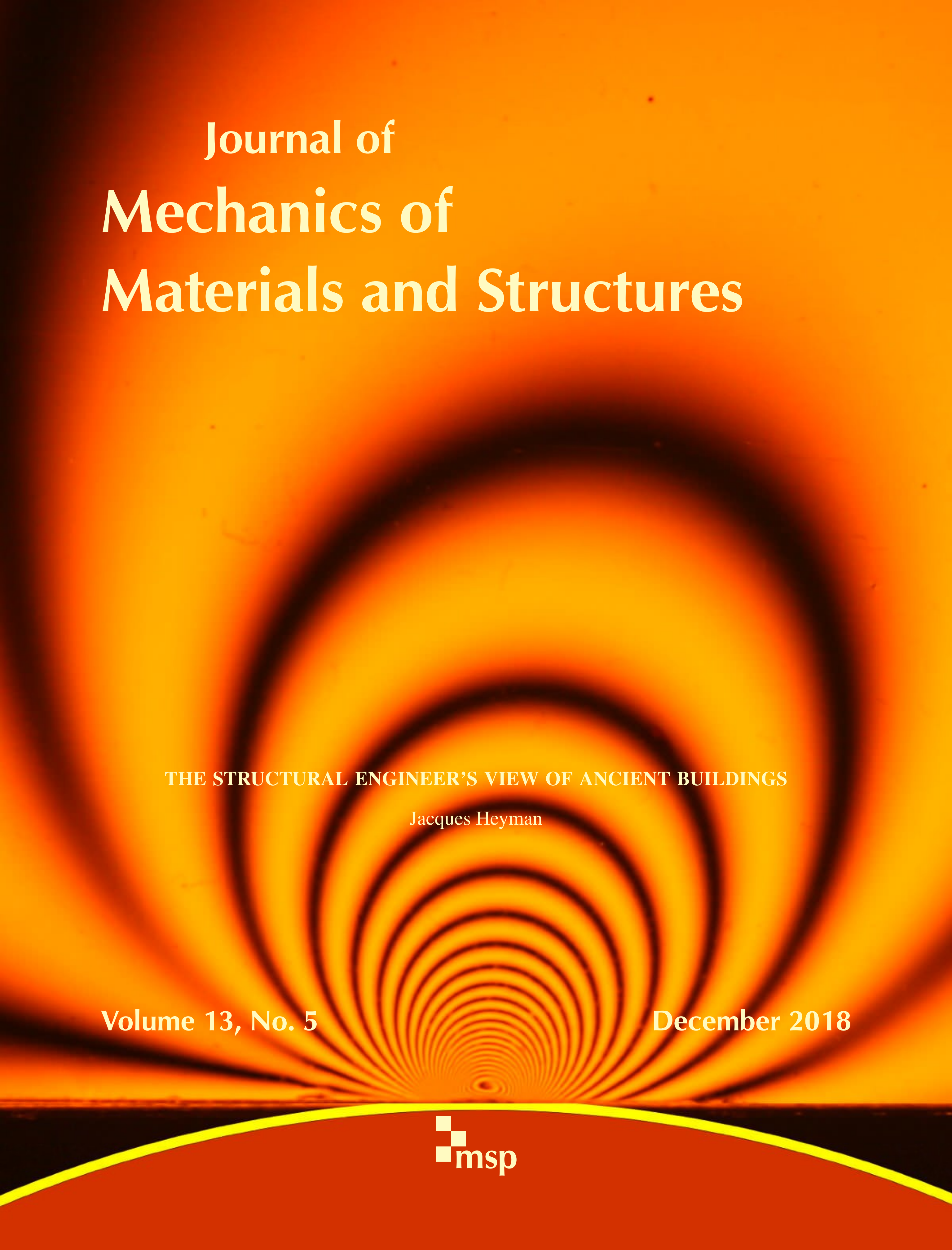

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Jacques Heyman

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# THE STRUCTURAL ENGINEER'S VIEW OF ANCIENT BUILDINGS

JACQUES HEYMAN

Engineers, called on to advise on the repair of an old building — a cathedral, say — will usually have learned their skills in the design of modern buildings using modern materials — steel and reinforced concrete. Care must be taken in transferring those skills to ancient structures. In particular, the engineer is used to provide precise answers (for example, values of stresses) in order to satisfy criteria imposed by accepted practice.

Such an engineer will not have had occasion to consider the fact that a precise description cannot be given for the behaviour of any structure, modern or ancient. The states of all structures are critically dependent on unknown, and unknowable, defects in construction, and, above all, on unknowable movements of the environment. The footing of a column in a steel skyscraper, and the foundation of a pier carrying a tower in a cathedral, will in reality not be in the precise locations assumed by the engineer, and even small "defects" of this sort can have a very large influence on the structural state of the buildings being analysed.

Although unequivocal and unique answers cannot be given to questions that arise in the analysis and repair of old buildings, it is at least possible to calculate states of equilibrium with which a structure is "comfortable". Although such states will not be observed in practice, their existence satisfies one of the basic theorems of plastic theory — if any one such state can be found, then this gives assurance that the structure is in fact safe. Further, it may be possible to calculate minimum and maximum values for important structural quantities.

## 1. The engineer and the surveyor

In England, national legislation requires every cathedral to appoint an architect, whose duties include the preparation of a report, every five years, on the state of the fabric of the church. Such architects are often called surveyors — the first surveyor of Westminster Abbey was Christopher Wren. Similarly, the 16,000 English parish churches each have an architect who performs similar duties. Such men and women (there have so far been four women surveyors for English cathedrals, and the architect for Cologne was best addressed as Dombaumeisterin) normally have immense experience of such buildings, and they are fully competent to devise remedial and repair work. Occasionally, however, they feel a need for technical advice from an engineer. Wren, despite being a scientist, a mathematician and an astronomer, had a rather poor grasp of structural mechanics, but he had the good fortune to have as a "partner" Robert Hooke, whose grasp of such matters was profound. A present-day cathedral is required to appoint an archaeologist, but there is no mention of a structural engineer.

The present-day architect will engage in dialogue with a modern structural engineer — one who, unlike Robert Hooke, has been taught in an accredited university, and whose qualifications have been accepted

*Keywords:* ancient buildings, limit analysis, geometrical factor of safety, masonry equilibrium.

by a professional institution as conferring chartered status. The practical expertise of such an engineer has been gained in the design of structures using modern materials such as steel, reinforced concrete, and aluminium, and also with traditional design in wood. Whatever the material, the route to acceptance of a design lies in the satisfaction of a "Code of Practice", established by some official body such as the British Standards Institution. These codes have clauses which have a common pattern. Calculations must be made, for example, of the values of stresses, which must not exceed some permissible value defined in the code, a value whose function might be to prevent the yield of a steel member, or to prevent the buckling of a column or the flange of a steel beam. Or again, deflections must not exceed certain specified limits, so that the function of the structure is not in some way impaired.

Traditional university courses equip the engineer with methods of calculation leading to numbers — values of stresses, for example — which will satisfy the clauses of the code. The calculations may sometimes be simple, and may occasionally involve some "advanced" mathematics. In either case it will not occur to the engineer that the calculations may not actually lead to a description of the state of the structure under examination. Similarly, the code does not acknowledge that such a description is not even possible.

A very simple example illustrates the difficulty. A table with four legs has a known weight placed on its top in a known position, and it will be supposed that the problem facing the structural engineer is the design of those four legs. To determine their size, it is evident that the engineer must calculate the forces in the legs. It turns out, technically, that this is a very difficult problem (and this is why the cathedral architect needs engineering advice). In simplistic terms, there are only three equations, and four leg forces must be determined. The matter is straightforward for a tripod; the addition of a fourth leg makes the table structurally redundant (hyperstatic). In order to solve the problem, further equations must be written which are no longer simple — they are indeed hideously complex. Much effort has been devoted, from the middle of the nineteenth century and through the twentieth century, to the solution of these equations, and ingenious and brilliant ways have been devised to obtain exact or remarkably accurate approximate solutions. It is this structural theory which forms the backbone of the university courses taught to the engineer.

The advent of the high speed computer has taken the drudgery out of the solution of the complex equations; apparently unequivocal values can now be obtained easily for the four leg forces. These values are indeed the correct solutions of the equations, but they are not in fact the values of the forces in the legs of the actual table. A nearly rigid table placed on the nearly rigid pavement outside a restaurant on a summer's night will, annoyingly for the diners and for the structural analyst, rock — one leg will be clear of the ground. If the leg is clear by only a fraction of a millimetre then the force in that leg is known, pace the computer printout — it must be zero. Moreover, it is not known in practice which leg is off the ground: a slight shift of the table will reduce the force in some other leg to zero. A friendly wine waiter will slice a cork diagonally into two wedges; one of these, placed under a leg will make the table more comfortable (anthropomorphically for itself, and also for the diners).

This simple and seemingly abstract example is a paradigm for the structural engineer; how can some meaningful statement be made about the forces in the legs if any one of the four may be clear of the ground, be supported by a rigid floor, or may be resting on a flexible foundation of unknown properties? The reason that the computer has given wrong answers to the problem is that the program has assumed, or rather the analyst using the program has assumed, that all four legs are in contact with a rigid floor. The modern engineer designing a steel frame has the same problem, although the problem is almost certainly unknown to him; how can a meaningful statement, that is a set of numbers (stresses, deflections and so on), be generated to satisfy the code, if the boundary conditions for the frame are unknown? None of the column footings, for example, will have been constructed exactly at the levels specified in the design, and, in any case, the footings will have suffered settlements of small, but unknown, extents. Even very small discrepancies between the design and the actual construction can lead to large variations in the force distribution in the frame.

It is the same sort of problem that faces the surveyor repairing a cathedral, and who wishes to make some more or less major intervention into the fabric. It may be necessary, for example, to take down and rebuild a flying buttress, in which case some temporary propping will be necessary — what loads will the temporary work have to bear? Or an external main buttress may be surmounted by a massive pinnacle: can this pinnacle be safely removed and then replaced? Or a high vault may need the replacement of several of its masonry components; what is the force distribution in the vault so that assurance can be given that a stone can be taken out and replaced? It is answers to such questions that the surveyor requires from the engineer.

Because the foundations of a cathedral have settled, since completion of construction, by unpredicted amounts, and since the masonry has distorted and cracked, then it is not possible, even if all these defects are surveyed and recorded, to calculate a unique structural state for the cathedral as a whole, or for individual elements such as flying buttresses. But it is of little help to say that exact and unequivocal answers cannot be given to the surveyor's questions. Some sort of answer must be found.

## 2. Plastic theory

Plastic theory has seemingly changed its objectives since it was formulated in the 1930s and 1940s. It was devised originally for the analysis and design of engineering structures constructed from ductile material, above all steel structures, but is in fact applicable to any "sensible" material (not cast iron or glass, for example). The original objectives are revealed in the early US term for the theory — limit design; the calculations referred to the evaluation of the greatest load a structure could carry. Thus the objective shifted from an examination of the way a structure might stand in comfort, to an examination of the way it might collapse, and different sorts of equations were involved.

Early emphasis was on the estimation of such collapse loads, and the calculations were assisted by the three basic theorems of the new theory. The unsafe theorem states that a calculated collapse load can, in general, never be achieved in practice. The safe theorem states that if a structure can achieve a "comfortable" state of equilibrium under a certain loading, then the structure can never collapse under that loading. The uniqueness theorem states that the largest safe loading is equal to the smallest unsafe loading.

The calculation of collapse loads (for steel frames, for example) is relatively straightforward, at least compared with conventional elastic analysis, and simple plastic designs were, and still are, made in this way, with a full awareness of their "unsafeness". But the surveyor of a cathedral is not, at least in the first instance, interested in the possible ways a flying buttress might collapse; the interest is in the value of thrust in the buttress with which it is "comfortable". Emphasis shifts to the use of the safe theorem, and in this context the term "plastic analysis" could well be replaced by the term "equilibrium analysis".

## 3. Equilibrium analysis

It is of interest that Coulomb, in the section on masonry arches in his 1773 “Essai”, does not attempt to calculate the “actual” state of such an arch. (Coulomb tackles the four classic structural engineering problems of the eighteenth century — the strength of beams, the strength of columns, the thrust of arches and the thrust of soil. This last topic is commonly regarded as laying the foundations of the science of soil mechanics). The full title of the paper contains the words “the application of the rules of maximum and minimum to some problems in architecture”; Coulomb was among the first to use the century-old calculus to the solution of some problems in civil engineering.

In a very simplified summary of Coulomb’s findings, his fundamental conclusion for the stability of a masonry arch is that the “line of thrust” must lie within the boundaries of the masonry of the arch. This is a purely geometrical statement, with no references to values of stress, although Coulomb is well aware that, exceptionally, crushing may occur. If the line of thrust should touch the surface of the arch, then a “hinge” will form between two voussoirs, and equations of statics will enable the determination of the value of the horizontal thrust which must be exerted by the arch on its abutments. The positions of such hinges are to be determined, and different arrangements of their locations will lead to different values of the abutment thrust; consideration of all possible values will give upper and lower bounds on the thrust. These are the maxima and minima of Coulomb’s title.

Coulomb makes no numerical calculations, nor indeed does he consider arches of any defined shape; the principles he establishes are general. Couplet, some fifty years earlier, does not discuss bounds on the value of thrust; instead, he considers a particular problem, that of a semicircular arch of constant ring thickness subject to its own weight. He shows (with a trivial numerical error) that such an arch must have a certain minimum ring thickness (just over ten percent of its radius), and his calculations are based upon the supposition that hinges form simultaneously in both the extrados and the intrados. He does not anticipate Coulomb’s approach, but, effectively, Couplet establishes the position of a thrust line which gives, simultaneously, Coulomb’s maximum and minimum — that is, his solution gives a unique value of thrust for the semicircular arch of least thickness.

This geometrical idea of smallest dimensions of a masonry structure makes it possible for the engineer to give more helpful answers to the surveyor. For example, a semicircular arch of the type studied by Couplet, but with a ring thickness of twenty percent of the radius (rather than the minimum ten percent), may be said to have a “geometrical factor of safety” of 2, and the engineer may well consider this to be appropriate for a particular masonry element. Equally, the engineer can provide numerical values for the least and greatest values of abutment thrust necessary to maintain stability of the arch.

As an example, a Gothic flying buttress often has a linear upper surface and a curved lower surface, and may be treated as an inclined arch conveying the thrust from the high vault of the church to the main external buttress outside the aisles. Since a straight line which lies completely within the masonry can be drawn from end to end of this flying buttress, and since this implies that the corresponding straight line of thrust could maintain a very large (theoretically infinite) outward force exerted by the high vault, then the strength of the flier would be determined only by the crushing strength of the masonry.

By contrast, should the external buttress move outwards, away from the main body of the church, then the flying buttress will attempt conversion in the usual way to a three-pin arch, by the formation of hinges between the stones of the masonry. Such a three-pin arch is subject to a calculable value of thrust at its ends, and this is the minimum value of thrust for stability of the flying buttress. The thrust exerted by the high vault must exceed this minimum.

Thus the engineer can in fact give some sort of answer to the surveyor as to the forces within a flying buttress — the buttress will be stable if the thrust from the high vault exceeds a calculable value, and will in fact be able to sustain much higher thrusts if the flier is of a standard Gothic type.

The same sort of calculation can be made for the high vault itself. Since the masonry of such vaults is, relatively speaking, fairly "thin", and since the compressive forces necessary for stability of the vault must lie within the masonry, the lines of action of such forces, and their values, can be calculated with some certainty. The engineer can now be of more positive help to the surveyor — if a flying buttress has to be taken down and rebuilt, then the temporary propping to the high vault can be designed, also with some certainty.

## 4. The schooling of a structural engineer

This discussion of the actual behaviour of a structure, whether constructed from masonry, from steel, or from any other suitable material, has, by implication, exposed a severe pedagogic problem. How is the professor of structural engineering to teach a theory, any theory, which purports to describe the state of a structure under given loading, when it is known that that state is critically dependent on factors, notably boundary conditions, which are in practice unknown? Moreover those factors are not only unknown — they are, by their nature, unknowable. They depend, for example, on unknown errors of manufacture and assembly, on the variability of the construction materials, and, above all, on random movements of the environment from which the structure is supported.

The professor must somehow make clear to the students that, no matter how counter-intuitive this may seem, there is no calculable actual state of a structure, since there is in fact no unique such state. However, if it is possible to determine a state which satisfies every requirement of internal and external equilibrium, and with which the structure is comfortable (for example, stresses below specified limits), then this is a demonstration that the structure is safe. Thus although it is not possible to calculate the value of the force actually transmitted by a flying buttress, it is at least possible to determine limits between which that value must lie.

The newly qualified structural engineer, called upon to design a more or less conventional structure in steel or concrete, will be forced to provide calculations leading to numbers which satisfy some prescriptive code. As has been noted, such codes do not conceive the possibility that there is not a calculable actual state of a structure — on the contrary, they imply that calculations (normally by implication elastic calculations) can be made so that various clauses of the code can be shown to be satisfied. Until there is a complete revolution in the way in which such codes are assembled, the professor will fail the students if they are not equipped with a mastery of nineteenth and twentieth century elastic theory of structures. The best the professor can do, having ruined the students' faith in elastic calculations, is to reassure them that such calculations do in fact give numbers describing states with which the structure is comfortable.

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## Guest editors' note

Jacques Heyman has been one of the main contributors to the development of structural theory in the 20th century. After graduating he joined the distinguished Cambridge Team directed by J. F. Baker that established the basis for plastic analysis of steel frame structures.

He soon took on great responsibilities in the team and in 1949 he obtained his doctoral degree. He then travelled to the United States to work with William Prager. Having spent a postdoctoral year at Brown University, from 1949 to 1950, he returned to that school for his first sabbatical year, from 1958 to 1959. Already early in his career he had acquired a fundamental theoretical base. In 1956 he coauthored with J. F. Baker and M. R. Horne the first book rigorously covering plastic theory for the analysis of steel frames: "The steel skeleton, II: Plastic behaviour and design". The book summarised all the work carried out by the Cambridge team over the previous decade and, for the first time in a book about analysis, the fundamental theorems of limit analysis are stated and applied for design purposes.

Plastic theory was developed for steel structures and, later, it was seen that it could be applied to reinforced concrete structures. Actually, plastic theory can be applied to any kind of structure that exhibits a ductile behaviour and does not have buckling problems. This fact that had been foreseen by some engineers since the beginning of the 20th century, was clearly and rigorously stated by Jacques Heyman. He is the first one to notice that the fundamental theorems meant a new paradigm that could be applied to all structures built with conventional materials. Jacques Heyman realised that the theorems could, also, be translated even for more heterogeneous materials such as stone or brick.

In 1966 he published his celebrated paper "The stone skeleton", which constituted a milestone in the development of the modern theory of masonry structures. This highly original and lucid article explains how plastic theory is adapted to the field of masonry architecture. Following a hint from Prager, he realised that, if certain properties are given to the material masonry, the fundamental theorems can be translated to suit this case of seemingly different structures. In the field of masonry, over 30 other articles and various books have followed his first article of 1966. In these publications he has applied the modern theory to the study of basic structural elements in masonry buildings (vaults, domes, flying buttresses, towers, spires, etc.).

In fact, his interpretations of Gothic theory close the debates about the structural behaviour of gothic vaults and cathedrals, ongoing since the mid-19th century, occupying the minds of academics such as Viollet-le-Duc, Ungewitter, Mohrmann, Abraham, etc. The deep meaning and the practical consequences of Jacques Heyman's discovery has not been yet really understood by many architects and engineers, and the present paper, which opens this special issue of JoMMS on the application of structural analysis to real historic masonry constructions, is addressed precisely to explain why.

For the reader's convenience, we add a selection of papers by Jacques Heyman that can help put the present paper into a larger perspective.

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JACQUES HEYMAN: jh10002@eng.cam.ac.uk
*Department of Engineering, University of Cambridge, Cambridge, United Kingdom*

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