Journal of
Mechanics of
Materials and Structures

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Volume 14, No. 3 May 2019

msp

JOURNAL OF MECHANICS OF MATERIALS AND STRUCTURES
Vol. 14, No. 3, 2019
dx.doi.org/10.2140/jomms.2019.14.393

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THERMOELASTIC FRACTURE INITIATION: THE ROLE OF RELAXATION AND CONVECTION

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An isotropic, thermoelastic solid is at rest at uniform (absolute) temperature, and contains a semi-infinite, closed plane crack. Thermal relaxation governs, and crack surfaces are subject to convection. In-plane and compressive point forces, applied to each face of the crack initiate transient 3D extension. Wiener–Hopf equations are formulated in integral transform space from expressions whose inverses are dynamically similar and valid for short times. The solutions, upon inversion, are subjected to the dynamic energy release rate criteria, with kinetic energy included. A differential equation for crack edge contour is produced, and demonstrates that a certain type of point-force time variation can indeed cause a constant extension rate. Calculations for the pure compression case show that variation in crack growth rate with convection is not necessarily monotonic. A finite measure of crack edge thermal response for pure compression is provided by the temperature norm. Calculations indicate even greater sensitivity to thermal convection.

Introduction

Crack edge location in a transient 3D study is defined by a (possibly non-rectilinear) contour in the crack plane. As an illustration, the semi-infinite, planar crack in an unbounded thermoelastic solid is treated in [Brock 2017]. Fracture is driven by mixed-mode, point force loading on the crack faces, and crack extension rate is constant and well below Rayleigh and body-wave speed. Fracture initiation is the focus, and is governed by dynamic energy release rate [Freund 1972; 1990] with kinetic energy included [Gdoutos 1993]. Therefore:

- Thermal relaxation [Ignaczak and Ostoja-Starzewski 2010] can be important.
- Asymptotic forms of the governing equations for thermal relaxation are viable.
- Only knowledge of solution behavior near the crack edge is required.

The possibility that discontinuities in temperature and heat flux, as well as in displacement, occur is considered. Therefore analysis is based on the related, but unmixed, boundary-value problem of such discontinuities prescribed on a plane in a crack-free solid. The analytical solution in transform space is obtained and asymptotic forms whose inverses are valid for short times used to address the fracture problem. The displacement discontinuity corresponding to crack extension direction can be resolved in crack-opening, (in-plane) sliding and (in-plane) tearing modes. The fracture problem can thus be reduced to four equations of the Wiener–Hopf type [Morse and Feshbach 1953] and two of them are coupled. Solutions to the equations are then inverted, and subjected to the dynamic energy release rate. A nonlinear, first-order differential equation for the (dimensionless) speed parameter that defines the crack

Keywords: thermoelastic, relaxation, transient, fracture, discontinuity, convection.

edge contour results. Study shows that, in particular, a certain point-force loading history produces a parameter that can vary with direction, but is time-independent.

It is noted that crack-surface thermal convection is not addressed in [Brock 2017]. Moreover the restriction that crack extension rates be "well below" critical values simplified analysis of the Wiener–Hopf equations, but is not required for their solution. This paper therefore also addresses the situation treated in [Brock 2017], but crack extension rate is only required to be constant and subcritical, and thermal convection is possible. Two sets of assumptions are now listed explicitly. If loading is only in-plane:

- Crack surface friction can be neglected.
- Across the crack plane, temperature is continuous and heat flux is allowed.

With or without in-plane loading, if compression loading is present:

- Thermal convection, subject to thermal relaxation, occurs on the crack faces.
- A temperature discontinuity between crack faces can exist.
- Net heat flux across the crack itself cannot occur.

It will be seen that the latter assumption set gives, in contrast to [Brock 2017], three uncoupled sets of equations in integral transform space. A single equation for the displacement discontinuity due to in-plane tearing comprises one set. Two coupled equations for crack opening involve displacement discontinuity and discontinuity in temperature of the two crack faces, and comprise the second set. The third set consists of two coupled equations for in-plane sliding that involve displacement discontinuity and the average of the two crack face temperatures. Equations are of the Wiener–Hopf type.

Problem statement

An unbounded, thermoelastic solid is at rest for time $t \leq 0$ and uniform (absolute) temperature T_0 prevails. In terms of Cartesian basis $\boldsymbol{x}_0 = \boldsymbol{x}_0(x_k^0)$, $k = (1, 2, 3)$ the closed, plane crack occupies region A_C ($x_3^0 = 0, x_1^0 = 0$), with rectilinear boundary C $(x_1^0, x_3^0) = 0$. Shear and compressive point forces appear for $t > 0$ on both crack faces ($x_1^0 = 0-$, $x_2^0 = 0$, $x_3^0 = 0\pm$). Brittle fracture is instantaneous, and the crack extends outward from $\boldsymbol{x}_0 = 0$. The crack now occupies region $A_C + \delta A$. Boundary C is assumed to now include a concave bulge centered on the point-force sites:

$$\sqrt{(x_1^0)^2 + (x_2^0)^2} = l(\psi, t), \qquad l(\psi, t) = V(\psi)t, \tag{1a}$$

$$0 < V < V^*, \qquad \psi = \tan^{-1}\frac{x_2^0}{x_1^0}\ (|\psi| < \pi/2). \tag{1b}$$

Equation (1) implies a dynamically similar fracture process, and (speed parameter) V is subcritical, i.e., lies below V^*, the minimum of Raleigh and body wave speeds.

Displacement $\boldsymbol{u}(u_k)$, traction $\boldsymbol{T}(\sigma_{ik})$ and Θ, the change in temperature from T_0, are field variables. For the solid with thermal relaxation governed by the Lord–Shulman (LS) model [Lord and Shulman

1967; Ignaczak and Ostoja-Starzewski 2010]:

$$\nabla\cdot\boldsymbol{T}-\rho D_0^2\boldsymbol{u}=0, \tag{2a}$$

$$\big(k_T\nabla^2-\rho C_E D_0 \mathrm{P}_0\big)\Theta+\mu\alpha_D T_0 D_0 \mathrm{P}_0(\nabla\cdot\boldsymbol{u})=0, \tag{2b}$$

$$\frac{1}{\mu}\boldsymbol{T}=\Big[\frac{2\nu}{1-2\nu}(\nabla\cdot\boldsymbol{u})\boldsymbol{1}-\alpha_D\Theta\Big]+\nabla\boldsymbol{u}+\boldsymbol{u}\nabla=0, \tag{2c}$$

$$\mathrm{P}_0=1+t_0 D_0. \tag{2d}$$

In (2) Θ and components (u_k, σ_{ik}) are functions of $(\boldsymbol{x}_0, t)$, and $(\nabla, \nabla^2, \boldsymbol{1})$ respectively are gradient and Laplacian operators and identity tensor. Symbol $(D_0 f, \dot{f})$ signifies time differentiation in basis $\boldsymbol{x}_0$, and t_0 is thermal relaxation time. It is noted that (2) describes the classical (Fourier model) solid [Boley and Weiner 1960] when $\mathrm{P}_0=1$. Constants (μ, ρ, ν) represent shear modulus, mass density and Poisson's ratio, and $(k_\mathrm{T}, C_E, \alpha_D)$ are thermal conductivity, specific heat at constant strain, and coefficient of (volumetric) thermal expansion. Homogeneity of (2a) and (2b) reflects the absence of thermal and mechanical body forces. In particular, the solid contains no internal heat source or sink.

For convenience temporal behavior is described in terms of variable $s=V_R t$, operator $D_0=V_R D$ and parameters:

$$\mathrm{P}_0=1+h_0 D, \tag{3a}$$

$$V_R=\sqrt{\frac{\mu}{\rho}},\qquad V_D=C_D V_R,\qquad C_D=\sqrt{2\frac{1-\nu}{1-2\nu}},\qquad \varepsilon=\frac{\mu T_0}{\rho C_E}\alpha_D^2, \tag{3b}$$

$$h=\frac{k_\mathrm{T}}{C_E\sqrt{\mu\rho}},\qquad h_0=V_R t_0. \tag{3c}$$

In (3) ε is the dimensionless thermal coupling constant, and (h, h_0) are thermoelastic characteristic lengths. Symbols (V_R, V_D) are, respectively, rotational speed and isothermal dilatational speed. In regard to subcritical speed, it will be seen that subsonic ($< V_R$) Rayleigh speeds exist. These depend on both material properties and the nature of the point forces. Equations (2a) and (2b) can be partially uncoupled and for $s>0$ give, in view of (3)

$$\boldsymbol{u}=\boldsymbol{u}_R+\boldsymbol{u}_D, \tag{4a}$$

$$(\nabla^2-D^2)\boldsymbol{u}_R=0,\qquad \nabla\cdot\boldsymbol{u}_R=0, \tag{4b}$$

$$\big(c_D^2\nabla^2-D^2\big)\boldsymbol{u}_D-\alpha_D\nabla\Theta=0, \tag{4c}$$

$$\big[\big(c_D^2\nabla^2-D^2\big)\big(h\nabla^2-D\mathrm{P}_0\big)-\varepsilon D\mathrm{P}_0\nabla^2\big](\boldsymbol{u}_D, \Theta)=0. \tag{4d}$$

For $x_3^0=0\pm$, $(x_1^0, x_2^0)\in A_C+\delta A$ $(s>0)$:

$$\sigma_{3k}=-F_k\,\delta(x_1^0)\,\delta(x_2^0),\qquad \partial_3\Theta=\mp\chi\mathrm{P}_0\Theta. \tag{5a}$$

For $x_3^0=0$, $(x_1^0, x_2^0)\notin A_C+\delta A$ $(s>0)$:

$$[u_k]=[\sigma_{3k}]=[\Theta]=[\partial_3\Theta]=0. \tag{5b}$$

In (5a) and (5b) $k=(1,2,3)$ and $\partial_k f=\partial f/\partial x_k^0$. Force F_k is a positive constant and χ is the (positive) convection constant, with dimensions of inverse length. Symbol δ in (5a) denotes Dirac function, and $[f] = f^{(+)} - f^{(-)}$ where $f^{(\pm)} = f(x_1^0, x_2^0, 0\pm, s)$. Moreover $[u_k]$ must vanish continuously on C, but $[\Theta]$ can exhibit (integrable) singular behavior. It is noted that two other relations also arise for $x_3^0 = 0$, $(x_1^0, x_2^0) \in A_C + \delta A$ $(s > 0)$:

$$[\partial_3\Theta] + 2\chi \mathrm{P}_0\langle\Theta\rangle = 0, \qquad \langle\partial_3\Theta\rangle + \chi \mathrm{P}_0[\Theta] = 0. \tag{5c}$$

In (5c) $\langle f\rangle = \frac{1}{2}(f^{(+)} + f^{(-)})$ is the average taken over $(x_1^0, x_2^0) \in A_C + \delta A$. For $s \le 0$ $(\boldsymbol{u}, \boldsymbol{T}, \Theta) \equiv 0$, and for finite $s > 0$ $(\boldsymbol{u}, \boldsymbol{T}, \Theta)$ must be bounded as $|\boldsymbol{x}_0| \to \infty$.

Discontinuity problem

A common practice for solving crack problems is to represent the relative motion of crack faces as unknown discontinuities in displacement; see, e.g., [Barber 1992]. To implement that procedure here, a more general problem is considered: The unbounded solid is again at rest at uniform (absolute) temperature T_0 but for $(x_3^0 = 0, s > 0)$ discontinuities $([u_k], [\Theta], [\partial_3\Theta])$ are imposed. For $(x_1^0, x_2^0) \notin A_C + \delta A$ and $(x_1^0, x_2^0) \notin A_C + \delta A$, respectively the discontinuities vanish and are continuous functions of (x_1^0, x_2^0, s). They vanish for $s \le 0$, and are bounded in $A_C + \delta A$ for $\sqrt{(x_1^0)^2 + (x_2^0)^2} \to \infty$ $(s > 0)$. Therefore, as in the crack problem, $(\boldsymbol{u}, \boldsymbol{T}, \Theta) \equiv 0$ for $s \le 0$, and are bounded as $|\boldsymbol{x}_0| \to \infty$ for finite $s > 0$.

Transform solution

An effective procedure (e.g., [Brock and Achenbach 1973]) for 2D transient study of semi-infinite crack extension at constant speed employs coordinates that translate with the crack edge, and unilateral temporal and bilateral spatial integral transform [Sneddon 1972]. In view of (1) a translating basis $\boldsymbol{x}$ is defined for $|\psi| < \pi/2$ as

$$x_1 = x_1^0 - [c(\psi)\cos\psi]\,s, \qquad x_2 = x_2^0 - [c(\psi)\sin\psi]\,s, \qquad x_3 = x_3^0, \tag{6a}$$

$$c(\psi) = \frac{V(\psi)}{V_S}, \qquad Df = \partial_S f - c(\psi)(\partial_1 f\cos\psi + \partial_2 f\sin\psi), \tag{6b}$$

$$\partial_S = \frac{\partial f}{\partial s}, \qquad \partial_k f = \frac{\partial f}{\partial x_k} \quad k = (1, 2, 3). \tag{6c}$$

The temporal Laplace transform operation is

$$L(f) = \hat{f} = \int f(s)\exp(-ps)\,ds. \tag{7a}$$

Integration is over positive real s and $\mathrm{Re}(p) > 0$. A double spatial integral transform and inversion, respectively, can be defined [Sneddon 1972] by

$$\tilde{f}(p, q_1, q_2) = \iint \hat{f}(p, x_1, x_2)\exp[-p(q_1x_1 + q_2x_2)]\,dx_1\,dx_2, \tag{7b}$$

$$\hat{f}(p, x_1, x_2) = \left(\frac{P}{2\pi i}\right)^2 \iint \tilde{f}(p, q_1, q_2)\exp[p(q_1x_1 + q_2x_2)]\,dq_1\,dq_2. \tag{7c}$$

Integration in (7b) is over real (x_1, x_2); integration in (7c) is along the imaginary (q_1, q_2)-axes. It is noted that $(\boldsymbol{x}, s)$ have dimensions of length, p has dimensions of inverse length, and (q_1, q_2) are dimensionless. Because (1) involves a speed that varies with direction, application of (7a) and (7b) to (2)–(4) and discontinuity restraints for $(x_3^0, x_3) = 0$ is complicated. Despite use of ψ the discontinuity problem is not axially symmetric. However, 3D studies of sliding and rolling contact [Brock 2012] and crack growth [Brock 2017] suggest transformations:

$$\operatorname{Im}(q_1) = \operatorname{Im}(q)\cos\psi, \qquad \operatorname{Im}(q_2) = \operatorname{Im}(q)\sin\psi, \tag{8a}$$

$$x_1 = x\cos\psi, \qquad x_2 = x\sin\psi. \tag{8b}$$

Here $\operatorname{Re}(q) = 0+$, $|\operatorname{Im}(q)|$, $|x| < 0$ and $|\psi| < \pi/2$. Parameters (x, ψ) and (q, ψ) resemble quasi polar coordinates, i.e.,

$$dx_1\, dx_2 = |x|\, dx\, d\psi, \qquad dq_1\, dq_2 = |q|\, dq\, d\psi. \tag{8c}$$

The uncoupling effect of (8) leads to the combination

$$\tilde{f}(p, q_1, q_2) \to \bar{f}(p, q, \psi), \tag{9a}$$

$$\hat{f}(p, x, \psi) = -\frac{p^2}{2\pi}\int \frac{|q|}{q}\,\bar{f}(p, q, \psi)\exp(pqx)\, dq. \tag{9b}$$

Integration is along the positive $\big(\operatorname{Re}(q) = 0+\big)$ side of the $\operatorname{Im}(q)$-axis.

In view of (6)–(8) and (9a), equation (4) gives a corresponding set in transform space by making formal substitutions:

$$\nabla \to (pq\cos\psi, pq\sin\psi, \partial_3), \qquad D \to pQ, \qquad \nabla^2 \to \partial_3^2 + p^2 q^2, \tag{10a}$$

$$\mathrm{P}_0 \to \bar{\mathrm{P}}_0 = 1 + h_0 pQ, \tag{10b}$$

$$Q = 1 - cq. \tag{10c}$$

Set elements that correspond to (4b)–(4d) are homogeneous, ordinary differential equations in x_3, with characteristic functions $pB(q)$ and $pA_\pm(p, q)$:

$$B(q) = \sqrt{Q^2 - q^2}, \tag{11a}$$

$$A_\pm(p, q) = \sqrt{\left(\frac{2Q}{\Gamma_\pm \pm \Gamma_-}\right)^2 - q^2}, \tag{11b}$$

$$\Gamma_\pm = \sqrt{\left(c_D \pm \sqrt{hpQ/\bar{\mathrm{P}}_0}\right)^2 + \varepsilon}. \tag{11c}$$

The solutions to the differential equations are

$$\bar{\boldsymbol{u}}_R = \left[U_1^{(\pm)}, U_2^{(\pm)}, (\pm)\frac{q}{B}\big(U_1^{(\pm)}\cos\psi + U_2^{(\pm)}\sin\psi\big)\right]\exp(-pB|x_3|), \tag{12a}$$

$$\bar{\boldsymbol{u}}_D = \bar{\boldsymbol{u}}_+ + \bar{\boldsymbol{u}}_-, \tag{12b}$$

$$\bar{\boldsymbol{u}}_\pm = [q\cos\psi, q\sin\psi, (\mp)A_\pm]\, U_\pm^{(\pm)}\exp(-pA_\pm|x_3|), \tag{12c}$$

$$\bar{\Theta} = \bar{\Theta}_+ + \bar{\Theta}_-, \tag{13a}$$

$$\bar{\Theta}_\pm = -C_\pm \frac{Q^2}{\alpha_D} p U_\pm^{(\pm)} \exp(-pA_\pm |x_3|), \tag{13b}$$

$$C_\pm = 1 - \left(\frac{2c_D}{\Gamma_+ \pm \Gamma_-}\right)^2, \qquad C_+ - C_- = \frac{\Gamma_+\Gamma_-}{hpQ} \bar{\mathrm{P}}_0. \tag{13c}$$

Here $(U_\pm^{(\pm)}, U_1^{(\pm)}, U_2^{(\pm)})$ are unknown functions of (p, q, ψ) and $(\pm)$ signifies $x_3 > 0(+)$, $x_3 < 0(-)$. Equations (12a), (12c) and (13b) are bounded for $\mathrm{Re}(p) > 0$ as $|x_3| \to \infty$ when $\mathrm{Re}(A_\pm, B) \geq 0$ in the cut q-plane. Imposition of discontinuities $([u_k], [\Theta], [\partial_3\Theta])$ for $(x_3^0, x_3) = 0$ leads to equations in transform space that can be solved for the unknown functions. The results are presented in Appendix A, where it proves convenient to use displacement discontinuities $(\Delta_O, \Delta_T, \Delta_S)$ that for given $|\psi| < \pi/2$, correspond to crack opening and in-plane sliding and tearing, respectively:

$$\Delta_O = [u_3], \qquad \begin{bmatrix} \Delta_S \\ \Delta_T \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi \\ \sin\psi & -\cos\psi \end{bmatrix} \begin{bmatrix} [u_1] \\ [u_2] \end{bmatrix}. \tag{14}$$

Asymptotic analysis

Focus in this paper is upon fracture initiation, i.e., small t (small s). The LS model [Lord and Shulman 1967] is robust in this regard. Indeed calculations [Brock 2009; Ignaczak and Ostoja-Starzewski 2010] indicate that $h \approx O(10^{-9})$ m and $h_0 \approx O(10^{-10})$ m, so that in view of (7a) transform expressions valid for $|h_0 p| \gg 1$ suffice, i.e., $s/h_0 \ll 1$. Therefore (12), (13) and entries in Appendix A are modified by employing asymptotic forms of (11b) and (11c):

$$A_\pm(p, q) \to A_\pm(q) = \sqrt{\frac{Q^2}{c_\pm^2} - q^2}, \tag{15a}$$

$$C_\pm = 1 - \frac{c_D^2}{c_\pm^2}, \qquad c_\pm = \frac{1}{2}(\Gamma_+ \pm \Gamma_-), \qquad C_+ - C_- = \lambda\Gamma_+\Gamma_-, \tag{15b}$$

$$\Gamma_\pm = \sqrt{\left(\frac{1}{\sqrt{\lambda}} \pm c_D\right)^2 + \varepsilon}, \qquad \lambda = \frac{h_0}{h}. \tag{15c}$$

Equation (11a) and dimensionless terms $c_\pm$ in (15) show that solution behavior involves body wave speeds $(V_R, V_\pm = c_\pm V_R)$, where $1 < c_- < c_+$. Data from, e.g., [Brock 2009; Ignaczak and Ostoja-Starzewski 2010] suggest moreover that $c_+ > c_D$, $c_- \approx c_D-$ so that V_+ is larger than isothermal dilatational wave speed $V_D = c_D V_R$ while V_- is approximately the same. Bounded behavior for $(\hat{u}_k, \hat{\Theta})$ as $|x_3| \to \infty$ requires in light of (12) and (13) that $\mathrm{Re}(A_\pm) > 0$ and $\mathrm{Re}(B) > 0$ in the q-plane with, respectively, branch cuts:

$$\mathrm{Im}(q) = 0, \qquad \frac{-1}{c_\pm - c} < \mathrm{Re}(q) < \frac{1}{c_\pm + c}, \tag{16a}$$

$$\mathrm{Im}(q) = 0, \qquad \frac{-1}{1 - c} < \mathrm{Re}(q) < \frac{1}{1 + c}. \tag{16b}$$

It is noted that (16) is valid only for $c < 1$; i.e., $V(\psi) < V_R$ $(|\psi| < \pi/2)$.

Application to fracture problem

In order that (12)–(15) and results in Appendix A represent the (asymptotic) transform solution for the fracture problem, the transforms of (5a) must be satisfied. It is noted that (5a) is incorporated in general formulas for ($s > 0$, $x_3 = 0\pm$):

$$\sigma_{3k} = \sigma^0_{3k} - F_k\,\delta(x^0_1)\,\delta(x^0_2), \tag{17a}$$

$$\partial_3\Theta = \partial_3\Theta_0 \mp \chi \mathrm{P}_0\,\Theta^{(\pm)}, \tag{17b}$$

$$\Theta = \Theta_0 + \Theta^{(\pm)}. \tag{17c}$$

Here σ^0_{3k} and $(\partial_3\Theta_0, \Theta_0)$ respectively represent σ_{3k} for $|x_3| = 0$, $x > 0$ and $(\partial_3\Theta, \Theta)$ for $x > 0$ in a region generated behind wave front $c_+ s - x - cs > 0$. Thus the corresponding transforms exist for $\mathrm{Re}(q) > -1/(c_+ - c)$. The Dirac function term has transform

$$-\frac{F_k}{pQ}\big(\mathrm{Re}(q) < 1/c\big). \tag{17d}$$

Terms $(\Delta_O, \Delta_S, \Delta_T, \Theta^{(\pm)}, \partial_3\Theta^{(\pm)})$ and related terms $([\Theta], \langle\Theta\rangle, [\partial_3\Theta], \langle\partial_3\Theta\rangle)$ occur for $x < 0$ in a region generated behind wave front $c_+ s + x + cs > 0$. Thus the corresponding transforms exist for $\mathrm{Re}(q) < 1/(c_+ + c)$. These behaviors show that

$$(\bar\sigma^0_{3k}, \bar\partial_3\Theta_0, \bar\Theta_0) \quad \text{and} \quad (\bar\Delta_O, \bar\Delta_S, \bar\Delta_T, \bar\Theta^{(\pm)}, \bar\partial_3\Theta^{(\pm)}, F_k/pQ)$$

are analytic in halves of the q-plane that overlap in the strip $-1/(c_+ - c) < \mathrm{Re}(q) < 1/(c_+ + c)$. In view then, of (2c), (11)–(17) and Appendix A, three sets of transform equations of the Wiener–Hopf type can be generated. These are given in Appendix B where, in light of (14), it has proved convenient to introduce traction terms:

$$\sigma_O = \sigma^0_{33}, \qquad \begin{bmatrix}\sigma_S\\ \sigma_T\end{bmatrix} = \begin{bmatrix}\cos\psi & \sin\psi\\ \sin\psi & -\cos\psi\end{bmatrix}\begin{bmatrix}\sigma^0_{31}\\ \sigma^0_{32}\end{bmatrix}. \tag{18}$$

Coefficients $(M_O(q), M_S(q))$ in (B.2) and (B.3) exhibit behavior:

$$M_O\left(\frac{\pm 1}{c_O \pm c}\right) = 0, \qquad M_O \approx \frac{bR_O}{a_+ a_-}\sqrt{q}\,\sqrt{-q} \quad (|q| \to \infty), \tag{19a}$$

$$R_O = \frac{1}{c^2}\left[4a_+a_- - \frac{K^2}{\lambda\Gamma_+\Gamma_- b}(C_+a_+ - C_-a_-)\right], \qquad R_O(\pm c_O) = 0, \quad (0 < c_O < 1), \tag{19b}$$

$$M_S\left(\frac{\pm 1}{c_S \pm c}\right) = 0, \qquad M_S \approx \frac{R_S}{b}\sqrt{q}\,\sqrt{-q} \quad (|q| \to \infty), \tag{20a}$$

$$R_S = \frac{1}{c^2}\left[\frac{4b}{\lambda\Gamma_+\Gamma_-}(C_+a_+ - C_-a_-) - K^2\right], \qquad R_S(\pm c_S) = 0, \quad (0 < c_S < 1). \tag{20b}$$

Behavior of coefficients $(m_O(q), m_S(q), n_O(q), n_S(q))$ is given by

$$m_O \approx \mp i m_3 \quad (|q| \to \infty), \qquad m_3 = \frac{\chi h K}{c\Gamma_+\Gamma_-}\left(\frac{1}{a_+} - \frac{1}{a_-}\right), \tag{21a}$$

$$m_S \approx \pm i m_{12} \quad (|q| \to \infty), \qquad m_{12} = \frac{1}{\lambda c^2 \Gamma_+\Gamma_-}(a_+ - a_-), \tag{21b}$$

$$n_O \approx \mp i q n_3 \quad (|q| \to \infty), \qquad n_3 = \frac{\varepsilon K}{\Gamma_+\Gamma_-}\left(\frac{1}{a_+} - \frac{1}{a_-}\right), \tag{21c}$$

$$n_S \approx \pm i q^2 n_{12} \quad (|q| \to \infty), \qquad n_{12} = \frac{2\varepsilon}{\Gamma_+\Gamma_-}(a_+ - a_-). \tag{21d}$$

Notation $\pm i$ denotes $\mathrm{Im}(q) < 0$ and $\mathrm{Im}(q) > 0$, respectively, in expressions for (m_O, m_S, n_O, n_S). In (19)–(21):

$$a_\pm(c) = \sqrt{1 - \frac{c^2}{c_\pm^2}}, \qquad b(c) = \sqrt{1-c^2}, \qquad K(c) = c^2 - 2. \tag{22a}$$

Here $(b, a_\pm)$ arise as factors of $(B, A_\pm)$ for $|q| \to \infty$. Expressions (M_S, M_O) and (R_S, R_O) are Rayleigh functions of respectively, q and c. Data from, e.g., [Brock 2009; Ignaczak and Ostoja-Starzewski 2010] indicate that in general

$$0 < c_O < c_S < 1 < c_- < c_+, \quad c_- \approx c_D - . \tag{22b}$$

In addition to body wave speeds, therefore, solution behavior for the fracture problem is influenced by Rayleigh speeds $(V_O = c_O V_R,\ V_S = c_S V_R)$. In light of (22b) subcritical speed is defined as $V(\psi) < V_O$ $(|\psi| < \pi/2)$.

Solution: Wiener–Hopf problem (tearing mode)

Solution of Wiener–Hopf equation (B.1a) involves manipulations that produce left- and right-hand sides that are analytic in overlapping regions of the complex q-plane. That is, the two sides are analytic continuations of each other. To this end $(B, A_\pm)$ are written as products $(B^+B^-, A_\pm^+A_\pm^-)$ where

$$B^+ = \sqrt{1 + q(1-c)}, \qquad B^- = \sqrt{1 - q(1+c)}, \tag{23a}$$

$$A_\pm^+ = \sqrt{\frac{1}{c_\pm} + q\left(1 - \frac{c}{c_\pm}\right)}, \qquad A_\pm^- = \sqrt{\frac{1}{c_\pm} - q\left(1 + \frac{c}{c_\pm}\right)}. \tag{23b}$$

Factors B^+ and B^- are analytic in overlapping portions of the q-plane:

$$\mathrm{Re}(q) > \frac{-1}{1-c}, \qquad \mathrm{Re}(q) < \frac{1}{1+c}. \tag{23c}$$

In similar fashion factors $A_\pm^+$ and $A_\pm^-$ are analytic in overlapping portions:

$$\mathrm{Re}(q) > \frac{-1}{c_\pm - c}, \qquad \mathrm{Re}(q) < \frac{1}{c_\pm + c}. \tag{23d}$$

For $|q| \to \infty$ $(B^{\pm}, A^{+}_{\pm}, A^{-}_{\pm})$ generate factors

$$b^{\pm}(c) = \sqrt{1 \mp c}, \qquad a^{+}_{\pm}(c) = \sqrt{1 - \frac{c}{c_{\pm}}}, \qquad a^{-}_{\pm}(c) = \sqrt{1 + \frac{c}{c_{\pm}}}. \tag{24}$$

Manipulations of (B.1a) in view of (23a) and (23c) lead to

$$\frac{\bar{\sigma}_T}{B^+} - \frac{F_T}{pQ}\left(\frac{1}{B^+} - \sqrt{c}\right) = -\mu p B^- \bar{\Delta}_T + \sqrt{c}\,\frac{F_T}{pQ}. \tag{25}$$

Analytic continuation requires that the two sides of (25) be equal to the same entire function. Restrictions on $[u_k]$ noted in connection with (5) imply that $pq\bar{\Delta}_T$, and therefore the right-hand side of (25), vanish for $|q| \to \infty$. In light of Liouville's theorem [Morse and Feshbach 1953] the entire function must vanish. Equation (25) then gives

$$\bar{\sigma}_T = \frac{F_T}{pQ} - \sqrt{c}\,B^+\frac{F_T}{pQ}, \qquad p\bar{\Delta}_T = \frac{\sqrt{c}}{p^2 Q B^-}\,\frac{F_T}{\mu}. \tag{26a}$$

Imposition of fracture criteria such as dynamic energy release rate [Freund 1990] requires only knowledge of $(\sigma_T, D_0\Delta_T)$ near crack contour C, i.e., $\sqrt{x^2 + x_3^2} \approx 0$, $|\psi| < \pi/2$. Therefore transform behavior for $|q| \to \infty$ suffices and, in view of (6b), (26a) gives

$$\bar{\sigma}_T \approx \frac{F_T b^+}{p\sqrt{qc}}, \qquad pQ\bar{\Delta}_T \approx \frac{-F_T}{\mu p b^- \sqrt{-qc}}. \tag{26b}$$

Solution: Wiener–Hopf problem (crack-opening mode)

Two coupled equations, (B.2a) and (B.2b), are involved in this instance. In view of (19)–(21), (M_O, m_O, n_O) can be expressed as products $(M^+_O M^-_O, m^+_O m^-_O, n^+_O n^-_O)$. The factors are analytic in overlapping halves $\mathrm{Re}(q) > -1/(c_+ - c)(+)$ and $\mathrm{Re}(q) < 1/(c_+ + c)(-)$ of the complex q-plane. Based on a standard procedure [Morse and Feshbach 1953; Achenbach 1976] the factors are found to be

$$M^+_O = \frac{B^+ G^+_O}{A^+_+ A^+_-}\left(\frac{1}{c_O - c} + q\right), \qquad M^-_O = R_O \frac{B^- G^-_O}{A^-_+ A^-_-}\left(\frac{1}{c_O + c} - q\right), \tag{27a}$$

$$m^+_O = \frac{A^+_-}{a^+_- G^+}, \qquad m^-_O = -m_3 \frac{a^-_- G^-}{A^-_-}, \tag{27b}$$

$$n^+_O = \frac{A^+_-}{a^+_- G^+}, \qquad n^-_O = -n_3 q \frac{a^-_- G^-}{A^-_-}. \tag{27c}$$

Term $(G^{\pm}_O, G^{\pm})$ is given in Appendix C, and it is noted that $(M^+_O, m^+_O, n^+_O) \geq 0$. Equation (B.2a) can therefore be put in the form

$$\frac{\bar{\sigma}_O}{P_O^+} - \frac{F_3}{pQ}\left(\frac{1}{p_O^+} - \frac{c}{P_3}\right) = -\mu p \bar{\Delta}_O \frac{M_O^-}{2}\lambda_O^+ + \mu\alpha_D\langle\bar{\Theta}\rangle\frac{m_O^-}{\lambda_O^+} + \frac{F_3}{pQP_O}, \tag{28a}$$

$$P_O^+ = \sqrt{M_O^+ m_O^+}, \qquad \lambda_O^+ = \sqrt{\frac{M_O^+}{m_O^+}}, \tag{28b}$$

$$P_3 = \sqrt{c}\,P_O^+\left(\frac{1}{c}\right) = \frac{1}{\sqrt{c_O - c}}\sqrt{\frac{g_O^+ c_O}{g^+ a_-^+}}, \qquad g_O^+ = G_O^+\left(\frac{1}{c}\right), \qquad g^+ = G^+\left(\frac{1}{c}\right). \tag{28c}$$

The left-hand side of (28a) is analytic for $\mathrm{Re}(q) > -1/(c_+ - c)$. Equations (28b), (28c), (C.2) and (C.3) and behavior expected for σ_O suggest that this side vanishes for $|q| \to \infty$. Setting the right-hand side of (28a) to zero leads to a quadratic equation in λ_O^+. The solution is itself an equation of the Wiener–Hopf type; i.e., λ_O^+ is set equal to a combination of terms that are analytic in the overlapping region $\mathrm{Re}(q) < 1/(c_+ + c)$. Both sides must be analytic continuations of the same bounded entire function. For $|q| \to \infty$:

$$\lambda_O^+ \to J_3 = \sqrt{\frac{b^+}{a_+^+ a_-^+}}. \tag{29}$$

Equation (29) identifies this function as a constant, so that (28a) now takes the classic [Morse and Feshbach 1953] form:

$$\frac{\bar{\sigma}_O}{P_O^+} - \frac{F_3}{pQ}\left(\frac{1}{P_O^+} - \frac{c}{P_3}\right) = -\mu p \bar{\Delta}_O \frac{M_O^- J_3}{2} + \mu\alpha_D\langle\bar{\Theta}\rangle\frac{m_O^-}{J_3} + \frac{cF_3}{pQP_3}. \tag{30}$$

In view of the behavior noted for the left-hand side of (28a), the bounded entire function for (30) vanishes. Thus (30) defines $\bar{\sigma}_O$ and provides a linear equation for $(\bar{\Delta}_O, \langle\bar{\Theta}\rangle)$. Use of that in (B.2b) gives

$$\alpha_D\bar{\Theta}_O = -\frac{n_O}{J_3 M_O^-}\frac{\sqrt{c}\,F_3}{\mu pQP_3} + \alpha_D\langle\bar{\Theta}\rangle W, \qquad W = N_O - \frac{n_O m_O^-}{J_3^2 M_O^-}. \tag{31}$$

Rearrangement of (31) into a form analogous to (30) is possible, but coefficient W leads to a complicated expression. For $|q| \to \infty$ however, the resulting form, and its counterpart for (30), combine to give more tractable forms:

$$\bar{\sigma}_O \approx \frac{J_3 F_3}{P_3 p\sqrt{qc}}, \qquad pQ\bar{\Delta}_O \approx -\frac{F_3 D_3}{\mu p\sqrt{-q}}, \tag{32a}$$

$$\alpha_D\bar{\Theta}_0 \approx \frac{\sqrt{c}\,J_3 F_3}{m_3 P_3}\frac{h\chi}{\Gamma_+\Gamma_-}\left(\frac{C_-}{a_-} - \frac{C_+}{a_+}\right)\frac{1}{\mu p\sqrt{q}}, \qquad \alpha_D\langle\bar{\Theta}\rangle \approx \frac{F_3 E_3}{\mu p\sqrt{-q}}\exp(\mp i\Psi_3), \tag{32b}$$

$$D_3 = \frac{2a_+ a_-}{bR_O}\frac{\sqrt{c}\,J_3}{P_3}\cos\Psi_3, \qquad E_3 = \frac{h\chi}{\Gamma_+\Gamma_-}\frac{cJ_3}{m_3 P_3}\left(\frac{C_+}{a_+} - \frac{C_-}{a_-}\right)\cos\Psi_3, \tag{32c}$$

$$\Psi_3 = \tan^{-1}\frac{h\chi}{\Gamma_+\Gamma_-}\left[\left(\frac{C_+}{a_+} - \frac{C_-}{a_-}\right)c + \frac{\varepsilon K^2}{c\Gamma_+\Gamma_-}\frac{(a_+ - a_-)^2}{ba_+ a_- R_O}\right]. \tag{32d}$$

In (32b) $(\mp)$ signifies $\mathrm{Im}(q) > 0$ and $\mathrm{Im}(q) < 0$ respectively.

Solution: Wiener–Hopf problem (sliding mode)

Equations (B.3a) and (B.3b) govern in this instance, but the method of solution closely mirrors that for the crack-opening mode. For $|q| \to \infty$:

$$\bar{\sigma}_S \approx \frac{-\sqrt{b^+}\,F_S}{P_{12}\,p\sqrt{q}}, \qquad pQ\bar{\Delta}_S \approx \frac{F_S D_{12}}{\mu p\sqrt{-q}}, \tag{33a}$$

$$\alpha_D \bar{\partial}_3\Theta_0 \approx \frac{-F_S}{2\mu m_{12}}\left(\frac{C_+}{a_+} - \frac{C_-}{a_-}\right)\frac{pq}{\sqrt{b^+}\,P_{12}\,p\sqrt{q}}, \qquad \alpha_D|\bar{\Theta}| \approx -\frac{F_S F_{12}}{\mu p\sqrt{-q}}, \tag{33b}$$

$$D_{12} = \frac{2b}{P_{12}}\sqrt{\frac{c}{b^+}}\frac{C_+a_+ - C_-a_-}{2bm_{12}n_{12}\lambda\Gamma_+\Gamma_- + R_S(C_+a_+ - C_-a_-)}, \qquad E_{12} = \frac{1}{P_{12}m_{12}\sqrt{b^+}}. \tag{33c}$$

Term (P_{12}, m_{12}, n_{12}) in (33) correspond to (P_3, m_3, n_3) and are given by

$$P_{12} = \frac{1}{\sqrt{c_S - c}}\sqrt{\frac{g_S^+ c_S}{g^+ a_-^+}}, \qquad g_S^+ = G_S^+\left(\frac{1}{c}\right), \tag{34a}$$

$$m_{12} = \frac{a_+ - a_-}{\lambda c^2\Gamma_+\Gamma_-}, \qquad n_{12} = \frac{2\varepsilon}{\Gamma_+\Gamma_-}(a_+ - a_-). \tag{34b}$$

Term $G_S^{\pm}$ is defined in Appendix C.

Solution behavior in crack plane near C

Equations (26b), (32) and (33) involve linear combinations of three types of transform. The types and corresponding inverses are given in Appendix D. It proves convenient to now introduce some generality by considering point-force loads that are not temporal step-functions. That is $F_k \to F_k(s)$ $F_k(0) = 0$. It is also noted that $\bar{D}_0 f = V_R\bar{D}f = V_R pQ\bar{f}$. Thus ahead of the extending crack $(x \to 0+, |\psi| < \pi/2)$ (26b), (32), (33) and (D.3) give by convolution:

$$\sigma_O \approx \frac{-J_3\mathrm{K}_O}{\pi P_3\sqrt{cx}}, \qquad \sigma_S \approx \frac{\sqrt{b^+}\,\mathrm{K}_S}{\pi P_{12}\sqrt{cx}}, \qquad \sigma_T \approx \frac{-b^+\mathrm{K}_T}{\pi\sqrt{cx}}, \tag{35a}$$

$$\alpha_D\Theta_0 \approx \frac{c^2J_3}{\pi KP_3}\frac{C_+a_- - C_-a_+}{a_+ - a_-}\frac{\mathrm{K}_O}{\mu\sqrt{cx}}, \tag{35b}$$

$$\alpha_D\partial_3\Theta_0 \approx -\frac{\partial}{\partial x_0}\frac{c}{\pi P_{12}\sqrt{b^+}}\frac{C_+a_+ - C_-a_-}{2\lambda\Gamma_+\Gamma_-}\frac{\mathrm{K}_S}{\mu\sqrt{cx}}. \tag{35c}$$

For $(x \to 0-, |\psi| < \pi/2)$:

$$D_0\Delta_O \approx \frac{D_3}{\pi}\frac{V_R\mathrm{K}_O}{\mu\sqrt{-x}}, \qquad D_0\Delta_S \approx -\frac{D_{12}}{\pi}\frac{V_R\mathrm{K}_S}{\mu\sqrt{-x}}, \qquad D_0\Delta_T \approx \frac{1}{\pi b^-\sqrt{c}}\frac{V_R\mathrm{K}_T}{\mu\sqrt{-x}}, \tag{36a}$$

$$\alpha_D\langle\Theta\rangle \approx \frac{-E_3}{\pi}\cos\Psi_3\frac{V_R\mathrm{K}_O(s)}{\mu\sqrt{-x}}, \qquad \alpha_D|\Theta| \approx \frac{E_{12}}{\pi}\frac{V_R\mathrm{K}_S(s)}{\mu\sqrt{-x}}. \tag{36b}$$

In (35) and (36):

$$\mathrm{K}_O = \frac{d}{ds}\int \frac{dF_3}{du}\frac{du}{\sqrt{s-u}}, \qquad \mathrm{K}_S = \frac{d}{ds}\int \frac{dF_S}{du}\frac{du}{\sqrt{s-u}} \quad (0 < u < s), \tag{37a}$$

$$\mathrm{K}_T = \frac{d}{ds}\int \frac{dF_T}{du}\frac{du}{\sqrt{s-u}} \quad (0 < u < s). \tag{37b}$$

Velocity and temperature change near C

In regard to solution behavior near C for $|x_3| \geq 0$, temperature change Θ and particle velocity in terms of components $(D_0 u_S, D_0 u_T, D_0 u_3)$ can be obtained from expressions (12)–(14) that are evaluated for $|q| \to \infty$ in terms (A.1)–(A.3), (15), (26), (32), (33) and relation

$$D_0\begin{bmatrix} u_S \\ u_T \end{bmatrix} = D_0\begin{bmatrix} \cos\psi & \sin\psi \\ \sin\psi & -\cos\psi \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \tag{38}$$

The resulting expressions for $(\bar{D}_0 u_S, \bar{D}_0 u_T, \bar{D}_0 u_3)$ are linear combinations of two transform types. The types and corresponding inversions are given in Appendix D. Response near C is made clearer in terms of local coordinates (r, ψ, ϕ), where $(r \to 0+, |\psi| < \pi/2, |\phi| < \pi)$ and

$$x = r\cos\phi, \qquad x_3 = r\sin\phi. \tag{39}$$

In light of (39) term $1/\sqrt{x - i\omega}$ in Appendix D gives for $\omega = (b, a_\pm)$ respectively:

$$\frac{1}{\sqrt{2r}}(\mathrm{B}^{[+]} + i\mathrm{B}^{[-]}), \qquad \mathrm{B}^{[\pm]} = \frac{1}{B_\Phi}\sqrt{B_\Phi[\pm]\cos\phi}, \qquad B_\Phi = \sqrt{1 - c^2\sin^2\phi}, \tag{40a}$$

$$\frac{1}{\sqrt{2r}}\big(\mathrm{A}_\pm^{[+]} + i\mathrm{A}_\pm^{[\pm]}\big), \qquad \mathrm{A}_\pm^{[\pm]} = \frac{1}{A_\Phi^\pm}\sqrt{A_\Phi^\pm[\pm]\cos\phi}, \qquad A_\Phi^\pm = \sqrt{1 - \frac{c^2}{c_\pm^2}\sin^2\phi}. \tag{40b}$$

Generalization $F_k \to F_k(s)$, $F_k(0) = 0$ is again made, and it can then be shown in view of (40) that for $(r \approx 0+, |\psi| < \pi/2, |\phi| < \pi)$:

$$\begin{aligned}\alpha_D\Theta \approx{}& \frac{1}{2\pi\Gamma_+\Gamma_-}\frac{\mathrm{K}_S}{\mu\sqrt{2r}}\left[\frac{2\varepsilon}{c}D_{12}\big(\mathrm{A}_-^{[-]} - \mathrm{A}_+^{[-]}\big) + \frac{E_{12}}{\lambda}\big(C_-\mathrm{A}_-^{[-]} - C_+\mathrm{A}_+^{[-]}\big)\right] \\ &+ \frac{\varepsilon K D_3}{2\pi c\Gamma_+\Gamma_-}\frac{\mathrm{K}_O}{\mu\sqrt{2r}}\left(\frac{\mathrm{A}_+^{[+]}}{a_+} - \frac{\mathrm{A}_-^{[+]}}{a_-}\right) \\ &+ \frac{h\chi c E_3}{\pi\Gamma_+\Gamma_-}\frac{\mathrm{K}_O}{\mu\sqrt{2r}}\left[\left(\frac{\mathrm{A}_-^{[+]}}{a_-} - \frac{\mathrm{A}_+^{[+]}}{a_+}\right)\cos\Psi_3 + \left(\frac{\mathrm{A}_-^{[-]}}{a_-} - \frac{\mathrm{A}_+^{[-]}}{a_+}\right)\sin\Psi_3\right], \end{aligned} \tag{41}$$

$$D_0 u_T \approx -\frac{V_R}{\pi\mu}\sqrt{c}\,\frac{\mathrm{B}^{[-]}}{2b^-}\frac{\mathrm{K}_T}{\sqrt{2r}}, \tag{42a}$$

$$\begin{aligned} D_0 u_S \approx{}& \frac{V_R}{\pi\mu c^2}\left[bD_3\mathrm{B}^{[+]}\frac{\mathrm{K}_O}{\sqrt{2r}} - KD_{12}\mathrm{B}^{[-]}\frac{\mathrm{K}_S}{\sqrt{2r}}\right] \\ &+ \frac{V_R}{2\pi\mu c^2\lambda\Gamma_+\Gamma_-}\frac{\mathrm{K}_S}{\sqrt{2r}}\left[2D_{12}\left(C_-\mathrm{A}_+^{[-]} - C_+\mathrm{A}_-^{[-]}\right) + cE_{12}\left(\mathrm{A}_-^{[-]} - \mathrm{A}_+^{[-]}\right)\right] \\ &+ \frac{V_R K D_3}{2\pi\mu c^2\lambda\Gamma_+\Gamma_-}\frac{\mathrm{K}_O}{\sqrt{2r}}\left(\frac{C_+}{a_-}\mathrm{A}_-^{[+]} - \frac{C_-}{a_+}\mathrm{A}_+^{[+]}\right) \\ &+ \frac{V_R h\chi\lambda E_3}{\pi\mu\Gamma_+\Gamma_-}\frac{\mathrm{K}_O}{\sqrt{2r}}\left[\left(\frac{\mathrm{A}_-^{[+]}}{a_-} - \frac{\mathrm{A}_+^{[+]}}{a_+}\right)\cos\Psi_3 + \left(\frac{\mathrm{A}_-^{[-]}}{a_-} - \frac{\mathrm{A}_+^{[-]}}{a_+}\right)\sin\Psi_3\right], \end{aligned} \tag{42b}$$

$$\begin{aligned} D_0 u_3 \approx{}& \frac{V_R}{\pi\mu c^2}\left[\frac{K}{b}D_{12}\mathrm{B}^{[+]}\frac{\mathrm{K}_S}{\sqrt{2r}} + D_3\mathrm{B}^{[-]}\frac{\mathrm{K}_O}{\sqrt{2r}}\right] \\ &+ \frac{V_R}{2\pi\mu c^2\lambda\Gamma_+\Gamma_-}\frac{\mathrm{K}_S}{\sqrt{2r}}\left[a_-\mathrm{A}_-^{[+]}(2C_-D_{12} - cE_{12}) + a_+\mathrm{A}_+^{[+]}(2C_+D_{12} - cE_{12})\right] \\ &+ \frac{V_R K D_3}{2\pi\mu c^2\lambda\Gamma_+\Gamma_-}\frac{\mathrm{K}_O}{\sqrt{2r}}\left(C_+\mathrm{A}_-^{[-]} - C_-\mathrm{A}_+^{[-]}\right) \\ &+ \frac{V_R h\chi\lambda E_3}{\pi\mu\Gamma_+\Gamma_-}\frac{\mathrm{K}_O}{\sqrt{2r}}\left[\left(\mathrm{A}_+^{[-]} - \mathrm{A}_-^{[-]}\right)\cos\Psi_3 + \left(\mathrm{A}_-^{[+]} - \mathrm{A}_+^{[+]}\right)\sin\Psi_3\right]. \end{aligned} \tag{42c}$$

Preliminary comments

The coupling of (χ, Ψ_3) with K_O in (35), (36), (41) and (42) shows that crack opening (and therefore convection) indeed occurs only when compressive load $F_3(s)$ is present. These equations also show that introduction of components that align with coordinates (x, ψ, x_3) allow an uncoupling into three modes of fracture. However classical definitions [Freund 1990] of in-plane modes are made in terms of the normal and tangent to the crack edge, and designated as Modes II and III, respectively. Here crack edge orientation is controlled by $V(\psi)$. In terms of (35a) and (36a) for example

$$\begin{bmatrix}\sigma_{II}\\ \sigma_{III}\end{bmatrix} = \boldsymbol{M}_C\begin{bmatrix}\sigma_S\\ \sigma_T\end{bmatrix}, \qquad \begin{bmatrix}D_0\Delta_{II}\\ D_0\Delta_{III}\end{bmatrix} = \boldsymbol{M}_C\begin{bmatrix}D_0\Delta_S\\ D_0\Delta_T\end{bmatrix}, \tag{43a}$$

$$\boldsymbol{M}_C = \begin{bmatrix}\cos\psi_C & -\sin\psi_C\\ \sin\psi_C & \cos\psi_C\end{bmatrix}, \qquad \psi_C = \tan^{-1}\frac{dc}{c\,d\psi}. \tag{43b}$$

Dynamic energy release rate criterion

Equation (43) need not be employed if the imposed fracture criterion is based on scalar products, i.e., dynamic energy release rate [Freund 1990]. If kinetic energy is included [Gdoutos 1993; Brock 2017] it can be shown that here the criterion can be written:

$$D_0 \iint_{\delta A} e_F \, dx_1^0 dx_2^0 - \iint_{\mathfrak{I}} \sigma_{3k}^0 D_0 \Delta_k \, dx_1^0 dx_2^0 - D_0 \iiint_{123} \frac{\rho}{2} D_0 u_k D_0 u_k \, dx_1^0 dx_2^0 dx_3^0 = 0, \tag{44a}$$

$$\sigma_{3k}^0 D_0 \Delta_k = \sigma_O D_0 \Delta_O + \sigma_S D_0 \Delta_S + \sigma_T D_0 \Delta_T, \tag{44b}$$

$$D_0 u_k D_0 u_k = (D_0 u_O)^2 + (D_0 u_S)^2 + (D_0 u_T)^2. \tag{44c}$$

In (44a) e_F is the surface energy per unit area in area δA, and is generally viewed as constant [de Boer et al. 1988; Skriver and Rosengaard 1992]. Fracture zone $\mathfrak{I}$ is a strip of infinitesimal thickness in the $x_1^0 x_2^0$-plane that straddles the portion of C that borders δA. In view of the singular behavior seen in (35) and (36) it can be shown [Freund 1972] that integration yields a finite value. Subscript 123 signifies integration over the solid, but the singular behavior exhibited in (42) demonstrates that the volume integral can be confined to a tube of radius $r_C \to 0$ that is centered on, and encloses, the crack edge C. Analysis [Brock 2017] shows that these produce a single integration with respect to ψ on the left-hand side of (44a). That is, (44a) is satisfied if the integrand vanishes for all $|\psi| < \pi/2$. However the integrand cannot, in general, vanish for constant e_F and time-invariant crack-extension rate; compare [Achenbach and Brock 1973]. An exception, featured in [Brock 2017], is case $3F_k(s) = 2 f_k s^{3/2}$, i.e.,

$$\mathrm{K}_O = \pi f_3, \qquad \mathrm{K}_S = \pi(f_1 \cos\psi + f_2 \sin\psi), \qquad \mathrm{K}_T = \pi(f_1 \sin\psi - f_2 \cos\psi). \tag{45}$$

Here f_k is constant and $f_3 \geq 0$. This analysis concerns fracture initiation, and appropriate asymptotic forms such as (15) have been employed. So, the exception is here taken to represent only the initial loading behavior. A focus is, moreover, on the role of crack surface convection. The observation concerning (K_O, Ψ_3) made above suggests that consideration of the pure-compression case ($f_1 = f_2 = 0$) is sufficient in this regard. In view of (35), (36), (42) and (45) formula (44a) produces the equation:

$$\frac{f_3^2 c}{2\pi\mu} \frac{J_3 D_3}{P_3} + \left[e_F + \frac{f_3^2}{(2\pi)^2 \mu} \int_\Phi (Q_O^2 + Q_S^2) \cos\phi \, d\phi \right] \sqrt{c^2 + \left(\frac{dc}{d\psi}\right)^2} = 0, \tag{46a}$$

$$Q_O = \frac{D_3}{c^2} \left[\mathrm{B}^{[-]} + \frac{K}{2\lambda \Gamma_+ \Gamma_-} (C_+ \mathrm{A}_-^{[-]} - C_- \mathrm{A}_+^{[-]}) \right] + \frac{h\chi\lambda}{\Gamma_+ \Gamma_-} \left[(\mathrm{A}_+^{[-]} - \mathrm{A}_-^{[-]}) \cos\Psi_3 + (\mathrm{A}_-^{[+]} - \mathrm{A}_+^{[+]}) \sin\Psi_3 \right], \tag{46b}$$

$$Q_S = \frac{D_3}{c^2} \left[b\mathrm{B}^{[+]} + \frac{K}{2\lambda \Gamma_+ \Gamma_-} \left(\frac{C_+}{a_-} \mathrm{A}_-^{[+]} - \frac{C_-}{a_+} \mathrm{A}_+^{[+]} \right) \right] + \frac{h\chi\lambda}{\Gamma_+ \Gamma_-} \left[\left(\frac{\mathrm{A}_-^{[+]}}{a_-} - \frac{\mathrm{A}_+^{[+]}}{a_+} \right) \cos\Psi_3 + \left(\frac{\mathrm{A}_-^{[-]}}{a_-} - \frac{\mathrm{A}_+^{[-]}}{a_+} \right) \sin\Psi \right]. \tag{46c}$$

Subscript Φ in (46a) signifies integration over range $|\phi| < \pi$. Absence of ψ in (46a) implies that $dc/d\psi = 0$; i.e., the crack edge forms a semicircle of radius cs about the point force. Equation (46a) then reduces to a transcendental algebraic relation for constant c:

$$e_F + \frac{f_3^2}{2\pi\mu} \left[\frac{J_3 D_3}{P_3} + \frac{1}{2\pi} \int_\Phi (Q_O^2 + Q_S^2) \cos\phi \, d\phi \right] = 0. \tag{46d}$$

Thermal response on C

Equation (41) describes unbounded temperature change along crack edge C. As with dynamic energy release rate a finite measure is possible, in this case by considering the norm of crack edge temperature change:

$$\|\Theta\| = \sqrt{\int_C \Theta^2 \, dl}. \tag{47}$$

The line integration in (47) for given $|\psi| < \pi/2$ is around the surface of the tube of radius $r_C \to 0$ that is involved in analysis of (44a). Thus (41) governs and $dl = r_C \, d\phi$. For pure compression ($f_1 = f_2 = 0$) equation (47) gives

$$\|\Theta\| = \frac{f_3}{\mu\alpha_D} \frac{1}{\sqrt{2}\,\Gamma_+\Gamma_-} \left[\int_\Phi d\phi \left(\frac{K D_3}{2c} Q_D + h\chi c E_3 Q_E \right)^2 \right]^{1/2}, \tag{48a}$$

$$Q_D = \frac{\mathrm{A}_+^{[+]}}{a_+} - \frac{\mathrm{A}_-^{[+]}}{a_-}, \qquad Q_E = \left(\frac{\mathrm{A}_-^{[-]}}{a_-} - \frac{\mathrm{A}_+^{[-]}}{a_+} \right) \sin\Psi_3 - Q_D \cos\Psi_3. \tag{48b}$$

Illustration of convection effect

Convection is represented in (46) and (48) by $h\chi$, a dimensionless constant that plays a role similar to that of the Biot number in classical thermoelasticity [Boley and Weiner 1960]. As noted above, results here are valid for subcritical speed $V(\psi)$, i.e., $c(\psi) < (c_3, c_{12})$. In contrast results in [Brock 2017] require that $c(\psi) < 0.3$. Imposing a similar requirement here, $c < 0.4$, does allow an explicit, asymptotic representation of convection effect. In particular, expansions of (46b), (46c) and (48b) in powers of c allow closed-form integration with respect to ϕ. Equations (46d) and (48a) become

$$\left(c_D^2 \mathrm{E}_0 + \mathrm{E}_1 + \mathrm{E}_2 h^2\chi^2\right)c^2 + \left[\frac{2\mu e_F}{\pi f_3^2} c_O \left(1 - \frac{1}{c_D^2}\right) - \frac{1}{c_O} + \frac{\sqrt{\lambda}}{2c_D}\Gamma_+ \right] c_D^2 c - c_D^2 \approx 0, \tag{49a}$$

$$\|\Theta\| \approx \frac{7c^{3/2}}{4\mu\alpha_D} \sqrt{\frac{\pi}{2}} \frac{f_3}{c_O} \left(\frac{\varepsilon\lambda}{c_D^2 - 1} + h\chi c^{3/2} \right). \tag{49b}$$

The (positive) coefficients ($\mathrm{E}_0, \mathrm{E}_1, \mathrm{E}_2$) are given in Appendix E. Equation (49) indicates that for $c \to 0$ convection parameter $h\chi$ diminishes in importance. More insight is possible by calculation of c and the corresponding $\|\Theta\|$. Convection parameters $h\chi$ are based on Biot parameter values featured in [Boley and Weiner 1960]. Values for loading parameter f_3 are based on those in [Brock 2017], as are the material constants for a generic solid at room temperature:

$$\begin{gathered}
\mu = 79\,\mathrm{GPa}, \qquad e_F = 2.2\,\mathrm{J/m^2}, \qquad V_R = 3094\,\mathrm{m/s}, \\
c_D = 2, \qquad c_+ = 4.5452, \qquad c_- = 1.997, \qquad c_O = 0.9332, \\
T_0 = 294\,\mathrm{K}, \qquad \alpha_D = 89.6 \cdot 10^{-6}\,\mathrm{K^{-1}}, \qquad \varepsilon = 0.05044, \\
h = 3.1862 \cdot 10^{-9}\,\mathrm{m}, \qquad h_0 = 1.547 \cdot 10^{-10}\,\mathrm{m}.
\end{gathered}$$

Combinations of ($h\chi$, f_3) chosen are such that quadratic (49a) yields solutions $0 < c < 0.4$. Calculations for (c, $\|\Theta\|$) are displayed in Tables 1 and 2. Entries in Table 1 indicate that c tends to increase by orders

f_3 (N/m$^{3/2}$)	$h\chi = 0$	$h\chi = 10$	$h\chi = 50$	$h\chi = 65$	$h\chi = 80$
$1 \cdot 10^4$	0.00129249	0.00129248	0.001292476	0.00129247	0.00129225
$2 \cdot 10^4$	0.00518312	0.00518309	0.00518323	0.00518174	0.00518103
$5 \cdot 10^4$	0.0327376	0.0327294	0.0325396	0.0323988	0.0322297
$1 \cdot 10^5$	0.123558	0.123168	0.115141	0.110192	0.10563
$1 \cdot 10^6$	0.381241	0.2869926	0.2869926	0.250029	0.2194334

Table 1. Values of c for various $(h\chi,\ f_3)$.

f_3 (N/m$^{3/2}$)	$h\chi = 0$	$h\chi = 10$	$h\chi = 50$	$h\chi = 65$	$h\chi = 80$
$1 \cdot 10^4$	$1.215 \cdot 10^{-11}$	$1.907 \cdot 10^{-10}$	$4.674 \cdot 10^{-10}$	$5.712 \cdot 10^{-10}$	$5.663 \cdot 10^{-8}$
$2 \cdot 10^4$	$1.951 \cdot 10^{-9}$	$1.088 \cdot 10^{-8}$	$4.657 \cdot 10^{-8}$	$5.99 \cdot 10^{-8}$	$7.325 \cdot 10^{-8}$
$5 \cdot 10^4$	$7.747 \cdot 10^{-8}$	$5.694 \cdot 10^{-6}$	$2.767 \cdot 10^{-5}$	$3.549 \cdot 10^{-5}$	$4.325 \cdot 10^{-5}$
$1 \cdot 10^5$	$1.136 \cdot 10^{-6}$	$5.997 \cdot 10^{-4}$	$2.446 \cdot 10^{-3}$	$2.787 \cdot 10^{-3}$	$2.974 \cdot 10^{-3}$
$1 \cdot 10^6$	$6.132 \cdot 10^{-5}$	0.16846	0.37872	0.32555	0.27085

Table 2. Values of $\|\Theta\|$ (K m$^{1/2}$) for various $(h\chi,\ f_3)$.

of magnitude with increases in f_3. Variation in c with $h\chi$ is not however monotonic for given f_3. Indeed, for higher f_3-values a marked decrease occurs in c for higher $h\chi$-values. Table 2 entries indicate that $\|\Theta\|$ also tends to increase with increasing f_3. $\|\Theta\|$ is even more sensitive than c to variations in $h\chi$, and especially in f_3. Another contrast: except at the highest f_3-value, monotonic increases in $\|\Theta\|$ occur with increasing $h\chi$. Variations noted in Tables 1 and 2 can be a matter of significant figures. The trends described seem however to be clear.

Some summary comments

This paper addresses a problem similar to that found in [Brock 2017]. However crack surface thermal convection is now considered and crack extension rate need only be subcritical, not well below Rayleigh and body wave speed. In addition formulation of the governing Wiener–Hopf equations in integral transform space differs. Because the requirement on speed is relaxed moreover, the equations yield solutions that are more robust. Analysis of the inverses that result, and calculations for the pure compression case, indicate that:

- Effect of convection is less important at low crack extension rates.
- Increase in point force magnitude does in general increase crack extension rate.
- For given force, variation in rate with convection may not be monotonic.
- At higher forces, increases in convection can decrease extension rate.
- Thermal response, in terms of crack edge temperature norm, is similar.
- Norm variation with changes in convection is however more pronounced.

Appendix A

$$U_1^{(\pm)} = \frac{qB}{Q^2}\bar{\Delta}_O \cos\psi (\pm)\left(\frac{T\bar{\Delta}_S}{2Q^2}\cos\psi + \frac{\bar{\Delta}_T}{2}\sin\psi\right), \tag{A.1a}$$

$$U_2^{(\pm)} = \frac{qB}{Q^2}\bar{\Delta}_O \sin\psi (\pm)\left(\frac{T\bar{\Delta}_S}{2Q^2}\sin\psi - \frac{\bar{\Delta}_T}{2}\cos\psi\right), \tag{A.1b}$$

$$U_+^{(\pm)} = \frac{hp}{Q\bar{\mathrm{P}}_0\Gamma_+\Gamma_- A_+}\left(\frac{\alpha_D}{p^2}[\bar{\partial}_3\Theta] + C_- T\bar{\Delta}_O\right)(\mp)\frac{hp}{Q\bar{\mathrm{P}}_0\Gamma_+\Gamma_-}\left(\frac{\alpha_D}{p}[\bar{\Theta}] + 2C_- q\bar{\Delta}_S\right), \tag{A.2a}$$

$$U_-^{(\pm)} = \frac{-hp}{Q\bar{\mathrm{P}}_0\Gamma_+\Gamma_- A_-}\left(\frac{\alpha_D}{p^2}[\bar{\partial}_3\Theta] + C_+ T\bar{\Delta}_O\right)(\pm)\frac{hp}{Q\bar{\mathrm{P}}_0\Gamma_+\Gamma_-}\left(\frac{\alpha_D}{p}[\bar{\Theta}] + 2C_+ q\bar{\Delta}_S\right), \tag{A.2b}$$

$$T = Q^2 - 2q^2. \tag{A.3}$$

In view of (5c), equations (A.1) and (A.2) are subject to constraints:

$$[\bar{\partial}_3\Theta] + 2\chi\bar{\mathrm{P}}_0\langle\bar{\Theta}\rangle = 0, \qquad \langle\bar{\partial}_3\Theta\rangle = \chi\bar{\mathrm{P}}_0[\bar{\Theta}]. \tag{A.4}$$

Appendix B

Tearing mode response is governed by

$$\bar{\sigma}_T - \frac{F_T}{pQ} = -\mu p\bar{\Delta}_T B, \tag{B.1a}$$

$$F_T = F_1\sin\psi - F_2\cos\psi. \tag{B.1b}$$

Crack-opening mode response is governed by the coupled set

$$\bar{\sigma}_O - \frac{F_3}{pQ} = -\mu p\bar{\Delta}_O\frac{M_O}{2} + \mu\alpha_D\langle\bar{\Theta}\rangle m_O, \tag{B.2a}$$

$$\alpha_D\bar{\Theta}_0 = -p\bar{\Delta}_O\frac{n_O}{2} + \alpha_D\langle\bar{\Theta}\rangle N_O. \tag{B.2b}$$

Sliding mode response is governed by the coupled set

$$\bar{\sigma}_S - \frac{F_S}{pQ} = -\mu p\bar{\Delta}_S\frac{M_S}{2} + \mu\alpha_D[\bar{\Theta}]m_S, \tag{B.3a}$$

$$\alpha_D\,\partial_3\bar{\Theta}_0 = p^2\bar{\Delta}_S\frac{n_S}{2} + p\alpha_D[\bar{\Theta}]N_S, \tag{B.3b}$$

$$F_S = F_1\cos\psi + F_2\sin\psi. \tag{B.3c}$$

Coefficients in (B.2) and (B.3) are

$$M_O = \frac{1}{Q^2}\left[4q^2 B + \frac{T^2}{\lambda\Gamma_+\Gamma_-}\left(\frac{C_+}{A_-} - \frac{C_-}{A_+}\right)\right], \tag{B.4a}$$

$$M_S = \frac{1}{Q^2}\left[\frac{T^2}{B} + \frac{4q^2}{\lambda\Gamma_+\Gamma_-}(C_+A_- - C_-A_+)\right], \tag{B.4b}$$

$$N_O = -1 + \frac{\chi h Q}{\Gamma_+\Gamma_-}\left(\frac{C_+}{A_+} - \frac{C_-}{A_-}\right), \tag{B.4c}$$

$$N_S = \frac{1}{2}\left[\chi h_0 Q + \frac{1}{\lambda\Gamma_+\Gamma_-}(C_-A_- - C_+A_+)\right], \tag{B.4d}$$

$$m_O = \frac{\chi T h}{\Gamma_+\Gamma_-\beta}\left(\frac{1}{A_+} - \frac{1}{A_-}\right), \qquad m_S = \frac{1}{\lambda\Gamma_+\Gamma_-}\frac{q}{Q^2}(A_+ - A_-), \tag{B.5a}$$

$$n_O = -\frac{\varepsilon T}{\Gamma_+\Gamma_-}\left(\frac{1}{A_+} - \frac{1}{A_-}\right), \qquad n_S = -\frac{2\varepsilon q}{\Gamma_+\Gamma_-}(A_+ - A_-). \tag{B.5b}$$

Appendix C

$$\ln G_O^{\pm}(q) = \frac{1}{\pi}\int \frac{\Phi_O\, du}{(u \mp c)(qu \pm Q)}, \qquad \ln G_S^{\pm}(q) = \frac{1}{\pi}\int \frac{\Phi_S\, du}{(u \mp c)(qu \pm Q)}. \tag{C.1a}$$

Integration is over range $1 < u < c_+$, where for $c_- < u < c_+$:

$$\Phi_O = \tan^{-1}\left(\frac{4a_+\beta\lambda}{C_-K^2}\Gamma_+\Gamma_- + \frac{C_+a_+}{C_-\alpha_-}\right), \qquad \Phi_S = \tan^{-1}\frac{4C_-a_+\beta}{4C_+\alpha_-\beta + \lambda K^2\Gamma_+\Gamma_-}. \tag{C.1b}$$

For $1 < c < c_-$:

$$\Phi_O = \tan^{-1}\frac{K^2}{4\lambda\beta\Gamma_+\Gamma_-}\left(\frac{C_+}{a_-} - \frac{C_-}{a_+}\right), \qquad \Phi_S = \tan^{-1}\frac{4\beta}{\lambda K^2\Gamma_+\Gamma_-}(C_-a_+ - C_+a_-). \tag{C.1c}$$

$$G^{\pm}(q) = \frac{1}{\pi}\int \tan^{-1}\frac{a_+}{\alpha_-}\frac{du}{(u \mp c)(qu \pm Q)} \quad (c_- < u < c_+). \tag{C.2}$$

In (C.1) and (C.2) $a_\pm = a_\pm(u)$ and [see (24)] $K = K(u)$. Moreover

$$\beta = \sqrt{u^2 - 1}, \qquad \alpha_- = \sqrt{\frac{u^2}{c_-^2} - 1}. \tag{C.3}$$

Appendix D

Equations (26b), (32) and (33) involve three basic types of transform function. These types, and the corresponding inverses generated by (9b) are

$$\frac{1}{p\sqrt{q}} \to -\sqrt{\frac{p}{\pi x}} \qquad (x > 0), \tag{D.1a}$$

$$\frac{1}{p\sqrt{-q}} \to -\sqrt{\frac{p}{\pi |x|}} \qquad (x < 0), \tag{D.1b}$$

$$\frac{\exp(\mp i\Psi_3)}{p\sqrt{-q}} \to -\sqrt{\frac{p}{\pi |x|}} \cos\Psi_3 \quad (x < 0). \tag{D.1c}$$

In view of (38) it can be shown that $(\bar{D}_0 u_S, \bar{D}_0 u_T, \bar{D}_0 u_O)$ are linear combinations of two types of transforms. The types, and their inversions generated by use of (9b) are

$$\left[\frac{1}{p\sqrt{q}}, \frac{(\pm)}{p\sqrt{-q}}\right] \exp(-p\omega\sqrt{q}\sqrt{-q}) \to -\sqrt{\frac{p}{\pi}}\,[\mathrm{Re}, \mathrm{Im}]\frac{1}{\sqrt{x - i\omega}}, \tag{D.2a}$$

$$\left[\frac{1}{p\sqrt{q}}, \frac{(\pm)}{p\sqrt{-q}}\right] \exp\bigl((\mp) i\Psi_3 - p\omega\sqrt{q}\sqrt{-q}\bigr) \to \sqrt{\frac{p}{\pi}}\,[\mathrm{Re}, \mathrm{Im}]\frac{\exp(-i\Psi_3)}{\sqrt{x - i\omega}}, \tag{D.2b}$$

$$\omega = (b, a_{\pm})|x_3|. \tag{D.2c}$$

On the left-hand (transform) sides of (D.2a) and (D.2b) $(\pm)$ signifies $\mathrm{Im}(q) > 0$ and $\mathrm{Im}(q) < 0$, respectively. In view of (7a) moreover $\sqrt{p}$ is the transform of

$$\frac{d}{ds}\left(\frac{1}{\sqrt{\pi s}}\right) \quad (s > 0). \tag{D.3}$$

Appendix E

$$\mathrm{E}_0 = \frac{1}{8(c_D^2 - 1)}\left[\frac{3}{4}\left(23 + \frac{3}{2}(13c_D^2)\right) + \frac{c_D^2 - 32}{c_D^2 - 1}\right], \tag{E.1a}$$

$$\mathrm{E}_1 = 4c_D^2 + \bigl(1 + \lambda c_F^2\bigr)\left[1 - \frac{1}{c_D^2 - 1}\left(\frac{1}{2} + \frac{1}{c_D^2}\right)\right] + \frac{2c_D^2}{c_O}\left(1 - \frac{\sqrt{\lambda}}{c_D}\Gamma_+\right) + \frac{c_D^2}{c_D^2 - 1}(7 + 3\lambda), \tag{E.1b}$$

$$\mathrm{E}_2 = (c_D\lambda)^2\left[1 - \frac{\varepsilon\lambda}{2c_D^2(c_D^2 - 1)}\right]^2. \tag{E.1c}$$

In (E.1b) $c_F = \sqrt{c_D^2 + \varepsilon}$, where $V_D = c_F V_R$ is dilatational wave speed in classical thermoelasticity; see, e.g., [Brock 2009].

References

[Achenbach 1976] J. D. Achenbach, *Wave propagation in elastic solids*, vol. 16, North-Holland Series in Applied Mathematics and Mechanics, North-Holland, Amsterdam, 1976.

[Achenbach and Brock 1973] J. D. Achenbach and L. M. Brock, "On quasistatic and dynamic fracture", pp. 529–541 in *Proceedings of an international conference on Dynamic Crack Propagation*, edited by G. C. Sih, Springer, Dordrecht, 1973.

[Barber 1992] J. R. Barber, *Elasticity*, vol. 12, Solid Mechanics and its Applications, Kluwer, Dordrecht, 1992.

[Boley and Weiner 1960] B. A. Boley and J. H. Weiner, *Theory of thermal stresses*, Wiley, New York, 1960.

[Brock 2009] L. M. Brock, "Basic problems of coupled thermoelasticity with thermal relaxation and pre-stress: aspects observed in exact and asymptotic solutions", *J. Therm. Stresses* **32**:6-7 (2009), 593–622.

[Brock 2012] L. M. Brock, "Two cases of rapid contact on an elastic half-space: sliding ellipsoidal die, rolling sphere", *J. Mech. Mater. Struct.* **7**:5 (2012), 469–483.

[Brock 2017] L. M. Brock, "Transient analysis of fracture initiation in a coupled thermoelastic solid", *J. Mech. Mater. Struct.* **12**:5 (2017), 667–688.

[Brock and Achenbach 1973] L. M. Brock and J. D. Achenbach, "Extension of an interface flaw under the influence of transient waves", *Int. J. Solids Struct.* **9**:1 (1973), 53–68.

[de Boer et al. 1988] F. R. de Boer, W. C. M. Mattens, R. Boom, A. R. Miedema, and A. K. Niessen, *Cohesion in metals*, North-Holland, Amsterdam, 1988.

[Freund 1972] L. B. Freund, "Energy flux into the tip of an extending crack in an elastic solid", *J. Elasticity* **2**:4 (1972), 341–349.

[Freund 1990] L. B. Freund, *Dynamic fracture mechanics*, Cambridge Monographs on Mechanics, Cambridge University Press, Cambridge, 1990.

[Gdoutos 1993] E. E. Gdoutos, *Fracture mechanics: an introduction*, vol. 14, Solid Mechanics and its Applications, Kluwer, Dordrecht, 1993.

[Ignaczak and Ostoja-Starzewski 2010] J. Ignaczak and M. Ostoja-Starzewski, *Thermoelasticity with finite wave speeds*, Oxford University Press, 2010.

[Lord and Shulman 1967] H. W. Lord and Y. Shulman, "A generalized dynamical theory of thermoelasticity", *J. Mech. Phys. Solids* **15**:5 (1967), 299–309.

[Morse and Feshbach 1953] P. M. Morse and H. Feshbach, *Methods of theoretical physics*, McGraw-Hill, New York, 1953.

[Skriver and Rosengaard 1992] H. L. Skriver and N. M. Rosengaard, "Surface energy and work function of elemental metals", *Phys. Rev. B* **46**:11 (1992), 7157–7168.

[Sneddon 1972] I. N. Sneddon, *The use of integral transforms*, McGraw-Hill, New York, 1972.

Received 28 Jan 2019. Revised 5 May 2019. Accepted 11 May 2019.

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JoMMS peer-review and production is managed by EditFlow® from Mathematical Sciences Publishers.

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Journal of Mechanics of Materials and Structures

Volume 14, No. 3 May 2019