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This study aims to investigate the effect of honeycomb structure filled with graded and nongraded polyurethane foam on reaction force during energy absorption under in-plane and out-of-plane loadings. Three types of aluminum AL5052 honeycomb structures without filling, with graded filling and with nongraded filling were manufactured and subjected to quasistatic compression loading. In order to investigate the effect of reaction force and energy absorption capacity, honeycomb cores with different densities were selected. Afterward, the behavior of honeycomb structures was numerically simulated in the ABAQUS software. The results of finite element analysis show that using foam filling in honeycomb structures increases energy absorption. The structures filled with graded foam, shows better performance with the rate of stiffness reduction from impact location compared to those filled with nongraded one. The energy absorption capacity of the structure under out-of-plane loading is much higher than in-plane loading, but its reaction force is very high. The results of empirical tests are greatly similar to that of numerical studies. Therefore, it is possible to use simulation in ABAQUS environment for solving more complex problems.

A list of symbols can be found on page 321.

1. Introduction

In the recent years according to the importance of energy absorption in different industries, impact absorbers, especially honeycomb structures have gained increased attention. Inspired from natural structures, it is possible to create optimized structures with higher energy absorption capabilities. Human and bird bone structures are among the most efficient natural impact absorbers. In bones, sponge-like structure leads to impact absorption and the damage lessening to joints [Koch 1917]. Another natural example of impact absorption is the banana structure and its peal which protects the soft core of the fruit from outside forces [Ali et al. 2008]. Due to the high strength to weight ratio and high energy absorption up to 70% of the initial height, honeycomb structures have gained increased importance in various industries, especially in the aerospace industry [Bitzer 1997]. Various structures and materials are used to manufacture honeycomb structures. The most common honeycomb structures are hexagonal structures made from aluminum and filled polymer foams. Polymer foams are among the cheapest materials and have characteristics such as heat resistance, waterproofing and soundproofing and are also cost-efficient.

Keywords: honeycomb structure, energy absorption, in-plane loading, out-of-plane loading, graded foam, ABAQUS, experimental test.

Some of the recent studies have investigated the mechanical properties of polymer foams. Various types of polymer foams have been investigated but polyurethane foams are less frequency used. Deshpande and Fleck [2001] investigated the behaviors of two types of PVC (polyvinyl chloride) foams for a wide range of tensile and compression strains. Seo et al. [2004] studied the effects of compression on small packages of hard polyurethane and polystyrene foams. They investigated foam characteristics at different densities using compression test and calculated stress-strain charts, Young modulus and Poisson coefficient of foams based on their density.

Some studies have investigated the filling of honeycomb structures with polymer foams. Akay and Hanna [1990] studied the behaviors of honeycomb structures and sandwich panels filled with foam using force-bending equipment and scanned foam samples using ultrasonic waves. Hanssen et al. [2000] created a program made from 96 tests for axial deformation and investigated the effects of foam density on energy absorption of structures with thin walls filled with aluminum foam. Suvorov and Dvorak [2005] investigated general deformation of sandwich structures under average impact speeds of 10 m/s and 20 m/s caused by the impact between boat and docks. They selected carbon-vinyl ester plates and used foam nucleus made from PVC H100 and flexible polyurethane foam between carbon-vinyl ester plates. They concluded that energy absorption is directly related to distance and initial velocity of a projectile. Song et al. [2010] investigated the dynamic compression behavior of three-dimensional foam structures with Voroni geometry using finite element analysis and empirical tests. They also investigated the effects of irregularities in cell structure, impact loading, relative compression and hardness strain on deformation of the structure. Galehdari et al. [2015] proposed an analytical equation for plateau stress using exponential hardening model in honeycomb structures. They also extracted the equation for specific energy absorption of honeycomb structures using locking strain and strain energy. In order to validate these equations, they simulated five different aluminum types with exponential hardening model in ABAQUS software. They also carried out an impact test on a graded honeycomb structure in order to validate the results of the numerical analysis. A comparison showed a good agreement between their numerical and empirical results. Also, Galehdari and Khodarahmi [2016] designed a graded honeycomb structure for shock absorption in helicopter seats during a crash-landing. They simulated this structure in ABAQUS environment. Alavi Nia and Sadeghi [2010] carried out an empirical study for investigating the response of empty and filled honeycomb structures under quasistatic loadings. They used five different empty and foam-filled honeycomb structures made from Al-5052-H39 alloy and concluded that use of foam filling can increase energy absorption up to 300%. Zarei Mahmoudabadi and Sadighi [2011] carried out an empirical investigation about the effect of filling honeycomb structures with polyurethane under out-of-plane conditions. They reported that increasing loading speed from quasistatic to dynamic increases stress level in the stress-strain chart of both empty and foam-filled honeycomb structures while filling under out-of-plane conditions has no significant effects on energy absorption. Mozafari et al. [2016] investigated foam-filled honeycomb sandwich panels under in-plane impact loading and analysed them by numerical methods. They used three different aluminum honeycombs filled with three different polyurethane foam and studied their energy absorption capacity by quasistatic compression test. Ebrahimi et al. [2018] have studied the energy absorption characterization of functionally graded foam (FGF) filled tubes under axial loading experimentally. The FGF tubes are filled axially by gradient layers of polyurethane foams of different densities. Finally, the results of experimental test show that an FGF filled tube has excellent energy absorption capacity compared to the ordinary uniform foam-filled with

the same weight. Shahravi et al. [2019] have designed a polyurethane foam-filled thin-walled aluminum grooved circular tubes. The tubes are shaped with the inner and the outer circular grooves at different positions along the axis. They investigated the effects of the grooves distance, tube diameter, grooves depth, foam density, and tube thickness on the crashworthiness parameters of grooved circular tubes. Also, Yu et al. [2018] studied static axial crushing and energy absorption of density-graded aluminum foam-filled square metal columns experimentally and theoretically. It was shown that the density-graded aluminum foad-filled square metal column is a novel topological structure with higher energy absorption, higher load-carrying capacity and much higher crushing force efficiency.

Regarding the above-mentioned researches, except for tubes, the graded foam has not been used in honeycomb structures under in-plane or out-of-plane loading in honeycomb energy absorbers. The current study aims to investigate aluminum honeycomb structures filled with graded and nongraded polyurethane foams.

2. Problem definition

Honeycomb structures have better performance when they are subjected to out-of-plane loading direction. In some cases, such as impact absorbers for protecting an occupant against the crash, impacts might occur from in-plane direction. Therefore, it is important to investigate the behavior of honeycomb structures for in-plane loading. Two types of loading are shown in Figure 1.

The utilized honeycomb structure is made from 16 separate rows with 10 cells in each row. The dimensions of the aluminum honeycomb structure are determined based on the MIL-C-7438G standard (Table 1).

The polyurethane is a closed-cell material created from isocyanate and polyol. This foam has different densities depending on the ratio of isocyanate and polyol which are mixed together under pressure and heat. The mixture then shows a volume increase of 20 to 30 times in a few seconds creating a compact, uniform foam structure with any desirable thickness (Figure 2, left).



Figure 1. Loading conditions: out-of-plane (left) and in-plane (right).

Foil thickness (mm)	Cell size (mm)	Height (mm)	Width (mm)	Length (mm)
0.018	3.175	12.7	50	50

 Table 1. Aluminum honeycomb structure dimensions.



Figure 2. Left: polyol and isocyanate combined for the production of polyurethane foam. Right: Santam test machine.

Polyol weight (g)	Isocyanate weight (g)	Polyol to isocyanate weight composition ratio	Density (kg/m ³)
11	11	1	70
9	13	0.69	80
7	14	0.5	90
5	15	0.33	100

Table 2. The weight of ingredients combined with polyurethane foam.

In order to determine the mechanical behavior of foams used to fill the structures, these foams were subjected to compression test. The Santam (STM-150) equipment was used for compression test of polyurethane foams (Figure 2, right).

To this end, foams were cut with dimensions of 24.5 mm \times 70 mm \times 70 mm in accordance with the ASTM C365 standard. The polyol and isocyanate parts were mixed with ratios presented in Table 2 to produce foam with densities of 70 kg/m³, 80 kg/m³, 90 kg/m³ and 100 kg/m³. The density of the foam increases with increase in the ratio of isocyanate to polyol. For example, a ratio of 1 : 1 (isocyanate : polyol) has a density of 70 kg/m³ while a ratio of 2 : 1 has a density of 90 kg/m³ and a ratio of 3 : 1 leads to a density of 100 kg/m³.

3. Numerical simulation

In order to investigate the energy absorption of different honeycomb structures, a finite element simulation was performed in ABAQUS software. The rows in the structure are made from Al 5052-O alloy with a density of 2680 kg/m^3 , elasticity modulus of 70.3 GPa and the Poisson ratio of 0.33. Mechanical properties of Al 5052-O alloy are determined using the ASTM B209M standard in which yield and ultimate strength are equal to 65 MPa and ($170 \sim 215$) MPa respectively. A finite element model of the structure under in-plane loading is shown in Figure 3.



Figure 3. Finite element model of the structure under in-plane loading.

Movement is applied through a rigid plate placed above the honeycomb structure. This structure is attached to another rigid plate on the bottom side. The force is applied at the reference point of the rigid plate and all degrees of freedom of this plate except in moving direction are the constraint. All degrees of freedom of the lower rigid plate are also fixed. A four-node shell S4R element was used for meshing of honeycomb structure while two-line, four-node R3D4 element was used for meshing of the above and below rigid plate plates. For the S4R element, shear strain is assumed to be constant along the thickness. Since the structure has regular geometry and is made from thin metal sheets, a four-node shell element is used. Kinematic and penalty surfaces to surface interaction were used for contact between the upper plate and lower one and the structure, respectively.

In order to investigate the effects of filling honeycomb structure with foam, light-weight polyurethane foam was used as the filling phase. Material properties including density, elastic behavior, and crushable foam were defined in the material properties module. Foam with a density of 100 kg/m^3 , elasticity modulus of 10.1 MPa and Poisson coefficient of zero was selected as nongraded foam. To model the foam crushable in ABAQUS environment, *h* and ν_{ρ} parameters must be defined. The first parameter, *h*, is the ratio of initial Mises to initial hydrostatic compression while the second parameter, ν_{ρ} , is the plastic Poisson coefficient. The final value of the compression yield stress ratio was equal to 1 while the plastic Poisson's ratio was 0. The assumption of full adhesion was used for determining the contact between foam and honeycomb structure. A linear, six-node C3D8R element was used for meshing of foams.

In order to investigate the structures with graded foams, different foam densities with properties shown in Table 2 were used. The order of graded foam in the structure is shown in Figure 4.

For out-of-plane loading on the aluminum honeycomb structure, the in-plane model was rotated for 90 degrees (Figure 5).

4. Experimental tests

In order to carry out an empirical investigation on the behavior of honeycomb structures, some test specimens were prepared for each of the five models. Al-5052-O aluminum honeycomb sample was



Figure 4. Graded foam arrangement order.



Figure 5. Out-of-plane loading on honeycomb structure model.

purchased from Hexcel Co. with dimensions of $50 \text{ mm} \times 50 \text{ mm}$ (specifications are shown in Table 1). Then, this structure was used to prepare three types of structures without foam (empty structures), a structure filled with nongraded foam and a structure filled with graded foam. Names of test specimen which are subjected under quasistatic loadings are listed in Table 3.

The test specimens were subjected to compression test with the loading rate 2 mm/min using Santam machine.

Loading type	Sample type	Sample name
In-plane	Without foam With nongraded foam With graded foam	H-NF-I-S H-FN-I-S H-FG-I-S
Out-of-plane	Without foam With nongraded foam	H-NF-O-S H-FN-O-S

Table 3. Names of the test specimen.

5. Results and discussion

The stress-strain graph of polyurethane foams resulted from compression tests for different densities is shown in Figure 6. Based on the compression test results, mechanical properties of polyurethane foam are presented in Table 4.

5.1. *Numerical results.* Numerical analyses were carried out using Dynamic/Explicit solver and results were presented in various graphs. One of the important parameters in energy absorbers is the magnitude



Figure 6. Stress-strain graph of polyurethane foam for different densities.

Elasticity Module, E (MPa)	Yield Stress, σ_y (MPa)	Density, ρ (kg/m ³)
3.3	0.21	70
5.1	0.5	80
5.5	0.7	90
10.1	0.9	100

Table 4. Mechanical echanical properties of the polyurethane foam.

of the structure's reaction force, as well as magnitude and duration of energy absorption. A suitable absorber needs to absorb the maximum amount of energy during the highest possible time with the minimum reaction force. Kinetic energy applied to the structure is transformed into its internal energy which is the sum of strain energy and plastic deformation energy. In order to achieve more accurate numerical results, mesh dependency is checked for all numerical simulations. As a sample, the load-displacement graph of nongraded foam filled honeycomb structure under in-plane quasistatic loading for different element sizes is shown in Figure 7.

According to Figure 7, the results have proper convergence for three sizes of element. So, 0.003 m element size is selected for numerical analysis. The displacement contours for in-plane quasi-loading of different structures are shown in Figure 8.

The deformation pattern of all three structures is X-mode. However, the X-mode deformation is wider for empty structure and its center is located at the structure's center of mass. However, in the nongraded foam-filled structure, the center of X is located higher than the structure's center and is again further



Figure 7. Load-displacement graph of nongraded foam-filled honeycomb structure under in-plane quasistatic loading for different element sizes.



Figure 8. Structure's deformation contour for in-plane quasistatic loading for without foam (left), nongraded foam (middle) and graded foam models (right).

away from the structure's center in graded foam-filled structure. Deformation contours acquired from the out-of-plane quasistatic simulation for both structures are presented in Figure 9.

The results of numerical simulation for five models (Table 3) under quasistatic static loads are shown in different graphs. In these graphs, the reaction on force is measured at the lower plate and displacement is measured at the upper plate. The force-displacement graph for in-plane quasistatic simulation is presented in Figure 10 (left).

For in-plane quasistatic loadings of the empty structure, force increases in a smooth pattern while this increase in the nongraded foam-filled structure is irregular and step by step and shows a larger increase. On the other hand, this increase in the graded foam-filled structure shows smaller variations. In quasistatic, in-plane loading, the behaviors of all three structures are close to one another but the foam-filled structures absorb more energy and have a higher reaction force. The area under the graph for the graded foam-filled structure is also lower than nongraded structure. Force-displacement graph for quasistatic simulation in out-of-plane conditions is shown in Figure 10 (right).



Figure 9. Structure's deformation contour for out-of-plane quasistatic loading for without foam (left) and nongraded foam models (right).



Figure 10. Left: the in-plane quasistatic reaction force-displacement graph. Right: the out-of-plane quasistatic force-displacement graph.

For out-of-plane quasistatic loading, the empty structure shows a sudden increase in force and small oscillations at the end. The nongraded foam-filled structure also shows a sudden increase in force but experiences larger oscillations at the end. For this case, behaviors of both structures are close to one another but the foam-filled structure causes higher reaction forces.

5.2. *Experimental results.* The final deformation modes of three structures for in-plane quasistatic tests are shown in Figure 11.

The empty structure shows X-shaped deformation mode under in-plane quasistatic loads while the nongraded foam-filled structure shows uniform deformation and the graded foam-filled structure has only expanded on the lower side which shows a distribution of lower force toward the lower surface. This can be one of the advantages of this structure. The final deformation modes of structures for out-of-plane quasistatic tests are shown in Figure 12.

It can be seen that empty structure has wrinkled under out-of-plane loadings while the nongraded foam-filled structure shows uniform deformation. Force-displacement graph of in-plane quasistatic test for different structures is presented in Figure 13 (left).

The applied force in quasistatic loading for empty structure increases irregularly and step by step. This increase in the nongraded foam-filled structure is around 36 times higher and shows a harmonic increase but with a sharp slope. In graded form-filled structure, this value is almost half of nongraded structure



Figure 11. Final deformation modes of three structures for in-plane quasistatic tests for without foam (left), nongraded graded foam (middle) and graded foam models (right).





Figure 12. Final deformation modes of structures for out-of-plane quasistatic tests for without foam (left) and nongraded foam models (right).



Figure 13. Left: in-plane quasistatic force-displacement graph. Right: out-of-plane quasistatic force-displacement graph.

and this increase is significantly slower and occurs in four steps. This is due to using four different foams in this structure. Reaction force in the nongraded foam-filled structure is the largest and the force for the structure with graded foam is around half of the nongraded structure. However, the increase in the graded structure is slower which is one of the advantages of this structure. Force-displacement displacement graph for the out-of-plane quasistatic static test for different structures is shown in Figure 13 (right).

The force applied during quasistatic static loading for empty structure shows a 250 times increase compared to in-plane conditions and then continues with an attenuating peak. In the structure with nongraded foam filling, this increase is 3 times of empty structure but this increase had a lower slope. The force transferred in structure with nongraded foam is significantly higher and increases with a lower slope. This means that the foam-filled structure not only absorbs more energy but also transfers a lower amount of force.

5.3. *Comparison between numerical and experimental results.* Figure 14 shows a comparison between force-displacement results of in-plane quasistatic tests and the results of numerical simulation for different models.

The empirical and numerical results for the empty structure are almost similar to one another. The difference between results in the nongraded foam-filled structure is even smaller. The difference between empirical and numerical results for the graded foam-filled structure is also small. In general, the results of numerical and empirical studies show good agreement with each other. So, the numerical simulation method is verified and applicable to other models. The comparison between out-of-plane quasistatic empirical tests and numerical results for different models are presented in Figure 15.

These results are almost identical for empty structure. Addition of foam filling causes a small difference between numerical and empirical results. However, the results still have good agreement with one another.

5.4. *Comparison between in-plane and out-of-plane loadings.* The results of in-plane and out-of-plane plane loadings for different models are compared as shown in Figure 16.



Figure 14. Comparison between in-plane loading for experimental and numerical results: empty structure (left), nongraded foam-filled structure (middle) and graded foam-filled structure (right).



Figure 15. Comparison between the numerical and experimental results for out-of-plane loading for without foam (left) and nongraded foam models (right).



Figure 16. Comparison between the results of in-plane and out-of-plane loadings for without foam (left) and nongraded foam models (right).

In the out-of-plane loading, hallow structure applies a significantly higher force but shows a very high reaction force at the initial impact time while the nongraded foam-filled structure applies a force three times higher than the empty structure but reaction force reaches maximum magnitude at the later time. For the in-plane conditions, despite the fact that structure applies a significantly smaller force, this force decreases slowly overtime. This is more obvious in nongraded graded, foam-filled structure.

6. Conclusion

For in-plane loading, filling the structure with foam means that structure can absorb a higher amount of energy but simultaneously show a higher reaction force. Using graded filling in a way that structure stiffness increases downward and away from the location of the applied force increases energy absorption time and force transfer. This means that compared to structures with nongraded foam filling, graded structures absorb a lower amount of energy but this energy is absorbed with high reaction force during a longer time. The variation of the foam density is very important in energy absorption. It's better to increase the density of foam from the place of the impact load through the outer side of the energy absorber. For the structures with graded foam, the injury will be less regarding the structure with uniform density. When the energy absorber is used to protect a human, uniform reaction force and its transfer time are more important than the amount of absorbed energy. Therefore the application of honeycomb structures filled with graded foam is recommended. In out-of-plane loading, a foam-filled structure absorbs a higher amount of energy and also shows a milder reaction force.

In general, the results of the numerical simulation are close to that of empirical tests except in some parts of various graphs which require further investigation. The reasons for differences between numerical and empirical results can be manufacturing conditions and environmental factors. This means that numerical simulation results for behaviors of all five structures were validated using empirical tests. So, the numerical simulation method in ABAQUS software can be used to simulate the energy absorption of different honeycomb structures.

List of symbols

force (N)
graded foam-filled structure
nongraded foam-filled structure
empty structure

U displacement (mm)

- ε strain
- ρ density (kg/m³)
- σ stress (MPa)

[Akay and Hanna 1990] M. Akay and R. Hanna, "A comparison of honeycomb-core and foam-core carbon-fibre/epoxy sand-wich panels", *Composites* **21**:4 (1990), 325–331.

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