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A TWO-DIMENSIONAL PROBLEM IN MAGNETOTHERMOELASTICITY WITH LASER PULSE UNDER DIFFERENT BOUNDARY CONDITIONS

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This paper is concerned with the study of vibrations induced by a laser beam in the context of generalized magnetothermoelasticity. The basic governing equations for isotropic and homogeneous elastic solids are formulated under Green–Naghdi theory in the x-z plane. The temporal profile of the laser beam is considered as non-Gaussian. The governing nondimensional equations are solved using normal mode analysis. The obtained solution is then applied to two specific problems in the half-space, where the boundary is subjected to either a mechanical or thermal load. Numerical computations are performed for a specific model to calculate the displacement, temperature, and stress fields and the results are displayed. The effects of time and magnetic field on the variation of different field quantities are analyzed in the figures.

1. Introduction

The dynamical interactions between the thermal and mechanical fields in solids are important to many practical applications, such as modern aeronautics, nuclear reactors and high speed particle accelerators, etc. The classical theory of thermoelasticity finds stresses caused by a temperature field using the parabolic heat conduction equation. The absence of any elasticity term in the heat conduction equation for uncoupled thermoelasticity appears to be unrealistic, since, due to mechanical loading of an elastic body, the strain so produced causes variation in temperature field. Moreover, the parabolic nature of the heat conduction equation results in an infinite velocity of wave propagation, which also contradicts the actual physical phenomena.

Biot [1956] developed the coupled theory of thermoelasticity to overcome the paradox inherent in the uncoupled theory, that elastic changes have no effect on temperature. In this theory, the equations of elasticity and heat conduction are coupled. However, it shares the defect of the uncoupled theory in which it predicts an infinite speed of propagation for heat waves. Generalized thermoelastic theories have been developed with the objective of removing this defect of coupled theory. The development of these theories was accelerated by the advent of the experimental observation of the second sound effect in materials at very low temperatures by Ackerman et al. [1966] and Ackerman and Overton [1969]. In heat transfer problems involving very short time intervals and/or very high heat flux, the second sound effect in the coupled theory yields results that are realistic and very much different from those obtained from the classical theory of thermoelasticity.

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It is well known that there are three major generalizations of the theory of thermoelasticity. The first is that made by Lord and Shulman [1967], known as L-S theory, which involves one relaxation time for a thermoelastic process. The second is due to Green and Lindsay [1972] and is known as G-L theory. It takes into account two parameters as relaxation times. L-S theory only modifies Fourier's heat conduction equation, while G-L theory modifies both the energy equation and the equation of motion. Dhaliwal and Sherief [1980] extended L-S theory by including the anisotropic case. Later on, by providing sufficient basic modifications to the constitutive equations to follow thermodynamical principles, Green and Naghdi [1991; 1992; 1993] produced an alternative theory which was divided into three different parts, referred to as G-N theory of types I, II, and III. The constitutive assumptions for the heat flux vector are different in each theory. The nature of these three types of constitutive equations is such that when the respective theories are linearized, type I is same as classical heat conduction theory (based on Fourier's heat conduction law), type II predicts the finite speed of heat propagation involving no energy dissipation, and type III indicates the propagation of thermal signals with finite speed. Hetnarski and Ignaczak [1999] presented a survey of various representative theories in the range of generalized thermoelasticity. Ezzat et al. [2004] discussed a problem in generalized thermoelasticity theory for isotropic media with temperature-dependent moduli of elasticity under L-S, G-L, and coupled theories. Youssef [2006] studied two-dimensional generalized thermoelasticity problem with a spherical cavity subjected to thermal shock and ramp-type heating.

During pulsed laser heating, a thermoelastic wave is generated due to thermal expansion in the near-surface region and propagates into the target. Because of the extremely short heating time, the laser-induced thermoelastic wave has an extremely high strain rate, which in turn causes strong coupling between the strain rate and the temperature field. This coupling damps the stress wave during its propagation and induces a localized temperature variation [Wang and Xu 2001; 2002]. Chen et al. [2004] developed a problem in which three different approaches, ultrafast thermoelasticity, Lord–Shulman theory, and classical thermoelasticity, are used to investigate thermoelastic stress waves in a gold medium. Sun et al. [2008] studied the coupled thermoelastic vibrations of a microscale beam resonator induced by laser pulse heating. The vibrations of deflection and thermal moments were calculated using an analytical numerical technique based on the Laplace transformation. The effect of laser pulse energy depth, the size effect, and the effects of different boundary conditions were analyzed.

Laser-induced vibration of microbeam resonators has attracted considerable attention recently due to many important technological applications in microelectromechanical systems (MEMS) and nanoelectromechanical systems (NEMS). The field equations for coupled thermoelastic vibration of Rayleigh and Timoshenko beams have been derived by Jones [1966]. Many authors have studied the vibration and heat transfer process of beams [Kidawa-Kukla 1997; 2003; Fang et al. 2007].

So-called ultrashort lasers are those with a pulse duration ranging from nanoseconds to femtoseconds, in general. In the case of ultrashort-pulsed laser heating, high-intensity energy flux and ultrashort duration laser beams have introduced situations in which very large thermal gradients or an ultrahigh heating speed may exist on the boundaries. In such cases, as pointed out by many investigators, the classical Fourier model, which leads to an infinite propagation speed of thermal energy, is no longer valid [Joseph and Preziosi 1989; Özişik and Tzou 1994; Tzou 1997; Tang and Araki 1999]. The non-Fourier effect of heat conduction takes into account the effect of mean free time (thermal relaxation time) in the energy carrier's collision process, which can eliminate this contradiction. He et al. [2002] solved a boundary value problem for a one-dimensional semi-infinite piezoelectric rod with the left boundary subjected to a sudden heat flux using the theory of generalized thermoelasticity with one relaxation time. Youssef and Al-Felali [2012] investigated the induced temperature and stress fields when subjected to non-Gaussian laser heating in context with classical coupled thermoelasticity, Lord–Shulman theory and Green–Lindsay theory.

The theory of magnetothermoelasticity has received the attention of many researchers due to its applications in widely diverse fields such as geophysics, for understanding the effect of earth's magnetic field on seismic waves, damping of acoustic waves, emission of electromagnetic radiations from nuclear devices, optics, etc. The theory of magnetothermoelasticity was introduced by Knopoff [1955] and Chadwick [1957] and developed by Kaliski and Petykiewicz [1959]. The theoretical outline of the development of magnetothermoelasticity was discussed by Paria [1962]. Paria studied the propagation of plane magnetothermoelastic waves in an isotropic unbounded medium under the influence of a magnetic field acting transversely to the direction of propagation. Nayfeh and Nemat-Nasser [1972] studied the propagation of plane waves in a solid under the influence of an electromagnetic field. Sherief and Ezzat [1996] discussed a thermal shock problem in magnetothermoelasticity with thermal relaxation. Sherief and Helmy [2002] illustrated a two-dimensional half-space problem subjected to a nonuniform thermal shock in the context of electromagnetothermoelasticity theory. Ezzat and Youssef [2005] constructed a generalized magnetothermoelasticity problem in a perfectly conducting medium. Baksi et al. [2005] examined a magnetothermoelastic problem with thermal relaxation and a heat source in a three-dimensional, infinite rotating elastic medium. Deswal and Kalkal [2011] employed normal mode analysis to study a problem in the purview of magnetothermoviscoelasticity with diffusion.

The objective of present investigation is to study the phenomenon of wave propagation in generalized magnetothermoelasticity with pulsed heating of a microbeam. Normal mode analysis is employed for the general solution of the problem. The resulting formulation is then applied to the problem of an elastic half-space whose boundary is subjected to two types of loads, mechanical and thermal. Finally, a numerical example has been considered and the results are displayed graphically to highlight the effects of magnetic field and time on physical quantities. To the authors' best knowledge, the technique of normal mode analysis has never been applied to Green–Naghdi theory of type III. It is also pertinent that hardly any effort has been made to discuss the laser pulse problem in the above-mentioned theory. In addition, we have also studied magnetic effects on the field variables. The present model is not only of theoretical interest, but may have practical applications in various fields such as geophysics, plasma physics, and other related topics. The self-focusing of a circularly polarized laser pulse in the hot plasma is very much influenced by the application of an external magnetic field. The external magnetic field enhances self-focusing for right-hand polarization while for left-hand polarization it acts to reduce self-focusing [Javan and Nasirzadeh 2012].

2. Governing equations

The governing equations in the context of Green–Naghdi theory of type III with a laser pulse heat source and a magnetic field for a isotropic and homogeneous elastic medium are (see [Kumar and Mukhopadhyay 2009])

- the equation of motion

$$\rho \ddot{u}_i = \sigma_{ji,j} + F_i, \tag{1}$$

where u_i are the components of displacement vector $\vec{u}$, ρ is the density of the medium, σ_{ij} are the components of the stress tensor, and F_i are the components of the Lorentz body force vector;

- the heat conduction equation

$$k^*\theta_{,ii} + k\dot{\theta}_{,ii} = \rho C_E \ddot{\theta} + \beta_1 T_0 \ddot{u}_{i,i} - \frac{\partial Q}{\partial t}, \tag{2}$$

where $\theta = T - T_0$ with T the absolute temperature and T_0 is a reference temperature assumed to obey the inequality $|\theta/T_0| \ll 1$, C_E is the specific heat, k^* is material constant, k is the thermal conductivity, and $\beta_1 = (3\lambda + 2\mu)\alpha_t$ with α_t is the coefficient of linear thermal expansion;

- the constitutive relations

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e \delta_{ij} - \beta_1 \theta \delta_{ij}, \tag{3}$$

$$e_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i}), \tag{4}$$

where e_{ij} are the components of strain tensor, δ_{ij} is the Kronecker delta function, e is the cubical dilation, and λ and μ are the Lamé constants.

We take linearized Maxwell's equations governing the electromagnetic field for a perfectly conducting medium as

$$\operatorname{curl} \vec{h} = \vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}, \tag{5}$$

$$\operatorname{curl} \vec{E} = -\mu_0 \frac{\partial \vec{h}}{\partial t}, \tag{6}$$

$$\vec{E} = -\mu_0 \Big(\frac{\partial \vec{h}}{\partial t} \times \vec{H} \Big), \tag{7}$$

$$\operatorname{div} \vec{h} = 0, \tag{8}$$

where μ_0 is the magnetic permeability, ε_0 is the electric permittivity, $\vec{H}$ is the applied magnetic field, $\vec{h}$ is the induced magnetic field, $\vec{E}$ is the induced electric field, $\vec{J}$ is the current density vector, and Q is the laser pulse heat source.

3. Problem formulation

A rectangular cartesian coordinate system is chosen in such a way that the x-axis lies along the free boundary of a perfectly conducting homogeneous isotropic generalized thermoelastic half-space with a laser pulse heat source, subjected to a constant magnetic field $\vec{H}(0, H_0, 0)$ which produces an induced magnetic field $\vec{h}(0, h_2, 0)$ and induced electric field $\vec{E}(E_1, 0, E_3)$. Let the z-axis point vertically downward into the half-space so that it occupies the region $z \geq 0$. The surface ($z = 0$) of the half-space is subjected to mechanical and thermal loads, and all the considered quantities will be functions of the time variable t and of coordinates x and z. Also, the boundary plane ($z = 0$) of the half-space is heated

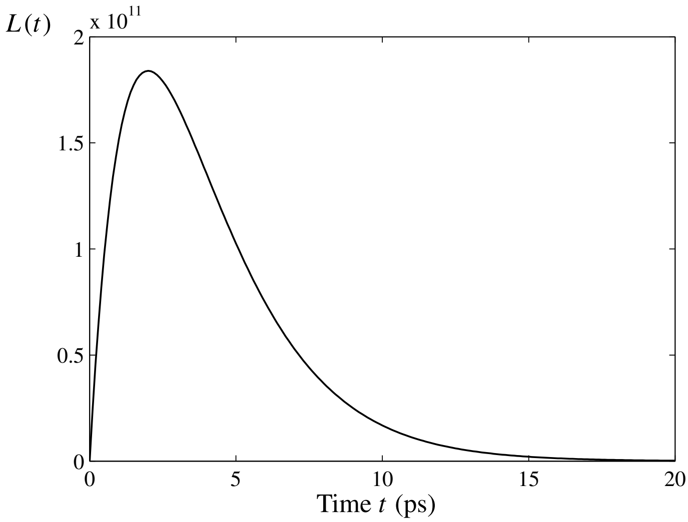

Figure 1. Temporal profile of $L(t)$ ($t_p = 2\,\text{ps}$).

uniformly by a laser pulse with non-Gaussian temporal profile [Sun et al. 2008]

$$L(t) = \frac{L_0 t}{t_p^2} \exp\Big(-\frac{t}{t_p}\Big), \tag{9}$$

where t_p is the time duration of a laser pulse and L_0 the laser intensity, which is defined as the total energy carried by a laser pulse per unit cross section of the laser beam. In the present study we take $t_p = 2\,\text{ps}$ as the time duration. According to [Tang and Araki 1999], the thermal conduction in the beam can be modeled as a one-dimensional problem with an energy source $Q(z, t)$ as

$$Q(z, t) = \frac{R_a}{\delta} \exp\Big(\frac{z - h/2}{\delta}\Big) L(t), \tag{10}$$

where δ is the absorption depth of the heating energy and R_a the absorptivity of the irradiated surface. For a two-dimensional problem in the x-z plane, the displacement components take the form

$$u = u(x, z, t), \quad v = 0, \quad w = w(x, z, t). \tag{11}$$

The strain components become

$$e_{xx} = \frac{\partial u}{\partial x}, \quad e_{zz} = \frac{\partial w}{\partial z}, \quad e_{xz} = \frac{1}{2}\Big(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\Big), \quad e_{xy} = e_{yz} = e_{yy} = 0. \tag{12}$$

The cubical dilatation e is thus given by

$$e = e_{xx} + e_{yy} + e_{zz} = \Big(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\Big). \tag{13}$$

The components of the initial magnetic field vector $\vec{H}$ are

$$H_x = 0, \quad H_y = H_0, \quad H_z = 0. \tag{14}$$

The electric intensity vector $\vec{E}$ is parallel to the current density vector $\vec{J}$, thus

$$E_x = E_1, \quad E_y = 0, \quad E_z = E_3, \tag{15}$$

$$J_x = J_1, \quad J_y = 0, \quad J_z = J_3. \tag{16}$$

From (5)–(8), we can obtain

$$E_1 = \mu_0 H_0 \frac{\partial w}{\partial t}, \qquad E_3 = -\mu_0 H_0 \frac{\partial u}{\partial t}, \tag{17}$$

$$h_1 = 0, \qquad h_2 = -H_0 e, \qquad h_3 = 0, \tag{18}$$

$$J_1 = -\varepsilon_0 \mu_0 H_0 \frac{\partial^2 w}{\partial t^2}, \quad J_3 = \varepsilon_0 \mu_0 H_0 \frac{\partial^2 u}{\partial t^2}. \tag{19}$$

The Lorentz's force $\vec{F}$ is given by the relation

$$\vec{F} = \mu_0 (\vec{J} \times \vec{H}). \tag{20}$$

Inserting (14) and (19) in (20), we can obtain the components of the Lorentz's force $\vec{F}$ as

$$F_x = -\varepsilon_0 \mu_0^2 H_0^2 \frac{\partial^2 u}{\partial t^2}, \quad F_y = 0, \quad F_z = -\varepsilon_0 \mu_0^2 H_0^2 \frac{\partial^2 w}{\partial t^2}. \tag{21}$$

Now, we will use the following nondimensional variables:

$$\begin{aligned} (x', z', u', w', \delta', h') &= c_0 \eta_0 (x, z, u, w, \delta, h), \qquad (t', t_p') = c_0^2 \eta_0 (t, t_p), \\ (\sigma_{ij}', p_1') &= \frac{1}{\lambda + 2\mu} (\sigma_{ij}, p_1), \qquad (\theta', p_2') = \frac{\beta_1}{(\lambda + 2\mu)} (\theta, p_2), \end{aligned} \tag{22}$$

where

$$c_0^2 = \frac{\lambda + 2\mu}{\rho}, \quad \eta_0 = \frac{\rho C_E}{k^*} \bar{\omega}, \quad \bar{\omega} = \frac{\rho C_E c_0^3}{k^* h^*},$$

and h^* is some standard length.

With the help of these nondimensional quantities, (1)–(3) take the following form (dropping the prime signs for convenience):

$$\alpha \frac{\partial^2 u}{\partial t^2} = \beta^2 \nabla^2 u + (1 - \beta^2) \frac{\partial e}{\partial x} - \frac{\partial \theta}{\partial x}, \tag{23}$$

$$\alpha \frac{\partial^2 w}{\partial t^2} = \beta^2 \nabla^2 w + (1 - \beta^2) \frac{\partial e}{\partial z} - \frac{\partial \theta}{\partial z}, \tag{24}$$

$$k^* \nabla^2 \theta + k (c_0^2 \eta_0) \nabla^2 \dot{\theta} = \rho C_E c_0^2 \frac{\partial^2 \theta}{\partial t^2} + \frac{\beta_1^2 T_0}{\rho} \frac{\partial^2 e}{\partial t^2} - \frac{\beta_1 c_0 \eta_0}{\rho} \frac{\partial Q}{\partial t}, \tag{25}$$

$$\sigma_{zx} = \beta^2 \Big(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \Big), \tag{26}$$

$$\sigma_{zz} = \frac{\partial w}{\partial z} + (1 - 2\beta^2) \frac{\partial u}{\partial x} - \theta, \tag{27}$$

where

$$\alpha = 1 + \frac{\varepsilon_0 \mu_0^2 H_0^2}{\rho}, \quad \beta^2 = \frac{\mu}{\lambda + 2\mu}.$$

Now, we introduce the displacement potentials $\phi(x,z,t)$ and $\psi(x,z,t)$, which are related to the displacement components as

$$u = \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial z}, \qquad w = \frac{\partial \phi}{\partial z} - \frac{\partial \psi}{\partial x}. \tag{28}$$

By simplifying (23)–(25) using (28) along with (9) and (10), we obtain the following equations:

$$\frac{\partial^2 \phi}{\partial t^2} = \frac{1}{\alpha}(\nabla^2 \phi - \theta), \tag{29}$$

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{1}{\alpha_0}\nabla^2 \psi, \tag{30}$$

$$k_1 \nabla^2 \theta + k_2 \nabla^2 \dot{\theta} = \frac{\partial^2 \theta}{\partial t^2} + \varepsilon_1 \nabla^2 \ddot{\phi} - \varepsilon_2 \frac{R_a L_0}{\delta t_p^2} \exp\left(\frac{z-h/2}{\delta} - \frac{t}{t_p}\right), \tag{31}$$

where

$$\alpha_0 = \frac{\alpha}{\beta^2}, \quad k_1 = \frac{k^*}{(\lambda+2\mu)C_E}, \quad k_2 = \frac{k}{k^*}\bar{\omega}, \quad \varepsilon = \frac{\beta_1^2}{(\lambda+2\mu)\rho}, \quad \varepsilon_1 = \frac{\varepsilon T_0}{C_E}, \quad \varepsilon_2 = \frac{\sqrt{\varepsilon}}{k^*}\bar{\omega}.$$

4. Normal mode analysis

The solution of the considered physical variables can be decomposed in terms of normal modes as

$$(u, w, \phi, \psi, \theta, \sigma_{ij})(x,z,t) = (u^*, w^*, \phi^*, \psi^*, \theta^*, \sigma_{ij}^*)(z) e^{(\omega t + \imath m x)}, \tag{32}$$

where ω is the complex time constant and m is the wave number in x-direction.

Using (32) in (29)–(31), we get

$$(D^2 - \varepsilon_3)\psi^* = 0, \tag{33}$$

$$(D^2 - \varepsilon_4)\phi^* = \theta^*, \tag{34}$$

$$(\varepsilon_5 D^2 - \varepsilon_6)\theta^* - \varepsilon_1 \omega^2 (D^2 - m^2)\phi^* + \varepsilon_2 \varepsilon_7 \exp\left(\frac{z-h/2}{\delta} - \frac{t}{t_p} - \omega t - \imath m x\right) = 0, \tag{35}$$

where

$$D \cong \frac{\partial}{\partial z}, \quad \varepsilon_3 = m^2 + \alpha_0 \omega^2, \quad \varepsilon_4 = m^2 + \alpha\omega^2, \quad \varepsilon_5 = k_1 + \omega k_2, \quad \varepsilon_6 = \varepsilon_5 m^2 + \omega^2, \quad \varepsilon_7 = \frac{R_a L_0}{t_p^2 \delta}.$$

Eliminating $\theta^*(z)$ between (34) and (35), we get the following fourth-order partial differential equation satisfied by $\phi^*(z)$:

$$(D^4 - AD^2 + B)\phi^*(z) = -CE \exp\left(\frac{z-h/2}{\delta} - \frac{t}{t_p} - \omega t - \imath m x\right), \tag{36}$$

where

$$A = \frac{\varepsilon_4 \varepsilon_5 + \varepsilon_1 \omega^2 + \varepsilon_6}{\varepsilon_5}, \quad B = \frac{\varepsilon_4 \varepsilon_6 + \varepsilon_1 m^2 \omega^2}{\varepsilon_5}, \quad C = \frac{\varepsilon_2 \varepsilon_7}{\varepsilon_5}, \quad E = 1 - \frac{(1+\omega t_p)t}{t_p}.$$

Using the solutions of (33), (34), and (36) (which are assumed to be bounded as $z \to \infty$), we can express $\phi(x, z, t)$, $\psi(x, z, t)$, and $\theta(x, z, t)$ in the following forms:

$$\phi(x, z, t) = \left(\sum_{i=1}^{2} L_i e^{-\lambda_i z}\right) e^{(\omega t + \iota m x)} - \varepsilon_8 f_1(z, t), \tag{37}$$

$$\psi(x, z, t) = L_3 e^{-\lambda_3 z} e^{(\omega t + \iota m x)}, \tag{38}$$

$$\theta(x, z, t) = \left(\sum_{i=1}^{2} L_i' e^{-\lambda_i z}\right) e^{(\omega t + \iota m x)} - \varepsilon_9 f_1(z, t), \tag{39}$$

where L_i, L_i' $(i = 1, 2)$, and L_3 are parameters depending on m and ω and

$$f_1(z, t) = CE \exp\left(\frac{z - h/2}{\delta} - \frac{t}{t_p}\right), \qquad L_i' = (\lambda_i^2 - \varepsilon_3) L_i \quad (i = 1, 2),$$

$$\varepsilon_8 = \left(\frac{\delta^4}{\delta^4 B - \delta^2 A + 1}\right), \qquad \varepsilon_9 = \frac{1 - \varepsilon_4 \delta^2}{\delta^2}.$$

Here, λ_i $(i = 1, 2)$ are the positive roots of the characteristic equation

$$\lambda^4 - A\lambda^2 + B = 0,$$

and λ_3 is the root of the characteristic equation

$$\lambda^2 - \varepsilon_3 = 0.$$

Similarly, applying normal mode analysis and using the solutions of (33), (34), and (36) in (26)–(28), we get

$$u = \left(\iota m \sum_{i=1}^{2} L_i e^{-\lambda_i z} - \lambda_3 L_3 e^{-\lambda_3 z}\right) e^{(\omega t + \iota m x)} - \iota m \varepsilon_8 f_1(z, t), \tag{40}$$

$$w = -\left[\left(\sum_{i=1}^{2} L_i \lambda_i e^{-\lambda_i z} + (\iota m) L_3 e^{-\lambda_3 z}\right) e^{(\omega t + \iota m x)} + \varepsilon_8 \frac{f_1(z, t)}{\delta}\right], \tag{41}$$

$$\sigma_{zx} = -\beta^2 \left[\left(2\iota m \sum_{i=1}^{2} L_i \lambda_i e^{-\lambda_i z} - (\lambda_3^2 + m^2) L_3 e^{-\lambda_3 z}\right) e^{(\omega t + \iota m x)} + f_2(z, t)\right], \tag{42}$$

$$\sigma_{zz} = \left(\sum_{i=1}^{2} (\eta + \varepsilon_4) L_i e^{-\lambda_i z} - 2\iota m \beta^2 \lambda_3 L_3 e^{-\lambda_3 z}\right) e^{(\omega t + \iota m x)} + f_3(z, t), \tag{43}$$

where $\eta = (1 - 2\beta^2) m^2$, $f_2(z, t) = \varepsilon_8 (2\iota m/\delta) f_1(z, t)$, and $f_3(z, t) = (\eta - 1/\delta^2 - \varepsilon_9) \varepsilon_8 f_1(z, t)$.

5. Applications

We consider a homogeneous, isotropic magnetothermoelastic solid with laser pulse heating occupying the half-space $z \geq 0$. The boundary $z = 0$ of the half-space is subjected to mechanical and thermal loads.

Case (i): *Mechanical load.* For an isothermal boundary plane $z = 0$, subjected to a normal mechanical load, the boundary conditions are given as

$$\sigma_{zz}(x,0,t)+\bar{\sigma}_{zz}(x,0,t) = -p_1(x,t), \tag{44}$$

$$\sigma_{zx}(x,0,t)+\bar{\sigma}_{zx}(x,0,t) = 0, \tag{45}$$

$$\theta(x,0,t) = 0, \tag{46}$$

where $p_1(x,t)$ is a given function of x and t, σ_{zj} is the mechanical stress, and $\bar{\sigma}_{zj}$ $(j = x, y, z)$ is the Maxwell stress, which is given as

$$\bar{\sigma}_{zj} = \mu_0[H_z h_j + H_j h_z - H_k h_k \delta_{zj}]. \tag{47}$$

Invoking the nondimensional form of (44)–(46) along with (26)–(28) and normal mode analysis as for (32), we obtain the system of equations

$$P_1L_1 + P_2L_2 + P_3L_3 = P, \tag{48}$$

$$Q_1L_1 + Q_2L_2 + Q_3L_3 = Q, \tag{49}$$

$$R_1L_1 + R_2L_2 = R, \tag{50}$$

whose coefficients are defined as follows, where $\eta_1 = \dfrac{\mu_0 H_0^2}{\lambda+2\mu}$:

$$\begin{aligned}
&P_i = \lambda_i^2\eta_1 + m^2(2\beta^2-\eta_1)+\alpha\omega^2, \quad Q_i = -2\iota m\lambda_i, \quad R_i = \lambda_i^2 - \varepsilon_4, \quad i = 1,2,\\
&P_3 = 2\iota m\lambda_3\beta^2, \qquad\qquad\qquad\qquad Q_3 = \lambda_3^2 + m^2,\\
&P = -p_1^* + \left(\frac{1}{\delta^2}(1+\eta_1) - \eta - m^2\eta_1 - \varepsilon_9\right)\varepsilon_8 f_1(z,t)e^{-(\omega t+\iota m x)},\\
&Q = \frac{2\iota m}{\delta}\varepsilon_8 f_1(z,t)e^{-(\omega t+\iota m x)}, \qquad R = \varepsilon_8\varepsilon_9 f_1(z,t)e^{-(\omega t+\iota m x)}.
\end{aligned} \tag{51}$$

The solution of the system of linear equations (48)–(50) can be expressed as

$$L_1 = \frac{\Delta_1}{\Delta}, \quad L_2 = \frac{\Delta_2}{\Delta}, \quad L_3 = \frac{\Delta_3}{\Delta}, \tag{52}$$

where

$$\Delta_1 = -PR_2Q_3 + QR_2P_3 + R(P_2Q_3 - Q_2P_3), \tag{53}$$

$$\Delta_2 = -P_1RQ_3 + Q_1RP_3 + R_1(PQ_3 - QP_3), \tag{54}$$

$$\Delta_3 = P_1(Q_2R - QR_2) - Q_1(P_2R - R_2P) + R_1(P_2Q - Q_2P), \tag{55}$$

$$\Delta = -P_1R_2Q_3 + Q_1R_2P_3 + R_1(P_2Q_3 - Q_2P_3). \tag{56}$$

Substituting the values of L_1, L_2 and L_3 from (52) into (39)–(43), we get the expressions for the displacement components, temperature distribution, and stress components as

$$u = \frac{1}{\Delta}\left(\iota m\sum_{i=1}^{2}\Delta_i e^{-\lambda_i z} - \lambda_3\Delta_3 e^{-\lambda_3 z}\right)e^{(\omega t+\iota m x)} - (\iota m)\varepsilon_8 f_1(z,t), \tag{57}$$

$$w = -\left[\frac{1}{\Delta}\left(\sum_{i=1}^{2}\Delta_i\lambda_i e^{-\lambda_i z} + \iota m\Delta_3 e^{-\lambda_3 z}\right)e^{(\omega t+\iota m x)} + \varepsilon_8\frac{f_1(z,t)}{\delta}\right], \tag{58}$$

$$\theta = \frac{1}{\Delta}\left(\sum_{i=1}^{2}(\lambda_i^2-\varepsilon_3)\Delta_i e^{-\lambda_i z}\right)e^{(\omega t+\iota m x)} - \varepsilon_8\varepsilon_9 f_1(z,t), \tag{59}$$

$$\sigma_{zx} = -\beta^2\left[\frac{1}{\Delta}\left(2\iota m\sum_{i=1}^{2}\Delta_i\lambda_i e^{-\lambda_i z} - \eta_2\Delta_3 e^{-\lambda_3 z}\right)e^{(\omega t+\iota m x)} + f_2(z,t)\right], \tag{60}$$

$$\sigma_{zz} = \frac{1}{\Delta}\left(\sum_{i=1}^{2}(\eta+\varepsilon_4)\Delta_i e^{-\lambda_i z} - 2\iota m\beta^2\lambda_3\Delta_3 e^{-\lambda_3 z}\right)e^{(\omega t+\iota m x)} + f_3(z,t), \tag{61}$$

where $\eta_2 = \lambda_3^2 + m^2$.

Case (ii): *Thermal load.* In this case the boundary conditions on the surface $z=0$ are given by

$$\sigma_{zz}(x,0,t) + \bar{\sigma}_{zz}(x,0,t) = 0, \tag{62}$$

$$\sigma_{zx}(x,0,t) + \bar{\sigma}_{zx}(x,0,t) = 0, \tag{63}$$

$$\theta(x,0,t) = p_2(x,t), \tag{64}$$

where $p_2(x,t)$ is a given function of x and t.

Adopting the same procedure as in Case (i), that is, using the required expressions in (62)–(64) (the dimensionless forms) and normal mode analysis, we can get a system of linear equations:

$$P_1L_1 + P_2L_2 + P_3L_3 = P', \tag{65}$$

$$Q_1L_1 + Q_2L_2 + Q_3L_3 = Q, \tag{66}$$

$$R_1L_1 + R_2L_2 = R', \tag{67}$$

where

$$\begin{aligned} P' &= \left(\frac{1}{\delta^2}(1+\eta_1) - \eta - m^2\eta_1 - \varepsilon_9\right)\varepsilon_8 f_1(z,t)e^{-(\omega t+\iota m x)}, \\ R' &= p_2^* + \varepsilon_8\varepsilon_9 f_1(z,t)e^{-(\omega t+\iota m x)}. \end{aligned} \tag{68}$$

The corresponding expressions for the displacement components u and w, temperature distribution θ, and stress components σ_{zx} and σ_{zz} are given by (57)–(61) with (P, R) replaced by (P', R') in (53)–(55).

6. Limiting cases

6.1. *Neglecting the laser pulse effect.*

Case (i): *Mechanical load.* To obtain the expressions for u, w, θ, σ_{zx}, and σ_{zz} in the context of the generalized theory of magnetothermoelasticity due to a mechanical load applied on the isothermal boundary $z=0$, we shall neglect the parameter corresponding to the laser pulse heat. For this, we put $L_0=0$, which implies that $\varepsilon_7=0$ and $f_1(z,t)=0$. Now, substituting $f_1(z,t)=0$ in (51), we get the following modifications in the expressions of P, Q, and R:

$$P = -p_1^*, \quad Q = 0, \quad R = 0. \tag{69}$$

Using (69) in (53)–(55), we get

$$\Delta_1 = -p_1^* R_2 Q_3, \quad \Delta_2 = -p_1^* R_1 Q_3, \quad \Delta_3 = -p_1^*(R_2 Q_1 - R_1 Q_2). \tag{70}$$

Hence, (57)–(61) take the form

$$u = \frac{1}{\Delta}\left(\iota m \sum_{i=1}^{2} \Delta_i e^{-\lambda_i z} - \lambda_3 \Delta_3 e^{-\lambda_3 z}\right) e^{(\omega t + \iota m x)}, \tag{71}$$

$$w = -\left[\frac{1}{\Delta}\left(\sum_{i=1}^{2} \Delta_i \lambda_i e^{-\lambda_i z} + \iota m \Delta_3 e^{-\lambda_3 z}\right) e^{(\omega t + \iota m x)}\right], \tag{72}$$

$$\theta = \frac{1}{\Delta}\left(\sum_{i=1}^{2} (\lambda_i^2 - \varepsilon_3) \Delta_i e^{-\lambda_i z}\right) e^{(\omega t + \iota m x)}, \tag{73}$$

$$\sigma_{zx} = -\beta^2\left[\frac{1}{\Delta}\left(2\iota m \sum_{i=1}^{2} \Delta_i \lambda_i e^{-\lambda_i z} - \eta_2 \Delta_3 e^{-\lambda_3 z}\right) e^{(\omega t + \iota m x)}\right], \tag{74}$$

$$\sigma_{zz} = \frac{1}{\Delta}\left(\sum_{i=1}^{2} (\eta + \varepsilon_4) \Delta_i e^{-\lambda_i z} - 2\iota m \beta^2 \lambda_3 \Delta_3 e^{-\lambda_3 z}\right) e^{(\omega t + \iota m x)}. \tag{75}$$

Case (ii): *Thermal load.* Similarly, for a thermal load, the corresponding expressions for the field variables under generalized magnetothermoelasticity are given by (71)–(75) with Δ_i replaced by Δ_i^*, where

$$P' = 0, \quad Q = 0, \quad R' = p_2^*, \tag{76}$$

and

$$\Delta_1^* = p_2^*(P_2 Q_3 - Q_2 P_3), \tag{77}$$

$$\Delta_2^* = -p_2^*(P_1 Q_3 - Q_1 P_3), \tag{78}$$

$$\Delta_3^* = p_2^*(P_1 Q_2 - P_2 Q_1). \tag{79}$$

6.2. ***Neglecting the magnetic effect.*** Case (i): *Mechanical load.* For a mechanical load applied on the isothermal boundary $z = 0$, we take $H_0 = 0$ and thus obtain $\alpha = 1$, which provides the following modifications in (51):

$$P_i = 2\beta^2 m^2 + \omega^2, \quad P = -p_1^* + \left(\frac{1}{\delta^2} - \eta - \varepsilon_9\right)\varepsilon_8 f_1(z,t) e^{-(\omega t + \iota m x)}, \quad i = 1, 2.$$

By considering these modifications in (53)–(56), we get the corresponding expressions for field variables from (57)–(61).

Case (ii): *Thermal load.* Similarly, in case of a thermal load, we assume that magnetic properties are absent from the medium. Then, by taking $H_0 = 0$ and $\alpha = 1$, we get the following changes in (68):

$$P_i = 2\beta^2 m^2 + \omega^2, \quad P' = \left(\frac{1}{\delta^2} - \eta - \varepsilon_9\right)\varepsilon_8 f_1(z,t) e^{-(\omega t + \iota m x)}, \quad i = 1, 2.$$

Following the same procedure as described earlier in Section 5(ii) and considering the above modifications, the corresponding expressions for u, w, θ, σ_{zx}, and σ_{zz} are given by (57)–(61).

7. Numerical results and discussions

In this section, we carry out computational work in order to illustrate the results derived in Section 5 and examine the behavior of the displacement components u and w, temperature distribution θ, and stress components σ_{zx} and σ_{zz}.

For this purpose, the material is chosen as copper and the values of the relevant parameters are taken as follows:

$$
\begin{aligned}
&k = 386\,\mathrm{W\,m^{-1}K^{-1}}, && T_0 = 293\,\mathrm{K}, && \rho = 8954\,\mathrm{kg\,m^{-3}}, && \alpha_t = 1.78\times 10^{-5}\,\mathrm{K^{-1}},\\
&C_E = 383.1\,\mathrm{J\,kg^{-1}K^{-1}}, && R_a = 0.5, && \mu_0 = 4\pi\times 10^{-7}\,\mathrm{H\,m^{-1}}, && \varepsilon_0 = (10^{-9}/36\pi)\,\mathrm{F\,m^{-1}},\\
&H_0 = (10^7/4\pi)\,\mathrm{A\,m^{-1}}, && h = 0.01, && \delta = 0.01, && L_0 = 1\times 10^{11}\,\mathrm{J\,m^{-1}},\\
&\lambda = 7.76\times 10^{10}\,\mathrm{kg\,m^{-1}s^{-2}}, && m = 1.2, && \omega = 1. && \mu = 3.86\times 10^{10}\,\mathrm{kg\,m^{-1}s^{-2}}.
\end{aligned}
$$

The computation is carried out for

$$t = 0.1, 0.3, 0.5; \quad p_1^* = 10;\ p_2^* = 10; \quad x = 1; \quad 0 \le z \le 4.5; \quad h^* = 10.$$

A comparison of the dimensionless form of the field variables for the cases of magnetothermoelasticity with a laser pulse (MTLP) and generalized thermoelasticity theory with a laser pulse (TLP) for three different values of time t, subjected to mechanical and thermal loads, is presented in Figures 2–9. The values of all the physical quantities for all the cases are displayed in the range $0 \le z \le 4.5$.

Case (i): *Mechanical load.* Figure 2 shows the variation of displacement component u with the distance z for MTLP and TLP for different values of t. We observe that all the curves show similar behavior, that is,

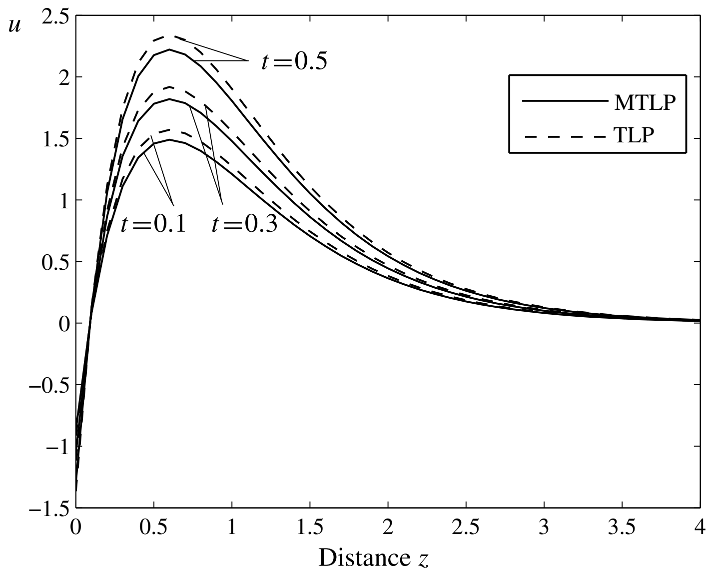

Figure 2. Variation of displacement u for a mechanical load.

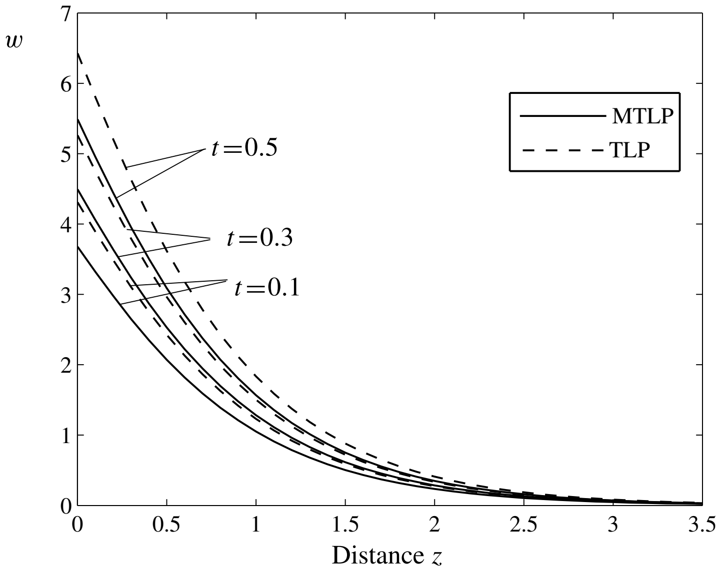

Figure 3. Variation of displacement w for a mechanical load.

all the curves start with negative values on the boundary of half-space, then rapidly increase to a maximal positive value and thereafter continuously decrease to zero value. Also, the effect of the magnetic field is significant for $0.5 \leq z \leq 2$ and the influence of time t is prominent in the range $0.3 \leq z \leq 3$.

Figure 3 shows the variation of displacement component w with distance z for MTLP and TLP for different values of t. We note that w starts with a positive value and then decreases continuously to zero value for all the cases in the range $0 \leq z \leq 3.5$. We see that the increment in time as well as the absence of magnetic field increase the magnitude of displacement component w. For MTLP and TLP at $t = 0.1, 0.3$, and 0.5, the effect is pronounced in the range $0 \leq z \leq 2$.

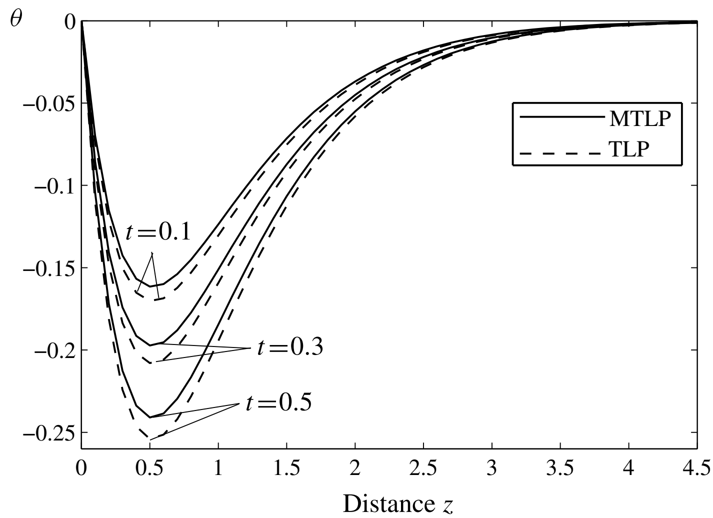

Figure 4. Variation of temperature distribution θ for a mechanical load.

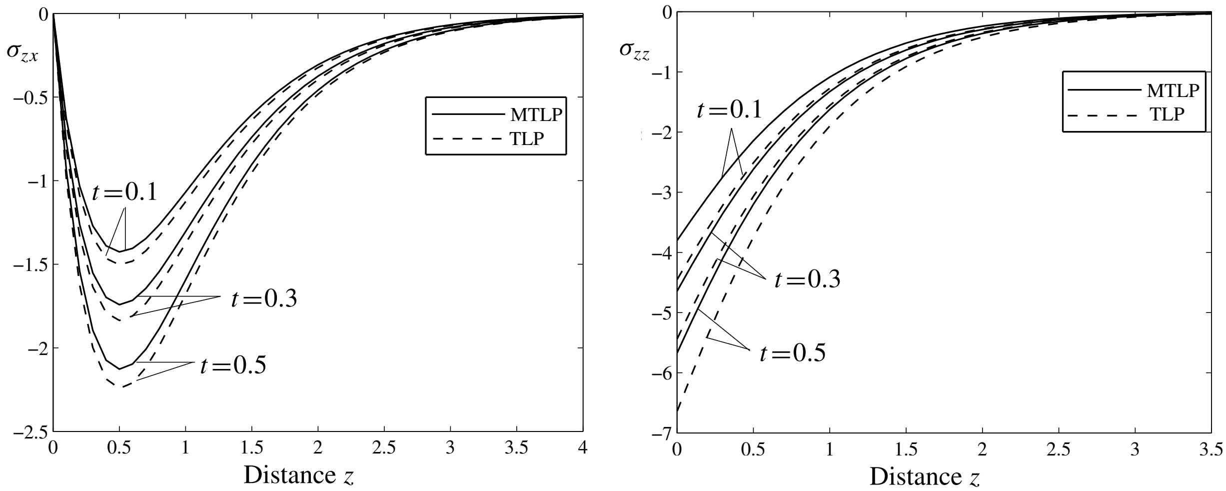

Figure 5. Variation of tangential stress σ_{zx} (left) and normal stress σ_{zz} (right) for a mechanical load.

Figure 4 shows the variation of θ with distance z for MTLP and TLP for different values of time t. It can be seen that the behavior of θ for all the three cases is similar: its magnitude increases in the range $0 \le z \le 0.5$ and then decreases in the range $0.5 \le z \le 4.5$. It is also seen that time strongly affects the temperature distribution θ; the difference is pronounced for both MTLP and TLP.

Figure 5 displays the variation of tangential stress σ_{zx} and normal stress σ_{zz} with distance z for MTLP and TLP for different values of t. In both cases the magnitude is greater for TLP than for MTLP, and it increases with t. It is also seen that all the curves show similar trends and the difference for time t is more pronounced than for a magnetic field.

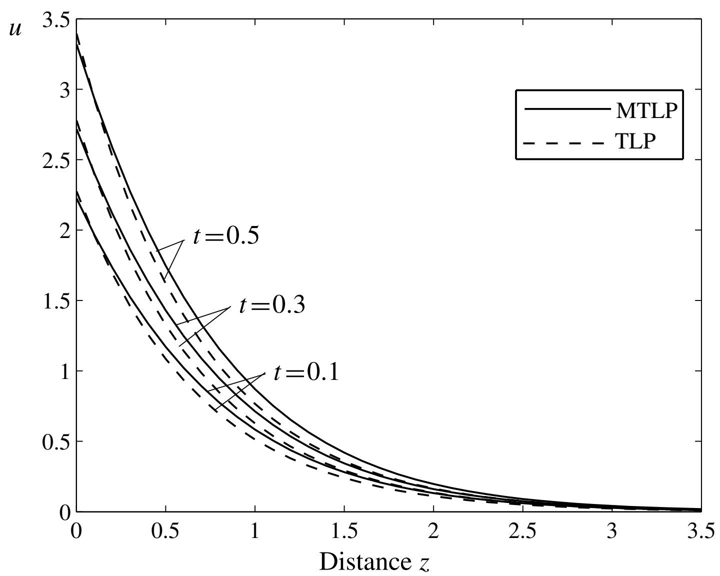

Figure 6. Variation of displacement u for a thermal load.

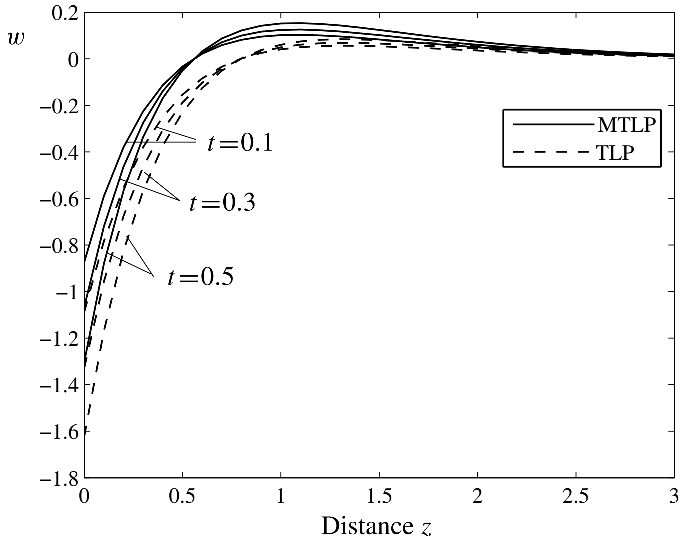

Figure 7. Variation of displacement w for a thermal load.

Case (ii): *Thermal load.* Figure 6 displays the variation of u with distance z for MTLP and TLP for different values of t. It is noticed that for all the cases the displacement component u behaves similarly. The values of u for TLP are found to be greater in the range $0 \le z \le 0.25$ and lesser in the range $0.25 \le z \le 3.5$, as compared to MTLP. Moreover, the value of u increases with time.

Figure 7 depicts the variation of w with distance z for MTLP and TLP for different values of t. The magnetic field acts to decrease the magnitude of displacement component w while an increment in time significantly enlarges the magnitude of w. Also, w shows a similar pattern for all the curves. The difference is clearly noticeable for MTLP and TLP in the range $0 \le z \le 2$.

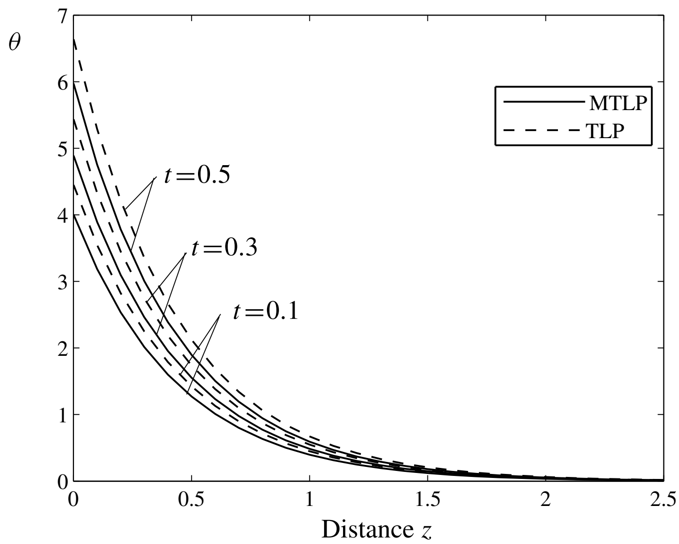

Figure 8. Variation of temperature distribution θ for a thermal load.

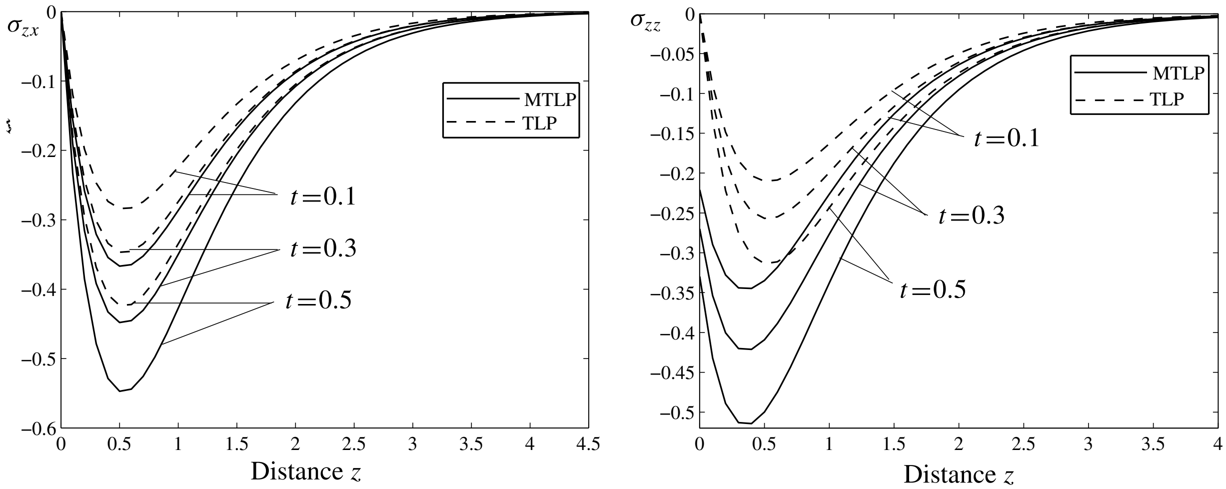

Figure 9. Variation of tangential stress σ_{zx} (left) and normal stress σ_{zz} (right) for a thermal load.

Figure 8 shows the variation of θ with distance z for MTLP and TLP for different values of t. We see that the trend of θ for all the cases is found to be similar. We notice that θ increases with time t while the presence of a magnetic field lowers the value of temperature. The effect of a magnetic field and time on temperature is prominent.

Figure 9 demonstrates the variation of σ_{zx} and σ_{zz} with distance z for MTLP and TLP for different values of t. We observe that σ_{zx} begins at zero value at $z = 0$ for all cases, then increases sharply to attain its highest value (in magnitude) at $z = 0.5$, and thereafter diminishes smoothly to zero. Hence, all the curves show similar trends. The magnitude of σ_{zx} for TLP is smaller than for MTLP; in both cases it increases with time. As for σ_{zz}, the curves for MTLP start at a negative value, while for TLP the curves begin with zero value. Normal stress shows significant sensitivity towards both factors. The magnitude of σ_{zz} decreases if we neglect the magnetic effect, and it increases with t.

8. Conclusions

The problem of investigating displacement components, temperature, and stress components in an infinite, homogeneous isotropic elastic half-space is studied in the purview of magnetothermoelasticity with a laser pulse. A normal mode analysis technique is employed to express the results mathematically. Theoretically obtained field variables are also exemplified through a specific model to present the results in graphical form.

The analysis of results permits some concluding remarks:

(1) It is clear from the figures that all the field variables have nonzero values only in the bounded region of space. Outside this region, the values vanish identically. This means that this outside region has not felt any thermal disturbance yet. Hence, all the results are in agreement with the generalized theory of thermoelasticity.

(2) The effect of the magnetic field is much pronounced in all the field variables except for the displacement component u and temperature field θ (however, it is still significant) in the case of a

thermal load. In case of a mechanical load being applied, the presence of a magnetic field decreases the magnitude of all the field variables, whereas it has both increasing and decreasing effects for thermal load.

(3) We see from the figures that the time t plays a significant role in all the field quantities. Changes in the value of time t cause significant changes in all the studied fields, and the magnitudes of all the field variables increase with an increase in time t.

(4) We can easily conclude from the figures that the curves for all the field variables show similar behaviors, for all the cases considered and for both type of loads applied.

(5) If the laser pulse effect is neglected, then the results are in agreement with [Das and Kanoria 2012] with appropriate modification in the boundary conditions.

(6) The temperature distribution θ shows a zero value for a mechanical load and a maximal value for a thermal load at the boundary of the surface, which is physically plausible and consistent with the theoretical boundary conditions of the problem.

The new model is employed in a homogeneous, isotropic thermoelastic medium as a new improvement in the field of thermoelasticity. The subject becomes more interesting because the use of a laser pulse with an extensive short duration or a very high heat flux has found numerous applications. The method used in this article is applicable to a wide range of problems in thermodynamics. By the obtained results, it is expected that the present model of equations will serve as more realistic and will provide motivation to investigate generalized magnetothermoelastic problems regarding laser pulse heat with high heat flux and/or short time duration.

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