrast with the existing classical studies, two important phenomena have been considered: thermoelastic damping and second sound. In this way the presented model much better describes thermomechanical processes running in "negative" materials of finite extent.

1. Introduction

Thermoelastic interactions in classical continuous media of finite extent have been investigated by many authors [Boley and Weiner 1960; Nowacki 1962; 1975; Noda et al. 2003]. However, the definite geometry of a body can also be the origin of certain unusual phenomena that do not occur in infinite media. One of these, among others, is so-called thermoelastic damping, which does not arise from eventual viscous features of the body. The extra energy dissipation resulting from that phenomenon, not observed in a pure elastic infinite medium, comes from an additional heat flux occurring in bodies of finite extent (in the case of a plate the flux is normal to its limiting surfaces). The origin of that extra heat flux is a specific deformation of the plate: if we consider a vibration process, for instance, the upper and lower fibers are alternatively extended and compressed. Thus, any thermoelastic problem in a plate in bending is 2D–3D (the latter because of the extra heat flux normal to the middle surface). Here nD stands for "n-dimensional". The first idea which pointed out one of the mechanisms of thermoelastic damping was based on the stress heterogeneities which give rise to fluctuations of temperature [Zener 1937]. Zener focused his attention on 1D bodies. While his theory has been successful in explaining the measurements of internal friction in reeds and wires, it is incomplete in two respects:

(a) It is not consistent with the modern theory of thermoelasticity.

(b) It does not describe the thermoelastic behavior of bodies of arbitrary form, especially if coupling occurs between different vibration modes.

The complete thermodynamical model of that phenomenon has been presented in [Alblas 1961; 1981]. There you can find the general theory of thermoelastic internal friction in 3D bodies of finite extent and its application to thermoelastic damping during vibrations of beams of various cross-sections (1D–3D problems) which proves its consistency. Following that model, in [Maruszewski 1992] damping during vibrations of circular plates has been considered. Research on thermoelastic damping has continued for

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many years as the phenomenon plays an important role in many applications. However, all the mentioned considerations were based on the classical irreversible thermodynamics. Within that model, the temperature distribution is described by the parabolic heat conduction equation [Bishop and Kinra 1993; 1994; Milligan and Kinra 1993; Kinra and Milligan 1994]. Since any physical signals propagate with finite velocity it is much better to base the above considerations on extended irreversible thermodynamics [Cattaneo 1958; Vernotte 1958; Chester 1963; Lebon 1982; Jou et al. 1988; Maruszewski 1988]. That idea has been, for the first time, applied to describe thermoelastic damping with the effect of second sound [Khisaeva and Ostoja-Starzewski 2006; Ignaczak and Ostoja-Starzewski 2010]. Such an approach is crucial in researching micro, nano, and macroengineering problems [Lifshitz and Roukes 2000; Nayfeh and Younis 2004; Vengallatore 2005; Prabhakar and Vengallatore 2008].

Fast technological development requires the use of unconventional materials with peculiar and unusual properties. Special interest has been recently focused on "negative" materials, that is, materials with negative Poisson's ratio, negative compressibility, negative stiffness, negative heat expansion coefficient, and the like [Almgren 1985; Lakes 1987; Wojciechowski 1989; Novikov and Wojciechowski 1999; Poźniak et al. 2010; Kołat et al. 2010; 2011]. Those unconventional features strongly influence the behavior of many mechanical structures, like bodies of finite extent, laminates, composite structures, weaved structures, and other multiphase structures. In contrast to [Khisaeva and Ostoja-Starzewski 2006], where the thermoelastic damping in a vibrating beam has been investigated with one relaxation time based only on the hyperbolic heterogeneous heat equations, we have focused in this paper on an analysis of the forced bending vibrations of an auxetic thermoelastic rectangular plate (a plate with negative Poisson's ratio) within coupled thermoelasticity. This research takes also into account thermoelastic damping and the relaxation features of the thermal field (second sound). The obtained results have been compared to those of the normal material of the plate and those without thermal relaxation.

2. Basic equations

Let us consider forced vibrations of a thermoelastic rectangular plate with $0 \le x_1 \le a$, $0 \le x_2 \le b$, $-h/2 \le x_3 \le h/2$, where h denotes the thickness, and $a, b \gg h$ (see Section 4). Following [Nowacki 1975; Alblas 1981; Ignaczak and Ostoja-Starzewski 2010] the basic equations for such a plate consisting of one relaxation time of the thermal field read

$$D_0 w_{,\alpha\alpha\beta\beta} + \rho h \ddot{w} + \frac{1}{1-\nu_T} M_{T,\alpha\alpha} = p, \tag{2-1}$$

$$\theta_{,ii} - \left(\tau \frac{\partial}{\partial t} + 1\right)\left(\frac{\rho c_v}{k}\dot{\theta} + \frac{m}{k} T_0 \dot{e}\right) = 0. \tag{2-2}$$

Here $w = w(x_1, x_2, t)$ and $\theta = \theta(x_1, x_2, x_3, t)$ with ranges of variables $0 \le x_1 \le a$, $0 \le x_2 \le b$, $-h/2 \le x_3 \le h/2$, and $0 \le t < \infty$, where α, $\beta = 1, 2$, $i = 1, 2, 3$, τ is the thermal relaxation time, ρ is the constant plate density, and

$$D_0 = \frac{E_T h^3}{12(1-\nu_T^2)}, \qquad m = \frac{E_T \alpha_T}{1-2\nu_T}.$$

Taking into account (2-1) on the one hand and (2-2) and (2-5) below on the other, we see that the problem is 2D–3D. The heat expansion coefficient is α_T, k is the heat conductivity, and the moment M_T due to the temperature distribution is given by

$$M_T = \alpha_T E_T \int_{-h/2}^{h/2} \theta(x_1, x_2, x_3, t) x_3 \, dx_3. \tag{2-3}$$

The general form of the dilatation e for the thermoelastic plate is as follows:

$$e = \frac{1-2\nu}{1-\nu}(u_{,1} + v_{,2} - x_3 w_{,\alpha\alpha}) + \alpha_T \frac{1+\nu}{1-\nu}\theta. \tag{2-4}$$

T denotes the absolute temperature of the plate and θ is a small temperature variation coming from reciprocal thermoelastic interactions (see (2-1) and (2-2)), so we assume that $\theta = T - T_0$, $|\theta/T_0| \ll 1$. T_0 is the constant reference temperature (see (2-9) and (2-10)) and c_v is the specific heat, while u and v denote displacements corresponding to the elongation of the middle surface and w is the deflection of the plate. However, we confine ourselves to a simplified form of e in the sequel, i.e.,

$$e = -\frac{1-2\nu}{1-\nu} x_3 w_{,\alpha\alpha}, \tag{2-5}$$

assuming that the contribution of the remaining terms in (2-4) can be neglected in the case considered, of pure small bending (see [Khisaeva and Ostoja-Starzewski 2006]). Note that the coefficients E_T (the Young's modulus) and ν_T (the Poisson's ratio) are isothermal. The Poisson's ratio ν in (2-4) and (2-5) has an effective value dependent on the vibrational mode but does not much differ from ν_T [Alblas 1981]. So, we assume that $\nu = \nu_T$ in the sequel. For the model of interactions taken in this paper we also assume that changes in temperature come only from mechanical vibrations of the plate. The mass forces and heat sources have been neglected.

To obtain an exact solution of the problem we have to pose proper boundary conditions for the set (2-1) and (2-2), assuming that

- the plate is simply supported at all edges,
- the temperature at lateral surfaces $T = T_0$, and
- at the upper and lower surfaces temperature changes result from alternate compression and extension of the plate fibers; free heat exchange across those surfaces has been assumed.

Hence we have [Boley and Weiner 1960]

$$w(0, x_2, t) = w(a, x_2, t) = w(x_1, 0, t) = w(x_1, b, t) = 0, \tag{2-6}$$

$$w_{,11} + \frac{1}{D_0(1-\nu_T)} M_T = 0, \quad \text{at } x_1 = 0, a, \tag{2-7}$$

$$w_{,22} + \frac{1}{D_0(1-\nu_T)} M_T = 0, \quad \text{at } x_2 = 0, b, \tag{2-8}$$

$$\theta = 0, \quad \text{at } x_1 = 0, a, \tag{2-9}$$

$$\theta = 0, \quad \text{at } x_2 = 0, b, \tag{2-10}$$

$$\frac{\partial\theta}{\partial x_3} \pm \eta\Big(\tau\frac{\partial}{\partial t} + 1\Big)\theta = 0, \quad \text{at } x_3 = \pm\frac{h}{2}, \tag{2-11}$$

where η denotes the surface heat exchange coefficient.

From (2-3), (2-9), and (2-10) follows that $M_T = 0$ at $x_1 = 0, a$ and $x_2 = 0, b$.

3. Forced vibrations

Since we are interested in the description of the forced vibrations of a rectangular plate accompanied by thermoelastic damping, the general solutions of (2-1) and (2-2) with the boundary conditions (2-6)–(2-11) are looked for in the forms

$$w(x_1,x_2,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{00mn}\sin\Bigl(\frac{m\pi}{a}x_1\Bigr)\sin\Bigl(\frac{n\pi}{b}x_2\Bigr)e^{i\omega t}, \tag{3-1}$$

$$\theta(x_1,x_2,x_3,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \theta_{00mn}(x_3)\sin\Bigl(\frac{m\pi}{a}x_1\Bigr)\sin\Bigl(\frac{n\pi}{b}x_2\Bigr)e^{i\omega t}. \tag{3-2}$$

These solutions concern the situation that our problem is 2D–3D, as was mentioned before.

For the plate vibrations having a forced character we assume that the upper surface $x_3=h/2$ is loaded by (see (2-1))

$$p(x_1,x_2,t)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} p_{00mn}\sin\Bigl(\frac{m\pi}{a}x_1\Bigr)\sin\Bigl(\frac{n\pi}{b}x_2\Bigr)e^{i\omega t}. \tag{3-3}$$

For the sake of simplicity we take into consideration only the first terms of expansions (3-1)–(3-3), meaning in the sequel that $w_{0011}=w_{00}$, $\theta_{0011}=\theta_{00}$, and $p_{0011}=p_{00}$, confining us to only one solution for $\theta_{00}(x_3)$.

Making use now of (3-1)–(3-3) with (2-5) in (2-1) and (2-2), we arrive at the following result for displacement w_{00}:

$$w_{00}=\frac{\beta B+\bar{p}_{00}}{\omega_0^2-\omega^2}, \tag{3-4}$$

where

$$\bar{p}_{00}=\frac{p_{00}}{\rho h},\quad \beta=\frac{\alpha_T E_T H}{(1-\nu_T)\rho h},\quad \omega_0^2=\frac{D_0 H^2}{\rho h},\quad H=\frac{\pi^2}{a^2}+\frac{\pi^2}{b^2}, \tag{3-5}$$

$$B=\int_{-h/2}^{h/2} x_3\theta_{00}\,dx_3=\bar{C}\int_{-h/2}^{h/2} x_3\tilde{\theta}_{00}\,dx_3=\bar{C}\bar{D}.$$

On using (3-4) in (2-2) the equation for $\theta_{00}(x_3)$ (which gives the solution for temperature distribution along the plate thickness within the third dimension) reads

$$\frac{\partial^2\theta_{00}}{\partial x_3^2}-\epsilon^2\theta_{00}=\bar{C}x_3. \tag{3-6}$$

Now, setting $\theta_{00}=\bar{C}\tilde{\theta}_{00}$, the solution of (3-6) is possible (see [Alblas 1981]) with the help of the modified condition (2-11) by the assumption that $\eta=+\infty$, that is, $(\tau\partial/\partial t+1)\theta=0$ for $0\le x_1\le a$, $0\le x_2\le b$, and $0\le t<\infty$ at $x_3=\pm h/2$, and postulating that $\theta_{00}(h/2)=\theta_{00}(-h/2)=0$:

$$\tilde{\theta}_{00}=\frac{-h\exp\bigl[\frac{1}{2}(h-2x_3)\epsilon\bigr]+h\exp\bigl[\frac{1}{2}(h+2x_3)\epsilon\bigr]+2x_3-2x_3\exp(h\epsilon)}{2\epsilon^2(-1+\exp(h\epsilon))}, \tag{3-7}$$

where $\epsilon=\sqrt{H+(i\omega-\tau\omega^2)\gamma}$ and $\gamma=\rho c_v/k$.

Thus way we have, from (3-5) and (3-7),

$$\bar{D} = -h\frac{12+h^2\epsilon^2-6h\epsilon\coth(h\epsilon/2)}{12\epsilon^4} \quad \text{and} \quad \bar{C} = \frac{\bar{p}_{00}(i\omega-\tau\omega^2)\delta}{\omega_0^2-\omega^2-\beta\delta(i\omega-\tau\omega^2)\bar{D}}, \tag{3-8}$$

where

$$\delta = \frac{E_T\alpha_T T_0 H}{k}\,\frac{1-2\nu}{1-2\nu_T}. \tag{3-9}$$

Therefore, the final solutions for the displacement amplitude w_{00} and temperature θ_{00} are

$$w_{00} = \frac{p_{00}}{\omega_0^2-\omega^2-\beta\bar{D}(i\omega-\tau\omega^2)}, \qquad \theta_{00} = \bar{C}\tilde{\theta}_{00}. \tag{3-10}$$

4. Numerical results

Let us analyze the results obtained for a plate with the following thermomechanical properties:

$$E_T = 10^{11}\,\frac{\mathrm{N}}{\mathrm{m}^2}, \quad \alpha_T = 3\times10^{-6}\,\mathrm{K}^{-1}, \quad \rho = 7860\,\frac{\mathrm{kg}}{\mathrm{m}^3}, \quad k = 58\,\frac{\mathrm{J}}{\mathrm{s\,m\,K}}, \quad c_v = 460\,\frac{\mathrm{J}}{\mathrm{kg\,K}}, \tag{4-1}$$

$$h = 0.005\,\mathrm{m}, \quad a = 1\,\mathrm{m}, \quad b = 1\,\mathrm{m}, \quad T_0 = 100\,\mathrm{K}, \quad p_{00} = 1000\,\mathrm{N/m^2}. \tag{4-2}$$

The characteristic frequency, assuming a Poisson's ratio $\nu_T = 0.3$, reads $\omega_0 = 106.5313\,\mathrm{s}^{-1}$. The first eigenfrequency, as the root of the denominator of (3-10), is then $\omega_R = 106.5324\,\mathrm{s}^{-1}$.

In Figure 1 we see that the resonance frequency for auxetics, ω_{R1}, is lower than that for normal materials, ω_{R2}, and both amplitudes $\mathrm{Re}(w_{00})$ (top plot) and $\mathrm{Im}(w_{00})$ (bottom) are bigger for auxetics. Moreover $\mathrm{Re}(w_{00})$ and $\mathrm{Im}(w_{00})$ are phase-shifted by $\frac{3}{2}\pi$ for auxetics and $\frac{\pi}{2}$ for normal materials. However, much more interesting are the real and imaginary parts of the temperature distribution θ_{00} across the thickness of the plate, because that distribution is the direct origin of the thermoelastic damping.

Figure 2, top, shows that the above temperature parts are phase shifted with each other and their distributions are completely different across the thickness. We see that thermoelastic damping dominates in the vicinity of the resonance frequency. Detailed depictions of the more spectacular situations indicated in the figure are presented in 3D form in Figure 3. Figure 2, bottom, shows details of the two top panels of Figure 2 around the first eigenfrequency $\omega_R = \omega_{R2}$.

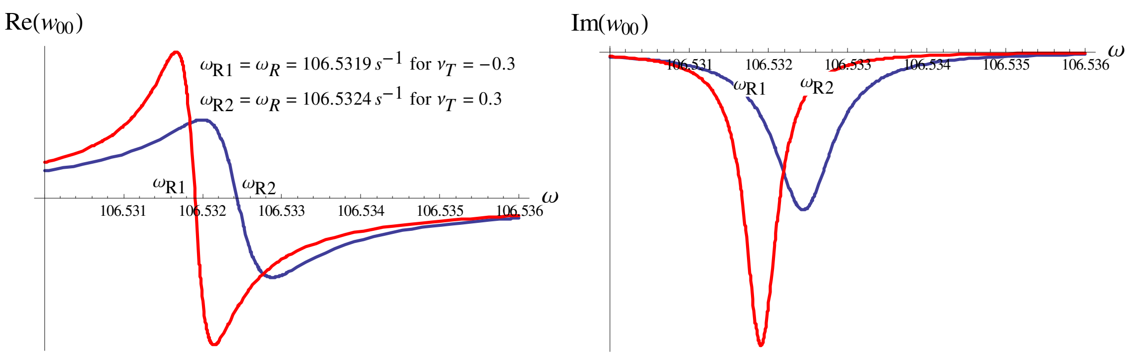

Figure 1. Bending displacement amplitudes w_{00} versus forcing frequency for $\nu_T = 0.3$ (blue) and $\nu_T = -0.3$ (red); $\tau = 10^{-10}$ s.

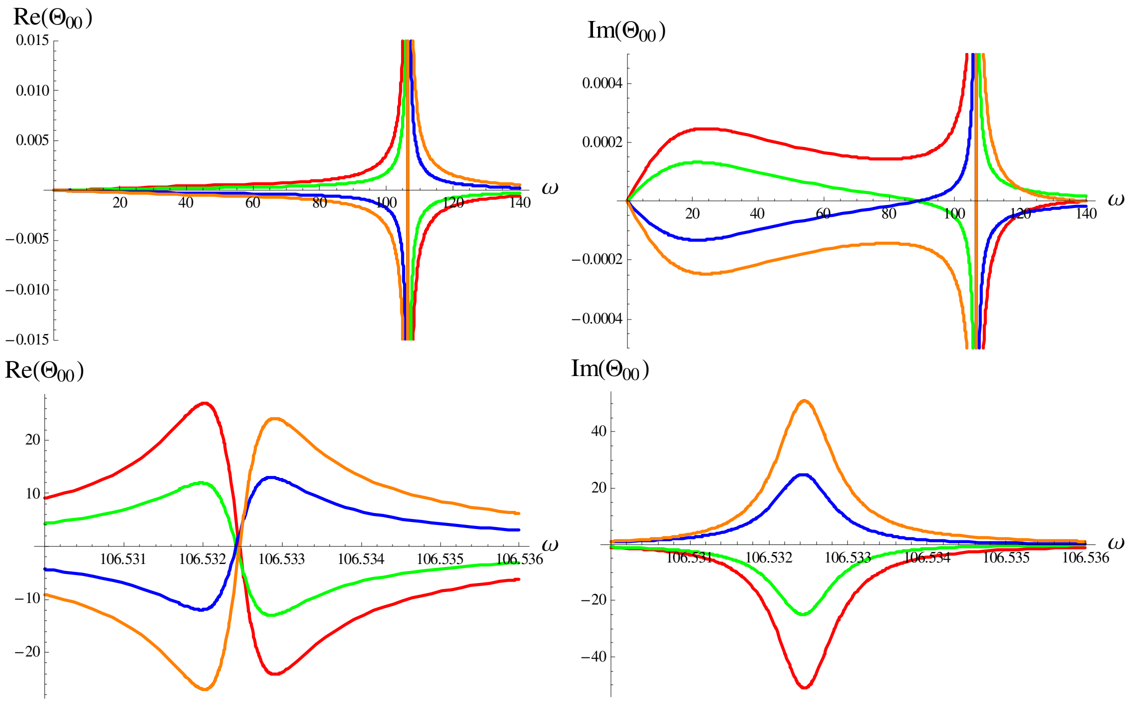

Figure 2. Temperature distribution amplitudes across plate thickness versus forcing frequency for $\nu_T = 0.3$ and $\tau = 10^{-10}$ s; $x_3 = -0.001$ m (red), $x_3 = -0.0005$ m (green), $x_3 = 0.0005$ m (blue), and $x_3 = 0.0001$ m (orange).

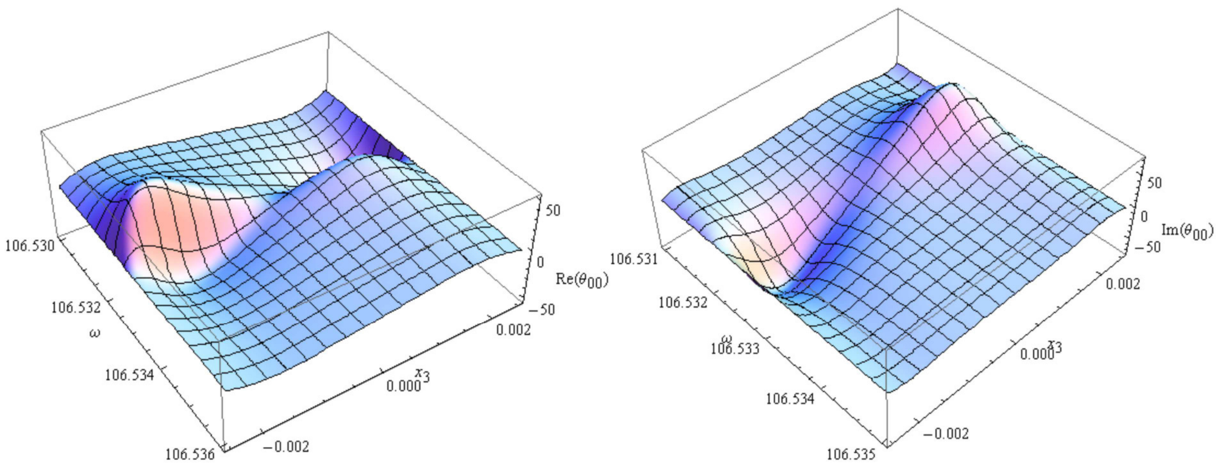

Figure 3. 3D presentation of real and imaginary parts of the temperature amplitude distribution across the thickness.

We remark that the results presented in Figures 2 and 3 are qualitatively comparable to those shown in [Khisaeva and Ostoja-Starzewski 2006].

We still have to analyze how the auxeticity and the relaxation of the thermal field influence mechanical bending and the amplitude distribution responsible for the thermoelastic damping temperature.

For greater legibility, Figures 4–7 show the amplitudes in limited ranges only, while the Poisson's ratios are shown in their full range. The amplitudes are actually represented by smooth functions, not asymptotic ones. The amplitude distributions are symmetric with respect to $\nu = 0$ only within the mathematical range, not within the physical one. For the first eigenfrequency $\omega < 101.625\,\mathrm{s}^{-1}$ there is no resonance in any normal and auxetic material. But for increasing ω if $\omega > 101.625\,\mathrm{s}^{-1}$ the resonance peaks follow increasing values of $|\nu|$. Beyond approximately $\omega = 120\,\mathrm{s}^{-1}$ the resonance occurs only for

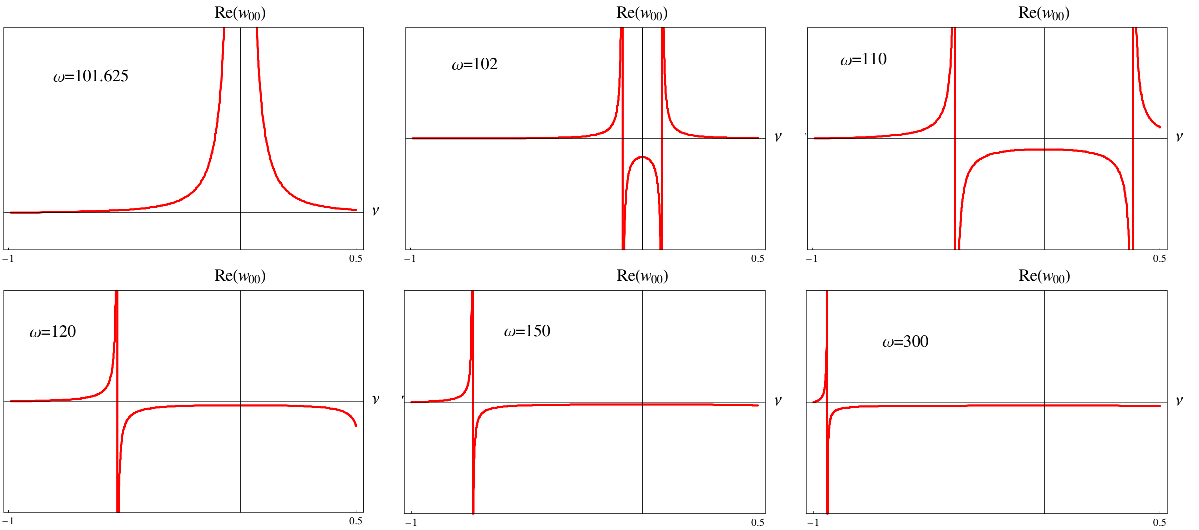

Figure 4. Distribution of w_{00} amplitudes versus Poisson's ratio for different ω.

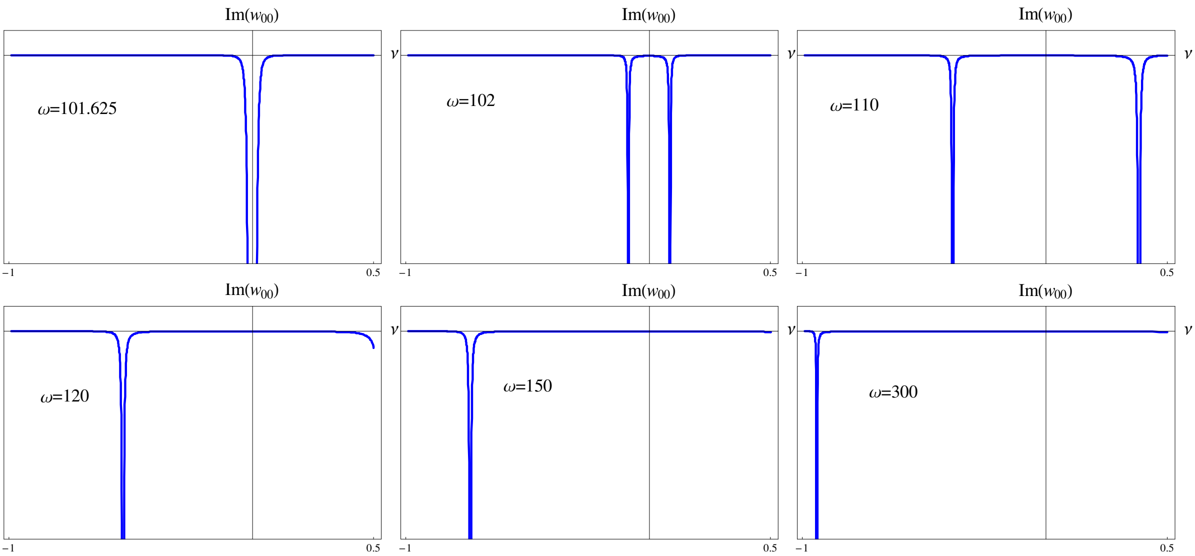

Figure 5. Bending amplitudes $\mathrm{Im}(w_{00})$ for different Poisson's ratios and different frequencies ω; phase-shifted $\pi/2$ with respect to $\mathrm{Re}(w_{00})$.

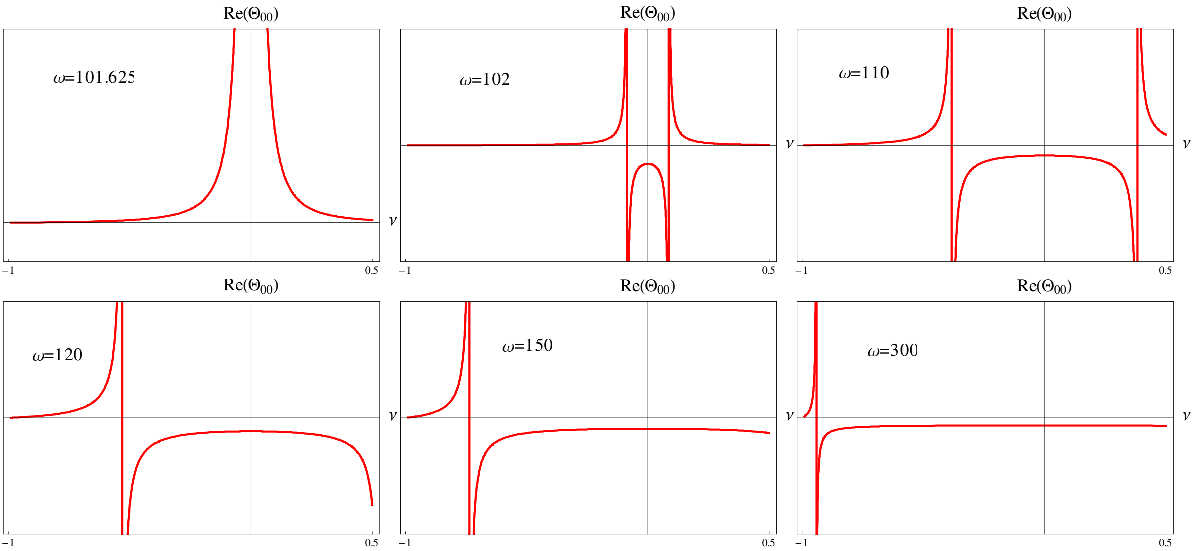

Figure 6. Temperature amplitudes $\mathrm{Re}(w_{00})$ for different forcing frequencies and various materials, for various Poisson's ratios ν.

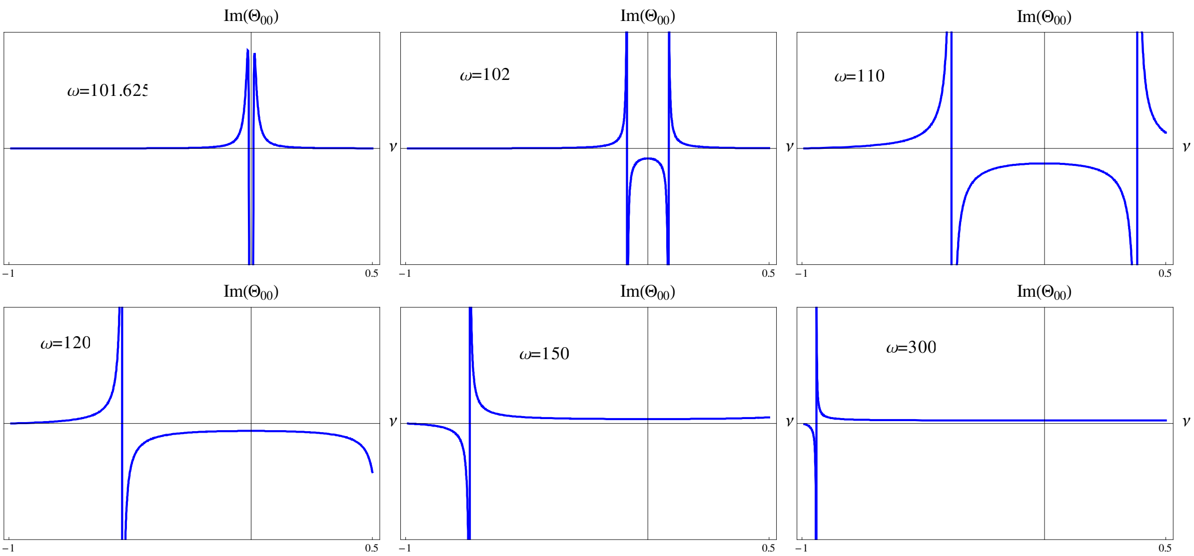

Figure 7. Temperature amplitudes $\mathrm{Im}(w_{00})$, phase-shifted $\pi/2$ with respect to $\mathrm{Re}(\theta_{00})$, for different frequencies and various materials, and various Poisson's ratios ν (continued on next page).

auxetics. Numerical analysis has indicated that for arbitrarily large values of $\omega > 101.625\,\mathrm{s}^{-1}$ resonances occur as the Poisson's ratio ν approaches -1. In the "normal material" ($\nu > 0$) sides of Figures 6 and 7 the character of the dependence temperature variations on the forcing frequency ω are also qualitatively comparable to the similar character of results presented in [Khisaeva and Ostoja-Starzewski 2006]. Note that the temperature distributions shown in Figures 6 and 7 have been calculated for fixed x_3.

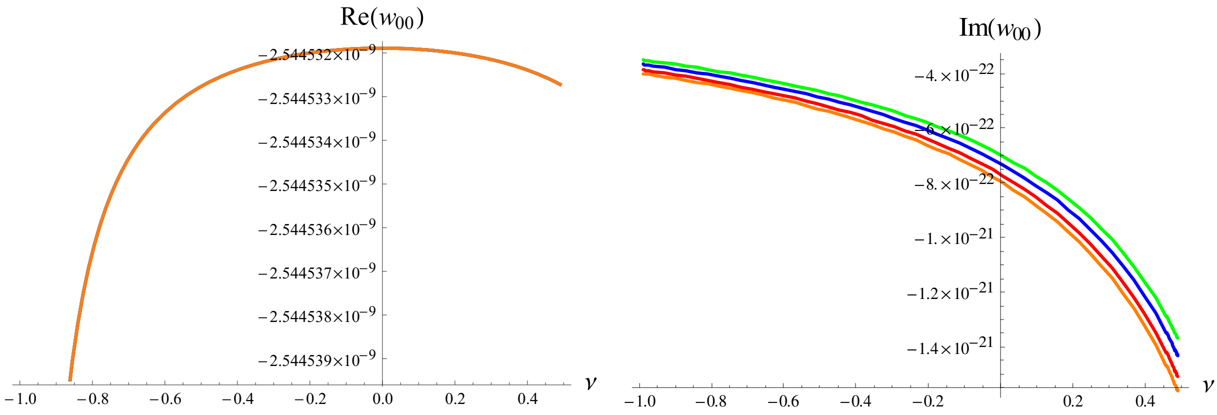

Figure 8. Real and imaginary parts of the bending w_{00} versus the Poisson's ratio for different thermal relaxation times, with high frequency $\omega = 10^5\ \mathrm{s}^{-1}$. On the left, the graph of $\mathrm{Re}(w_{00})$ remains essentially the same for $\tau = 10^{-10}$ s, $\tau = 10^{-8}$ s, $\tau = 10^{-6}$ s, and $\tau = 5 \times 10^{-6}$ s. On the right, $\tau = 10^{-8}$ s (blue), $\tau = 10^{-6}$ s (green), $\tau = 10^{-5}$ s (red), and $\tau = 5 \times 10^{-6}$ s (orange).

Analysis of these figures shows that thermoelastic damping decreases for increasing frequencies and comes from the thermoelastic damping occurring in the plate.

We see in Figure 8, left, that the mechanical vibrations described by the real part of the bending amplitude w_{00} are practically independent of the thermal relaxation time τ; its influence in that situation is negligible. Moreover, we see that amplitudes are very small because the frequency ω is high.

But for the mechanical vibrations phase-shifted $\pi/2$, described by the imaginary part of the bending amplitude w_{00}, the situation is different (see Figure 8, right). Although those amplitudes are extremely small, they depend on the thermal relaxation time τ and decrease if the Poisson's ratio ν approaches -1.

A similar situation occurs qualitatively for the temperature distribution $\mathrm{Re}(\theta_{00})$ (see Figure 9). This conclusion is very important because the distribution of $\theta_{00} = \theta_{00}(x_3)$ forms the origin of the thermoelastic damping. That damping increases for decreasing frequencies.

Comparison of Figures 8 and 9 shows that $\mathrm{Im}(w_{00})$ strongly depends on the Poisson's ratio being weakly dependent on the thermal relaxation time. But $\mathrm{Im}(\theta_{00})$ is constant for various Poisson's ratios which are strongly dependent on the thermal relaxation time.

5. Conclusions

The detailed analysis of thermoelastic damping during forced vibrations of an auxetic rectangular plate presented in this paper within the extended thermodynamical model shows that

- energy dissipation is lower in an auxetic material then in normal material,
- thermoelastic damping decreases if forcing frequency increases,
- unconventional behavior of materials occurs in the vicinity of the resonance frequency ω_R, and
- only the imaginary parts of the bending and temperature amplitudes depend on the thermal relaxation time.

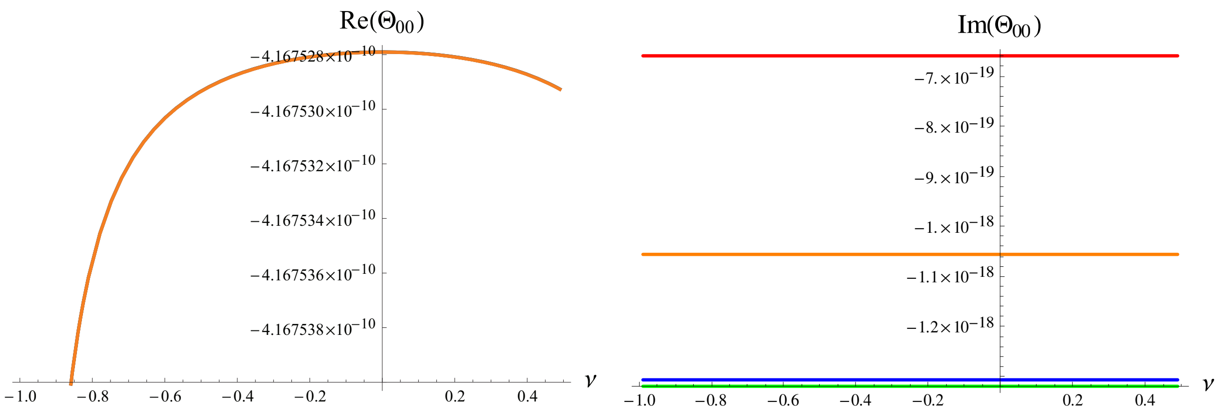

Figure 9. Real and imaginary parts of the temperature distribution θ_{00} versus the Poisson's ratio for different thermal relaxation times, with high frequency $\omega = 10^5\ \mathrm{s}^{-1}$. On the left, the graph of $\mathrm{Re}(\theta_{00})$ remains essentially the same for $\tau = 10^{-10}$ s, $\tau = 10^{-8}$ s, $\tau = 10^{-6}$ s, and $\tau = 5 \times 10^{-6}$ s. On the right, $\tau = 10^{-8}$ s (blue), $\tau = 10^{-6}$ s (green), $\tau = 10^{-5}$ s (red), and $\tau = 5 \times 10^{-6}$ s (orange).

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