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ELASTIC FOUNDATION**

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## WAVE PROPAGATION IN THREE-DIMENSIONAL GRAPHENE AEROGEL CYLINDRICAL SHELLS RESTING ON WINKLER–PASTERNAK ELASTIC FOUNDATION

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The objective of this work is to investigate the wave propagation characteristics of circular cylindrical shells made of three-dimensional graphene aerogel (3D-GA). Different distributions of 3D-GA inside the shells are taken into account. The first-order shear deformation (FSD) shell theory is utilized to model the present shells. Hamilton's principle is employed to drive the equations of motion, which governs the wave propagation behavior of 3D-GA cylindrical shells. The analytical wave dispersion relations with longitudinal and circumferential wave numbers are obtained. In addition, detailed parametric studies are conducted to emphasize the influences of the porosity distribution, the porosity coefficient, the radius-to-thickness ratio, the applied forces and the elastic foundation on wave propagation characteristics of 3D-GA cylindrical shells.

### 1. Introduction

Since the isolation of graphene films was first achieved by Novoselov et al. [2004], this type of carbon materials has been one of the most interesting materials owing to its extraordinary fracture strength, superior Young's modulus, extreme thermal conductivity and so on [Lee et al. 2008; Geim and Novoselov 2009; Chatterjee et al. 2012; Wang et al. 2012; Geim 2009]. Recently, based on several physical and chemical methods, the macroscopic architecture of three-dimensional graphene aerogel (3D-GA) was successfully fabricated by using the controlled micro/nano-scale graphene sheets as building blocks [Vickery et al. 2009; Xu et al. 2010; Chen et al. 2011; Huang et al. 2012; Kuang et al. 2013; Jiang and Fan 2014; Li et al. 2014; Sha et al. 2016; 2017; Strek et al. 2017]. Such creative design make 3D-GA possess many exciting properties, such as high compressibility, super elasticity, extremely low density and electrochemical stability [Chen et al. 2014; Wu et al. 2015; Xu et al. 2016; Qin et al. 2017; Qiu et al. 2017].

As one of the most novel developments in advanced porous structures, 3D-GA structures exhibit tremendous potentials and applications in biological, environmental, electric and chemical engineering. For instance, using 3D-GA structures as the conductive and biocompatible scaffold, it can support neural stem cells (NSCs) growth and keep NSCs at the positive proliferation state [Li et al. 2013]. With the abilities of reversible absorption and discharge of liquids and strong hydrophobicity, 3D-GA structures can be applied for the liquid transfer and environmental cleanup [Wu et al. 2015]. Owing to their high

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*Keywords:* three-dimensional graphene aerogel, cylindrical shell, wave propagation, Winkler–Pasternak elastic foundation, first-order shear deformation theory.

specific surface area, 3D-GA structures were proposed as recyclable and versatile sorbent with the efficient absorption of not only toxic solvents, but also fats and petroleum products [Bi et al. 2012]. It is worth mentioning that the porous structures could be analyzed by using the gradient elasticity [Xu et al. 2008; 2014; Askes and Aifantis 2009; Sun and Aifantis 2014; Aifantis 2016; Lurie et al. 2018].

Investigation of wave propagation characteristics in structures is of significance in practical engineering applications [Wang 2010; Zeighampour et al. 2017; 2018; Zhen 2017]. Zeighampour and Beni [2017] carried out the wave propagation analysis of functionally graded (FG) cylindrical shells reinforced by CNTs. Hu et al. [2008] investigated the transverse and torsional wave propagation in double- and single-walled CNTs. Ma et al. [2018] utilized the classical and FSD shell theories to study wave propagation in magneto-electro-elastic nanoshells. Based on the fast Fourier transform (FFT), Shakeri et al. [2006] investigated wave propagation characteristics in FG thick cylindrical shells subjected to dynamic load. Aminipour et al. [2018] utilized Reddy's higher-order shear deformation theory to study the wave propagation of FG anisotropic doubly-curved shells. Thorp et al. [2005] analyzed the attenuation of wave propagation in fluid-loaded cylindrical shells with periodic shunted piezoelectric rings. Using the three-dimensional theory of anisotropic elasticity, Talebitooti and Choudari Khameneh [2017] analyzed the wave propagation across double-walled laminated composite cylindrical shells along with air-gap. Sorokin and Ershova [2004] studied the plane wave propagation in periodic cylindrical shells with and without fluid loads. The wave propagation analysis in spherically symmetric shells made of laminated piezoelectric materials was carried out in [Dai and Wang 2005]. Here the electric excitation and the thermal shock loads were taken into account. By using the reverberation ray matrix method and the generalized ray method, Liu et al. [2011] investigated the transient elastic wave propagation in laminated composite circular cylindrical shells. The Flügge shell theory was utilized to investigate wave propagation of CNTs in [Wang and Varadan 2007]. Using the Cooper–Naghdi thick shell theory and Love's thin shell theory, Liew and Wang [2007] studied wave propagation in single- and double-walled CNTs.

In this study, wave propagation analysis of circular cylindrical shells made of 3D-GA is carried out for the first time. Different distributions of 3D-GA inside the shells are taken into account. The FSD shell theory and Hamilton's principle are employed to obtain the governing equations. The analytical wave dispersion relations for 3D-GA cylindrical shells are derived. In addition, the influences of several parameters are investigated on the wave propagation in 3D-GA cylindrical shells resting on the Winkler–Pasternak elastic foundation.

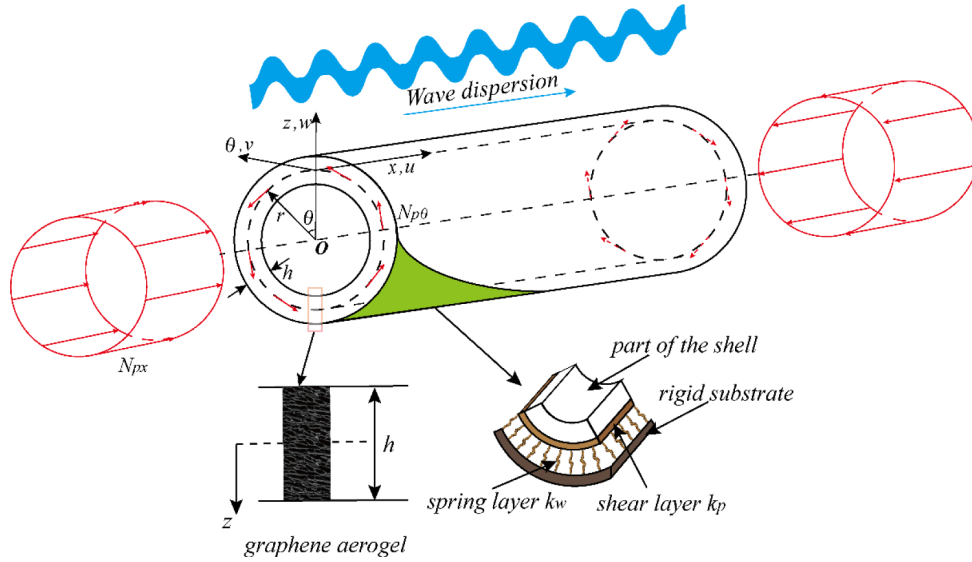
## 2. Material properties of 3D-GA cylindrical shells

A 3D-GA cylindrical shell with the middle-surface radius  $r$  and thickness  $h$ , subjected to applied axial and circumferential distributed forces  $N_{px}$  and  $N_{p\theta}$ , is shown in Figure 1. Suppose that the shell is resting on the Winkler–Pasternak elastic foundation with spring constant  $k_w$  and shear constant  $k_p$ . A cylindrical coordinate system  $(x, \theta, z)$  is set on the middle surface of the shell.

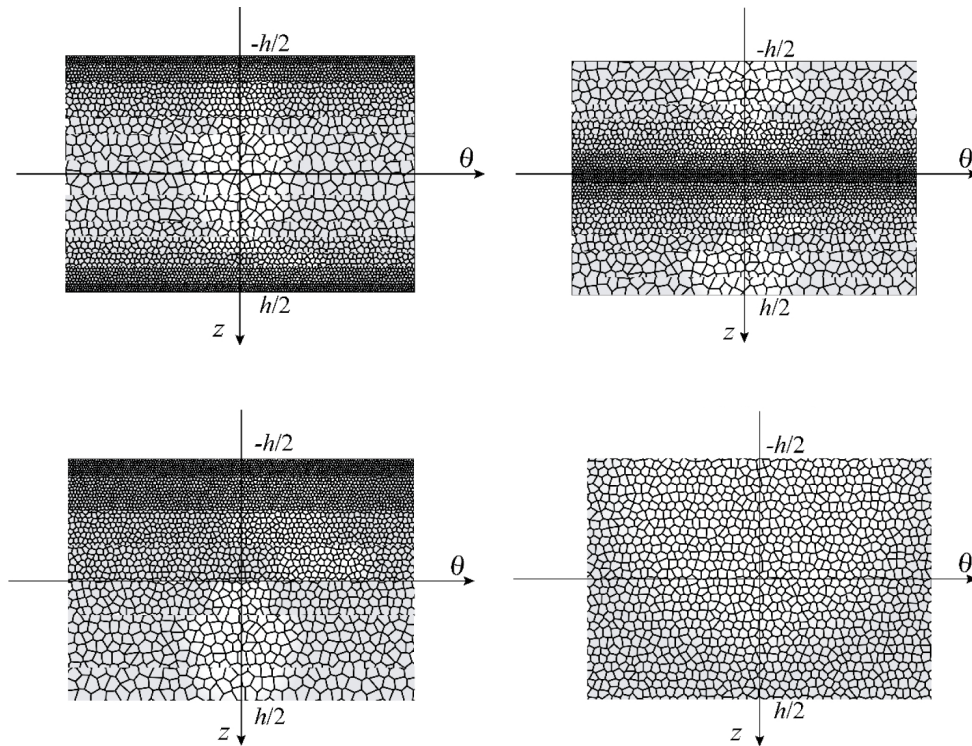
As illustrated in Figure 2, four types of porosity distribution across the shell thickness, namely, porosity-1, porosity-2, porosity-3 and porosity-4, are taken into account.

In the porosity-1 shell, the mass density and elastic moduli have the maxima on the inner and outer surfaces which are equal to corresponding material parameters of solid graphene without internal foams,





**Figure 1.** Schematic diagram of 3D-GA cylindrical shell.



**Figure 2.** Different types of porosity distribution: porosity-1 (top left), porosity-2 (top right), porosity-3 (bottom left) and porosity-4 (bottom right).

while the mass density and elastic moduli reach the minimum values on the middle-surface owing to the largest size of internal pores. In the porosity-2 shell, the mass density and elastic moduli are the minima on the inner and outer surfaces, while the maxima of mass density and elastic moduli are on the middle-surface. In the porosity-3 shell, mass density and elastic moduli are the minima on the outer surface and increase gradually to the maxima on the inner surface. In the porosity-4 shell, the mass density and elastic moduli remain constant.

Young's modules  $E(z)$ , shear modules  $G(z)$  and mass density  $\rho(z)$  for porosity-1, porosity-2, porosity-3 and porosity-4 shells are expressed below [Magnucki and Stasiewicz 2004; Jabbari et al. 2014; Chen et al. 2015; Yang et al. 2018; Wang et al. 2019].

Porosity-1 shell:

$$E(z) = E_g \left[ 1 - e_1 \cos \frac{\pi z}{h} \right], \quad (1a)$$

$$G(z) = G_g \left[ 1 - e_1 \cos \frac{\pi z}{h} \right], \quad (1b)$$

$$\rho(z) = \rho_g \left[ 1 - e_1^* \cos \frac{\pi z}{h} \right]. \quad (1c)$$

Porosity-2 shell:

$$E(z) = E_g \left\{ 1 - e_2 \left[ 1 - \cos \frac{\pi z}{h} \right] \right\}, \quad (2a)$$

$$G(z) = G_g \left\{ 1 - e_2 \left[ 1 - \cos \frac{\pi z}{h} \right] \right\}, \quad (2b)$$

$$\rho(z) = \rho_g \left\{ 1 - e_2^* \left[ 1 - \cos \frac{\pi z}{h} \right] \right\}. \quad (2c)$$

Porosity-3 shell:

$$E(z) = E_g \left[ 1 - e_3 \cos \left( \frac{\pi z}{2h} + \frac{\pi}{4} \right) \right], \quad (3a)$$

$$G(z) = G_g \left[ 1 - e_3 \cos \left( \frac{\pi z}{2h} + \frac{\pi}{4} \right) \right], \quad (3b)$$

$$\rho(z) = \rho_g \left[ 1 - e_3^* \cos \left( \frac{\pi z}{2h} + \frac{\pi}{4} \right) \right]. \quad (3c)$$

Porosity-4 shell:

$$E(z) = E_g \zeta, \quad (4a)$$

$$G(z) = G_g \zeta, \quad (4b)$$

$$\rho(z) = \rho_g \zeta^*, \quad (4c)$$

where the maximum value of Young's modules, shear modules and mass density are  $E_g$ ,  $G_g$  and  $\rho_g$ , respectively;  $e_1$ ,  $e_2$  and  $e_3$  ( $0 \leq e_1, e_2, e_3 < 1$ ) are porosity coefficients for the porosity-1, porosity-2 and porosity-3 shells, respectively; and the corresponding coefficients of mass density are  $e_1^*$ ,  $e_2^*$  and  $e_3^*$ ;  $\zeta$  and  $\zeta^*$  are corresponding coefficients for the porosity-4 shell. The shear modulus is given by

$$G(z) = \frac{E(z)}{2(1 + \mu)}, \quad (5)$$

where Poisson's ratio  $\mu$  is a constant.

For the graphene aerogel with open-cell foams, the relationship of the material properties is provided as [Qin et al. 2017]

$$\frac{E(z)}{E_g} = \left[ \frac{\rho(z)}{\rho_g} \right]^{2.73}. \quad (6)$$

Therefore, the relationships between porosity coefficients and mass density coefficients can be expressed as

$$1 - e_1^* \cos \frac{\pi z}{h} = \sqrt[2.73]{1 - e_1 \cos \frac{\pi z}{h}}, \quad (7)$$

$$1 - e_2^* \left[ 1 - \cos \frac{\pi z}{h} \right] = \sqrt[2.73]{1 - e_2 \left[ 1 - \cos \frac{\pi z}{h} \right]}, \quad (8)$$

$$1 - e_3^* \cos \left( \frac{\pi z}{2h} + \frac{\pi}{4} \right) = \sqrt[2.73]{1 - e_3 \cos \left( \frac{\pi z}{2h} + \frac{\pi}{4} \right)}, \quad (9)$$

$$\zeta^* = \sqrt[2.73]{\zeta}. \quad (10)$$

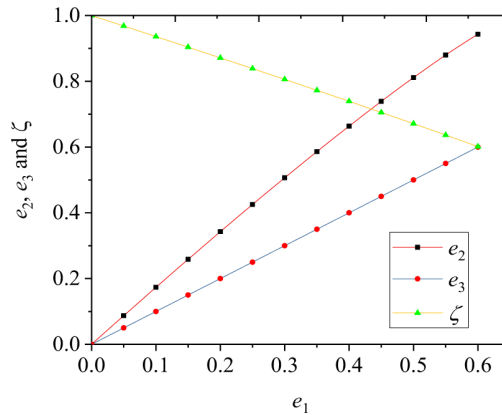
Without the loss of generality, let the masses of 3D-GA cylindrical shells with different porosity distributions keep equal to each other. Hence, we obtain [Yang et al. 2018]

$$\int_{-h/2}^{h/2} \sqrt[2.73]{1 - e_1 \cos \frac{\pi z}{h}} dz = \int_{-h/2}^{h/2} \sqrt[2.73]{1 - e_2 \left[ 1 - \cos \frac{\pi z}{h} \right]} dz, \quad (11)$$

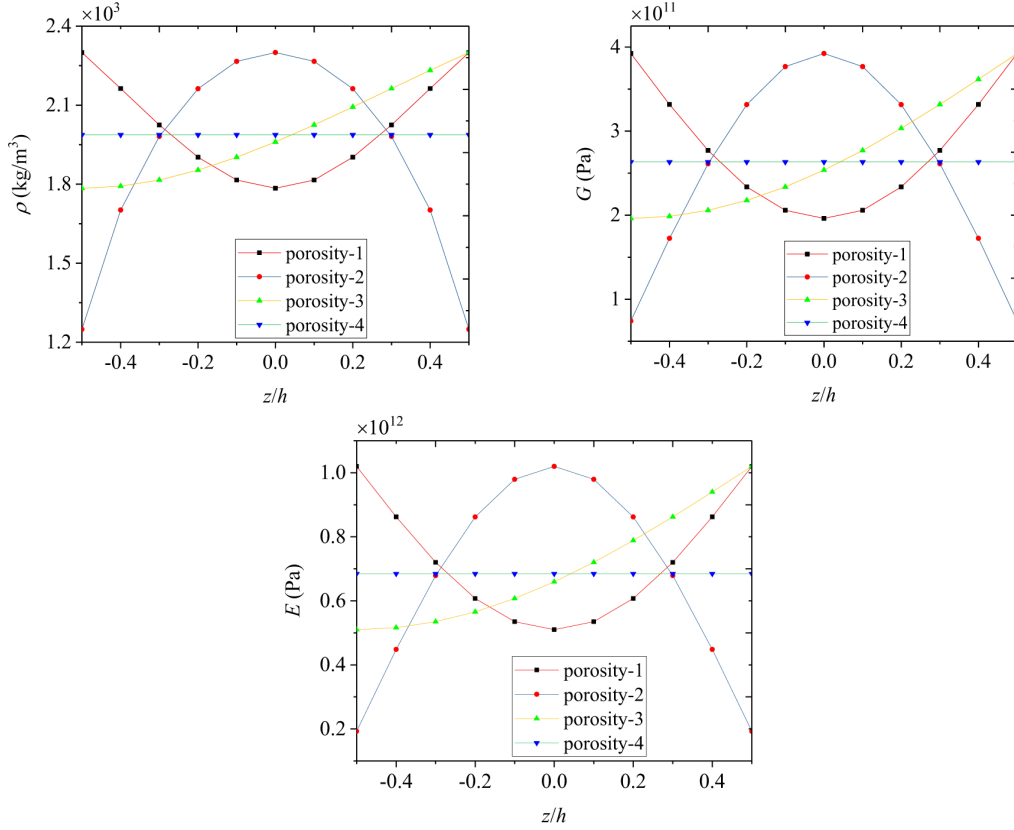
$$\int_{-h/2}^{h/2} \sqrt[2.73]{1 - e_1 \cos \frac{\pi z}{h}} dz = \int_{-h/2}^{h/2} \sqrt[2.73]{1 - e_3 \cos \left( \frac{\pi z}{2h} + \frac{\pi}{4} \right)} dz, \quad (12)$$

$$\int_{-h/2}^{h/2} \sqrt[2.73]{1 - e_1 \cos \frac{\pi z}{h}} dz = \int_{-h/2}^{h/2} \sqrt[2.73]{\zeta} dz, \quad (13)$$

which means that the values of  $e_2$ ,  $e_3$  and  $\zeta$  can be calculated by a given  $e_1$ . As observed in Figure 3, the increase of  $e_1$  leads to the increase of  $e_2$  and  $e_3$  and the decrease of  $\zeta$ . When  $e_1 = 0.6$ ,  $e_2$  approaches the upper limit. Therefore, the range of  $e_1$  ( $0 \leq e_1 \leq 0.6$ ) is selected in the following calculations.



**Figure 3.** Variations of porosity coefficients for different porosity distributions.



**Figure 4.** Variations of material properties along the radial direction: mass density (top left), shear modulus (top right) and Young's modulus (bottom).

In Figure 4, the changing curves of mass density, shear modulus and Young's modulus along the radial direction are depicted for porosity-1, porosity-2, porosity-3 and porosity-4 shells, respectively. Herein the following parameters are utilized:  $E_g = 1.02$  TPa,  $\mu = 0.3$ ,  $\rho_g = 2300$  kg/m<sup>3</sup>,  $e_0 = 0.5$ ,  $h = 0.1$  m,  $r = 0.3$  m.

### 3. Wave propagation via FSD shell theory

On the basis of the FSD shell theory [Reddy 2004], the displacement field of an arbitrary point in the shell along the  $x$ -,  $\theta$ - and  $z$ -axes, denoted by  $u_x(x, \theta, z, t)$ ,  $v_\theta(x, \theta, z, t)$  and  $w_z(x, \theta, z, t)$  are

$$u_x(x, \theta, z, t) = u(x, \theta, t) + z\phi_x(x, \theta, t), \quad (14)$$

$$v_\theta(x, \theta, z, t) = v(x, \theta, t) + z\phi_\theta(x, \theta, t), \quad (15)$$

$$w_z(x, \theta, z, t) = w(x, \theta, t), \quad (16)$$

where  $u(x, \theta, t)$ ,  $v(x, \theta, t)$  and  $w(x, \theta, t)$  are the displacements of a point at the midplane;  $t$  is time;  $\phi_x(x, \theta, t)$  and  $\phi_\theta(x, \theta, t)$  denote the rotations of a transverse normal about the  $\theta$ - and  $x$ -axes, respectively.



The relations between strains and displacements can be written as

$$\varepsilon_x = \frac{\partial u}{\partial x} + z \frac{\partial \phi_x}{\partial x}, \quad (17)$$

$$\varepsilon_\theta = \frac{1}{r} \left( \frac{\partial v}{\partial \theta} + w \right) + \frac{z}{r} \frac{\partial \phi_\theta}{\partial \theta}, \quad (18)$$

$$\gamma_{x\theta} = \frac{\partial v}{\partial x} + \frac{1}{r} \frac{\partial u}{\partial \theta} + z \left( \frac{\partial \phi_\theta}{\partial x} + \frac{1}{r} \frac{\partial \phi_x}{\partial \theta} \right), \quad (19)$$

$$\gamma_{xz} = \phi_x + \frac{\partial w}{\partial x}, \quad (20)$$

$$\gamma_{\theta z} = \phi_\theta + \frac{1}{r} \frac{\partial w}{\partial \theta} - \frac{v}{r}. \quad (21)$$

The relations between stresses and strains can be given by [Reddy 2004]

$$\sigma_x = \frac{E(z)}{1 - \mu^2} (\varepsilon_x + \mu \varepsilon_\theta), \quad (22)$$

$$\sigma_\theta = \frac{E(z)}{1 - \mu^2} (\varepsilon_\theta + \mu \varepsilon_x), \quad (23)$$

$$\tau_{x\theta} = G(z) \gamma_{x\theta}, \quad (24)$$

$$\tau_{xz} = G(z) \gamma_{xz}, \quad (25)$$

$$\tau_{\theta z} = G(z) \gamma_{\theta z}. \quad (26)$$

The resultant moments and forces are expressed as

$$\begin{Bmatrix} N_x \\ N_\theta \\ N_{x\theta} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \tau_{x\theta} \end{Bmatrix} dz, \quad (27)$$

$$\begin{Bmatrix} M_x \\ M_\theta \\ M_{x\theta} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \tau_{x\theta} \end{Bmatrix} z dz, \quad (28)$$

$$\begin{Bmatrix} Q_{\theta z} \\ Q_{xz} \end{Bmatrix} = \kappa_s \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{\theta z} \\ \tau_{xz} \end{Bmatrix} dz, \quad (29)$$

where the shear correction coefficient is  $\kappa_s = 5/6$  [Reddy 2004].

From (17)–(29), we obtain

$$N_x = A_{11} \frac{\partial u}{\partial x} + A_{12} \frac{\partial \phi_x}{\partial x} + A_{13} \frac{1}{r} \left( \frac{\partial v}{\partial \theta} + w \right) + A_{14} \frac{1}{r} \frac{\partial \phi_\theta}{\partial \theta}, \quad (30)$$

$$N_\theta = A_{11} \frac{1}{r} \left( \frac{\partial v}{\partial \theta} + w \right) + A_{12} \frac{1}{r} \frac{\partial \phi_\theta}{\partial \theta} + A_{13} \frac{\partial u}{\partial x} + A_{14} \frac{\partial \phi_x}{\partial x}, \quad (31)$$

$$N_{x\theta} = B_{11} \left( \frac{\partial v}{\partial x} + \frac{1}{r} \frac{\partial u}{\partial \theta} \right) + B_{12} \left( \frac{\partial \phi_\theta}{\partial x} + \frac{1}{r} \frac{\partial \phi_x}{\partial \theta} \right), \quad (32)$$

$$M_x = C_{11} \frac{\partial u}{\partial x} + C_{12} \frac{\partial \phi_x}{\partial x} + C_{13} \frac{1}{r} \left( \frac{\partial v}{\partial \theta} + w \right) + C_{14} \frac{1}{r} \frac{\partial \phi_\theta}{\partial \theta}, \quad (33)$$

$$M_\theta = C_{11} \frac{1}{r} \left( \frac{\partial v}{\partial \theta} + w \right) + C_{12} \frac{1}{r} \frac{\partial \phi_\theta}{\partial \theta} + C_{13} \frac{\partial u}{\partial x} + C_{14} \frac{\partial \phi_x}{\partial x}, \quad (34)$$

$$M_{x\theta} = D_{11} \left( \frac{\partial v}{\partial x} + \frac{1}{r} \frac{\partial u}{\partial \theta} \right) + D_{12} \left( \frac{\partial \phi_\theta}{\partial x} + \frac{1}{r} \frac{\partial \phi_x}{\partial \theta} \right), \quad (35)$$

$$Q_{xz} = \kappa_s B_{11} \left( \phi_x + \frac{\partial w}{\partial x} \right), \quad (36)$$

$$Q_{\theta z} = \kappa_s B_{11} \left( \phi_\theta + \frac{1}{r} \frac{\partial w}{\partial \theta} - \frac{v}{r} \right), \quad (37)$$

where the parameters  $A_{ij}$ ,  $B_{ij}$ ,  $C_{ij}$  and  $D_{ij}$  ( $i, j = 1, 2, 3, 4$ ) are given in Appendix A.

The strain energy of the 3D-GA shell is written as

$$S = \frac{1}{2} \int_0^L \int_0^{2\pi} \int_{-h/2}^{h/2} (\sigma_x \varepsilon_x + \sigma_\theta \varepsilon_\theta + \tau_{x\theta} \gamma_{x\theta} + \tau_{xz} \gamma_{xz} + \tau_{\theta z} \gamma_{\theta z}) r \, dz \, d\theta \, dx. \quad (38)$$

The kinetic energy is represented as

$$K = \frac{1}{2} \int_0^L \int_0^{2\pi} \int_{-h/2}^{h/2} \rho(z) \left[ \left( \frac{\partial u}{\partial t} + z \frac{\partial \phi_x}{\partial t} \right)^2 + \left( \frac{\partial v}{\partial t} + z \frac{\partial \phi_\theta}{\partial t} \right)^2 + \left( \frac{\partial w}{\partial t} \right)^2 \right] r \, dz \, d\theta \, dx. \quad (39)$$

The work done by the applied forces can be expressed as

$$W_F = \frac{1}{2} \int_0^L \int_0^{2\pi} \left[ N_{px} \left( \frac{\partial w}{\partial x} \right)^2 + N_{p\theta} \left( \frac{1}{r} \frac{\partial w}{\partial \theta} \right)^2 \right] r \, d\theta \, dx, \quad (40)$$

where the applied axial and circumferential distributed forces are given by

$$N_{px} = N_{p\theta} = N_p. \quad (41)$$

The additional strain energy results from the Winkler–Pasternak elastic foundation is written as [Winkler 1867; Pasternak 1954]

$$W_G = \frac{1}{2} \int_0^L \int_0^{2\pi} \left\{ k_w w^2 + k_p \left[ \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{1}{r} \frac{\partial w}{\partial \theta} \right)^2 \right] \right\} r \, d\theta \, dx. \quad (42)$$

By employing Hamilton's principle

$$\int_0^t [\delta K - (\delta S + \delta W_F + \delta W_G)] \, dt = 0, \quad (43)$$

the governing equations of the 3D-GA shell can be obtained as

$$\frac{\partial N_x}{\partial x} + \frac{1}{r} \frac{\partial N_{x\theta}}{\partial \theta} = I_1 \frac{\partial^2 u}{\partial t^2} + I_2 \frac{\partial^2 \phi_x}{\partial t^2}, \quad (44)$$

$$\frac{\partial N_{x\theta}}{\partial x} + \frac{1}{r} \frac{\partial N_\theta}{\partial \theta} + \frac{Q_{\theta z}}{r} = I_1 \frac{\partial^2 v}{\partial t^2} + I_2 \frac{\partial^2 \phi_\theta}{\partial t^2}, \quad (45)$$

$$\frac{\partial Q_{xz}}{\partial x} + \frac{1}{r} \frac{\partial Q_{\theta z}}{\partial \theta} - \frac{N_\theta}{r} + N_{px} \frac{\partial^2 w}{\partial x^2} + N_{p\theta} \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} - k_w w + k_p \left( \frac{\partial^2 w}{\partial x^2} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) = I_1 \frac{\partial^2 w}{\partial t^2}, \quad (46)$$

$$\frac{\partial M_x}{\partial x} + \frac{1}{r} \frac{\partial M_{x\theta}}{\partial \theta} - Q_{xz} = I_2 \frac{\partial^2 u}{\partial t^2} + I_3 \frac{\partial^2 \phi_x}{\partial t^2}, \quad (47)$$

$$\frac{\partial M_{x\theta}}{\partial x} + \frac{1}{r} \frac{\partial M_\theta}{\partial \theta} - Q_{\theta z} = I_2 \frac{\partial^2 v}{\partial t^2} + I_3 \frac{\partial^2 \phi_\theta}{\partial t^2}, \quad (48)$$

where the coefficients  $I_1$ ,  $I_2$  and  $I_3$  take the form of

$$I_1 = \int_{-h/2}^{h/2} \rho(z) \, dz, \quad (49)$$

$$I_2 = \int_{-h/2}^{h/2} \rho(z) z \, dz, \quad (50)$$

$$I_3 = \int_{-h/2}^{h/2} \rho(z) z^2 \, dz. \quad (51)$$

By substituting (30)–(37) into (44)–(48), it yields

$$\begin{aligned} A_{11} \frac{\partial^2 u}{\partial x^2} + A_{12} \frac{\partial^2 \phi_x}{\partial x^2} + \frac{A_{13}}{r} \left( \frac{\partial w}{\partial x} + \frac{\partial^2 v}{\partial x \partial \theta} \right) + \frac{A_{14}}{r} \frac{\partial^2 \phi_\theta}{\partial x \partial \theta} + B_{11} \left( \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r} \frac{\partial^2 v}{\partial x \partial \theta} \right) \\ + B_{12} \left( \frac{1}{r^2} \frac{\partial^2 \phi_x}{\partial \theta^2} + \frac{1}{r} \frac{\partial^2 \phi_\theta}{\partial x \partial \theta} \right) = I_1 \frac{\partial^2 u}{\partial t^2} + I_2 \frac{\partial^2 \phi_x}{\partial t^2}, \end{aligned} \quad (52)$$

$$\begin{aligned} \frac{A_{11}}{r^2} \left( \frac{\partial^2 v}{\partial \theta^2} + \frac{\partial w}{\partial \theta} \right) + \frac{A_{12}}{r^2} \frac{\partial^2 \phi_\theta}{\partial \theta^2} + \frac{A_{13}}{r} \frac{\partial^2 u}{\partial x \partial \theta} + \frac{A_{14}}{r} \frac{\partial^2 \phi_x}{\partial x \partial \theta} + B_{12} \left( \frac{1}{r} \frac{\partial^2 \phi_x}{\partial x \partial \theta} + \frac{\partial^2 \phi_\theta}{\partial x^2} \right) \\ + B_{11} \left( \frac{1}{r} \frac{\partial^2 u}{\partial x \partial \theta} + \frac{\partial^2 v}{\partial x^2} + \frac{\kappa_s}{r^2} \frac{\partial w}{\partial \theta} + \frac{\kappa_s}{r} \phi_\theta - \frac{\kappa_s}{r^2} v \right) = I_1 \frac{\partial^2 v}{\partial t^2} + I_2 \frac{\partial^2 \phi_\theta}{\partial t^2}, \end{aligned} \quad (53)$$

$$\begin{aligned} A_{11} \frac{1}{r^2} \left( w + \frac{\partial v}{\partial \theta} \right) + A_{12} \frac{1}{r^2} \frac{\partial \phi_\theta}{\partial \theta} + A_{13} \frac{1}{r} \frac{\partial u}{\partial x} + A_{14} \frac{1}{r} \frac{\partial \phi_x}{\partial x} \\ + \kappa_s B_{11} \left( \frac{\partial \phi_x}{\partial x} + \frac{\partial^2 w}{\partial x^2} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} + \frac{1}{r} \frac{\partial \phi_\theta}{\partial \theta} - \frac{1}{r^2} \frac{\partial v}{\partial \theta} \right) + N_{px} \frac{\partial^2 w}{\partial x^2} + \frac{N_{p\theta}}{r^2} \frac{\partial^2 w}{\partial \theta^2} \\ - k_w w + k_p \left( \frac{\partial^2 w}{\partial x^2} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) = I_1 \frac{\partial^2 w}{\partial t^2} \end{aligned} \quad (54)$$

$$\begin{aligned}
& -B_{11}\kappa_s \left( \phi_x + \frac{\partial w}{\partial x} \right) + C_{11} \frac{\partial^2 u}{\partial x^2} + C_{12} \frac{\partial^2 \phi_x}{\partial x^2} + C_{13} \frac{1}{r} \left( \frac{\partial w}{\partial x} + \frac{\partial^2 v}{\partial x \partial \theta} \right) + C_{14} \frac{1}{r} \frac{\partial^2 \phi_\theta}{\partial x \partial \theta} \\
& + D_{11} \left( \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} + \frac{1}{r} \frac{\partial^2 v}{\partial x \partial \theta} \right) + D_{12} \left( \frac{1}{r^2} \frac{\partial^2 \phi_x}{\partial \theta^2} + \frac{1}{r} \frac{\partial^2 \phi_\theta}{\partial x \partial \theta} \right) = I_2 \frac{\partial^2 u}{\partial t^2} + I_3 \frac{\partial^2 \phi_x}{\partial t^2}, \quad (55)
\end{aligned}$$

$$\begin{aligned}
& \kappa_s B_{11} \left( \frac{v}{r} - \frac{1}{r} \frac{\partial w}{\partial \theta} - \phi_\theta \right) + \frac{C_{11}}{r^2} \left( \frac{\partial w}{\partial \theta} + \frac{\partial^2 v}{\partial \theta^2} \right) + \frac{C_{12}}{r^2} \frac{\partial^2 \phi_\theta}{\partial \theta^2} + \frac{C_{13}}{r} \frac{\partial^2 u}{\partial x \partial \theta} + \frac{C_{14}}{r} \frac{\partial^2 \phi_x}{\partial x \partial \theta} \\
& + D_{11} \left( \frac{1}{r} \frac{\partial^2 u}{\partial x \partial \theta} + \frac{\partial^2 v}{\partial x^2} \right) + D_{12} \left( \frac{1}{r} \frac{\partial^2 \phi_x}{\partial x \partial \theta} + \frac{\partial^2 \phi_\theta}{\partial x^2} \right) = I_2 \frac{\partial^2 v}{\partial t^2} + I_3 \frac{\partial^2 \phi_\theta}{\partial t^2}. \quad (56)
\end{aligned}$$

The wave propagation solutions of (52)–(56) are expressed as

$$u(x, \theta, t) = U e^{i(kx+n\theta-\omega t)}, \quad (57)$$

$$v(x, \theta, t) = V e^{i(kx+n\theta-\omega t)}, \quad (58)$$

$$w(x, \theta, t) = W e^{i(kx+n\theta-\omega t)}, \quad (59)$$

$$\phi_x(x, \theta, t) = \Phi_x e^{i(kx+n\theta-\omega t)}, \quad (60)$$

$$\phi_\theta(x, \theta, t) = \Phi_\theta e^{i(kx+n\theta-\omega t)}, \quad (61)$$

where  $i = \sqrt{-1}$ ;  $k$  and  $n$  are the wave numbers in the longitudinal and circumferential directions, respectively;  $\omega$  is the frequency of wave motion;  $U$ ,  $V$ ,  $W$ ,  $\Phi_x$  and  $\Phi_\theta$  are the amplitudes of wave motion.

Substituting (57)–(61) into (52)–(56) yields a generalized eigenvalue problem:

$$(L_{5 \times 5} - \omega^2 H_{5 \times 5}) \begin{Bmatrix} U \\ V \\ W \\ \Phi_x \\ \Phi_\theta \end{Bmatrix} = \{0\}, \quad (62)$$

where the elements  $L_{ij}$  and  $H_{ij}$  ( $i, j = 1, 2, \dots, 5$ ) in the matrix  $L_{5 \times 5}$  and  $H_{5 \times 5}$  can be found in Appendix B.

The dispersion relation derived from (62) takes the form of

$$\text{Det}[L_{5 \times 5} - \omega^2 H_{5 \times 5}] = 0. \quad (63)$$

The relation between the wave frequency  $\omega$  and phase velocity  $v$  is [Wang and Varadan 2007]

$$v = \omega / k. \quad (64)$$

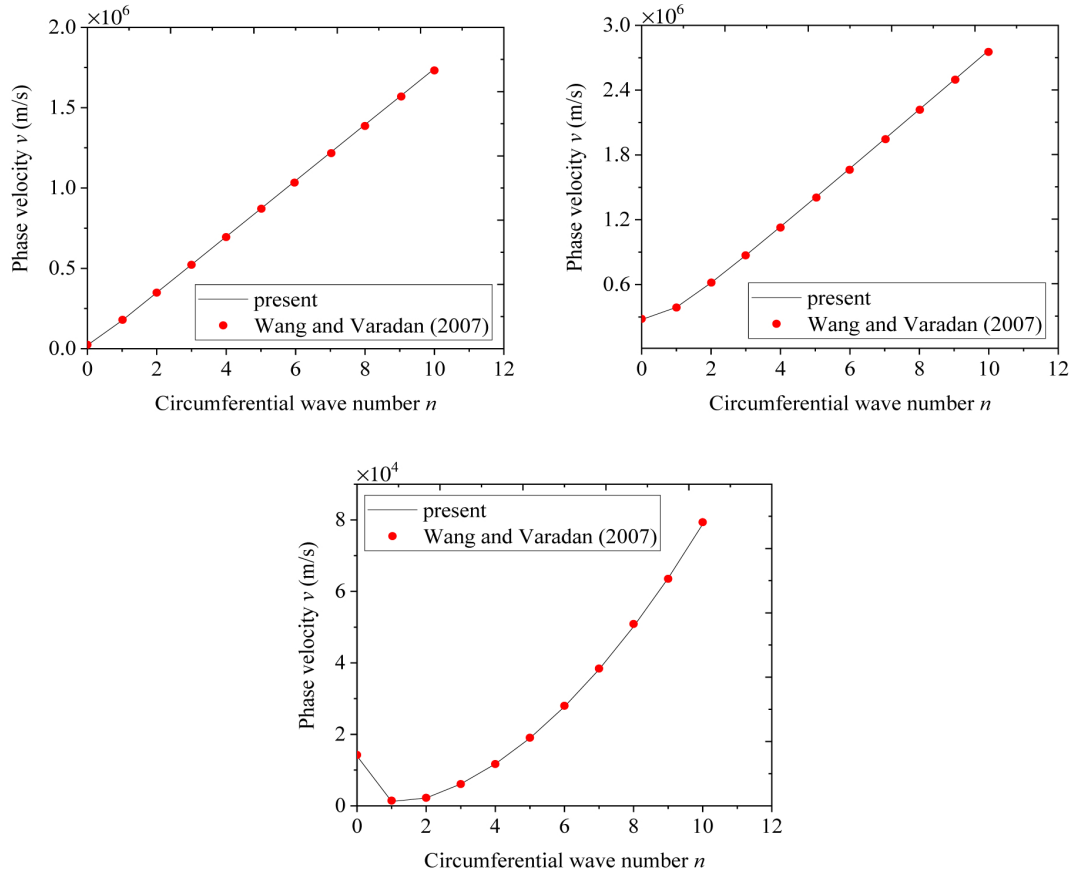
According to (63) and (64), five positive analytical solutions of wave phase velocity can be obtained. The first three low-value solutions which correspond to the coupled longitudinal, radial and circumferential (L-R-C) modes are discussed hereinbelow.



#### 4. Results and discussion

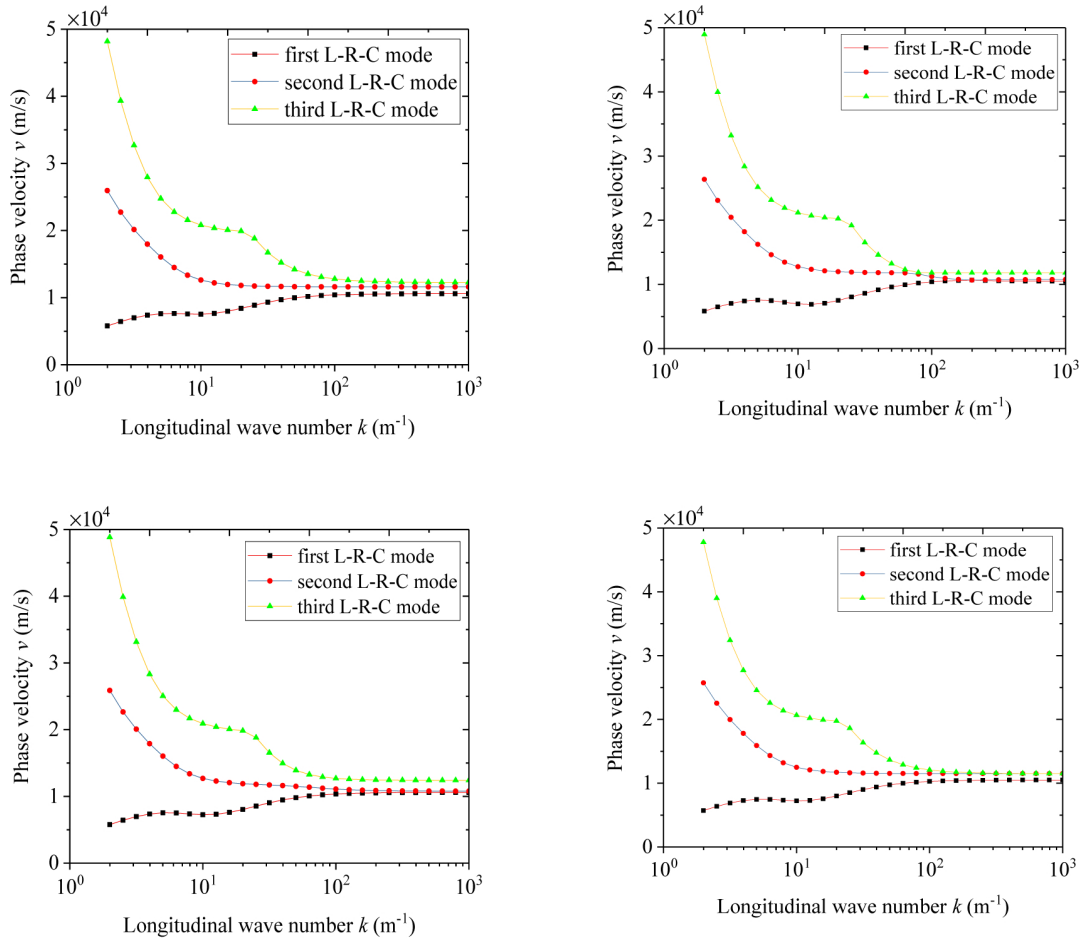
For the purpose of demonstrating the effectiveness of the present analysis, Figure 5 presents a comparative study of the dispersion relation between phase velocity and circumferential wave number for a homogeneous cylindrical shell. The present 3D-GA cylindrical shell model can be simplified to a homogeneous cylindrical shell by setting  $N_p = 0$  N,  $k_w = 0$  N/m<sup>3</sup>,  $k_p = 0$  N/m and  $e_1 = 0$ . The thickness, the middle-surface radius, Poisson's ratio and the mass density of the cylindrical shell are  $h = 0.34$  nm,  $r = 5$  nm,  $\mu = 0.2$  and  $\rho = 2.27$  g/cm<sup>3</sup>, respectively. The bending rigidity is  $D = 2$  eV and the in-plane stiffness is  $Eh = 360$  J/m<sup>2</sup>. The results from the classical shell theory were given in [Wang and Varadan 2007]. The present results have good consistency with those in the literature, manifesting the effectiveness of the present analysis.

Hereinafter, the wave propagation analysis in 3D-GA cylindrical shells is conducted. If not specified, the following geometric and material parameters are utilized:  $h = 0.1$  m,  $r = 0.3$  m,  $E_g = 1.02$  TPa,  $\rho_g = 2300$  kg/m<sup>3</sup> and  $\mu = 0.3$ .



**Figure 5.** Comparisons of wave characteristics in a homogenous cylindrical shell ( $k = 8 \times 10^6$  m<sup>-1</sup>,  $e_0 a = 0$ ): the first mode (top left), the second mode (top right) and the third mode (bottom).

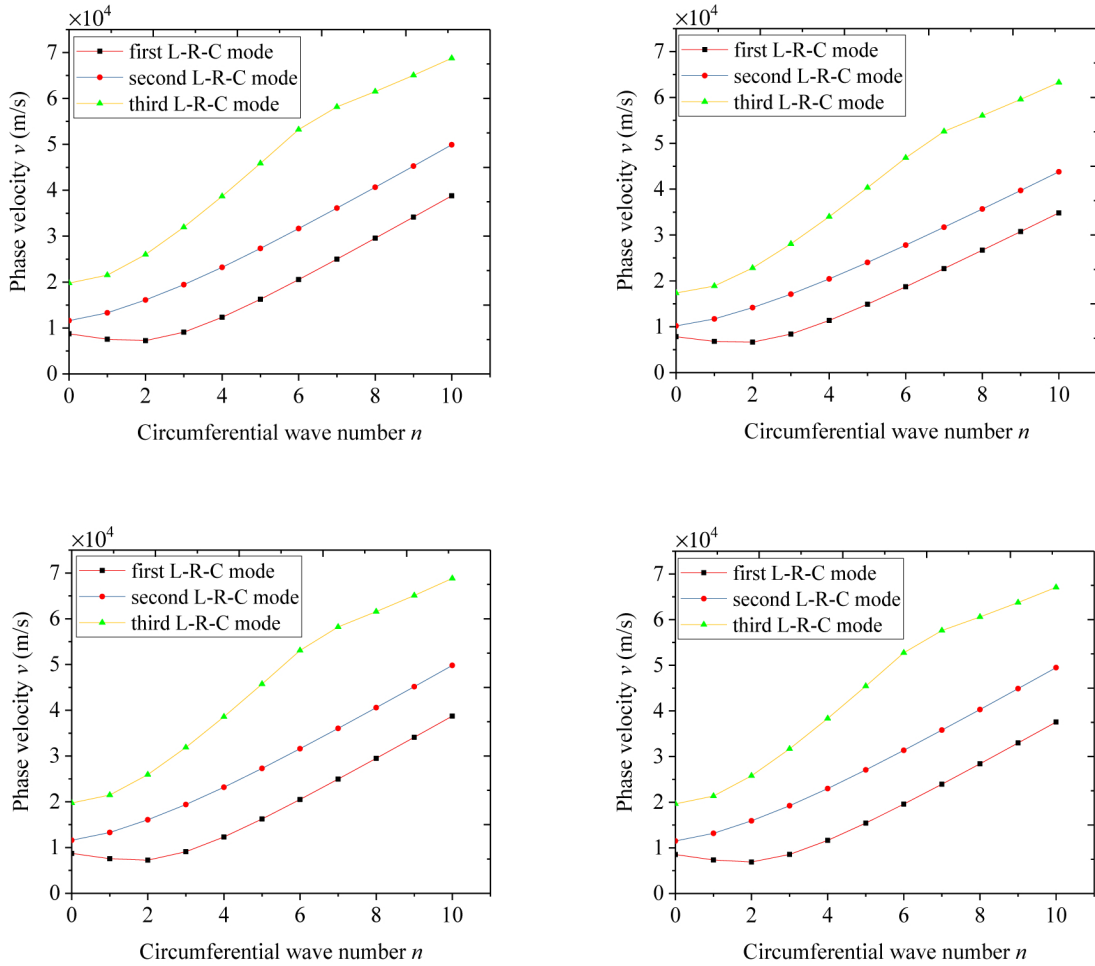
Figure 6 demonstrates the curves of phase velocity versus longitudinal wave number  $k$  of 3D-GA shells for the L-R-C modes. Dispersion relations are investigated for porosity-1, porosity-2, porosity-3 and porosity-4 shells. For clearer discussion, we define the mode number by the numerical order of phase velocity in the following results, which is different from Figure 5. As can be observed, for all porosity distributions, the phase velocity for the first L-R-C mode fluctuates initially and then tends to be constant. As for the second and third L-R-C modes, the phase velocities initially exhibit a decreasing trend and then tend to be constant. The nonlinear variation of phase velocities is because the coupling exists among the longitudinal, radial and circumferential wave modes, which arises from the coupled displacement fields. It is worth mentioning that at the larger longitudinal wave number  $k > 10^2 \text{ m}^{-1}$ , or at corresponding smaller longitudinal wavelength  $\lambda = 1/k < 0.01 \text{ m}$ , all the phase velocities for the L-R-C modes are close to each other.



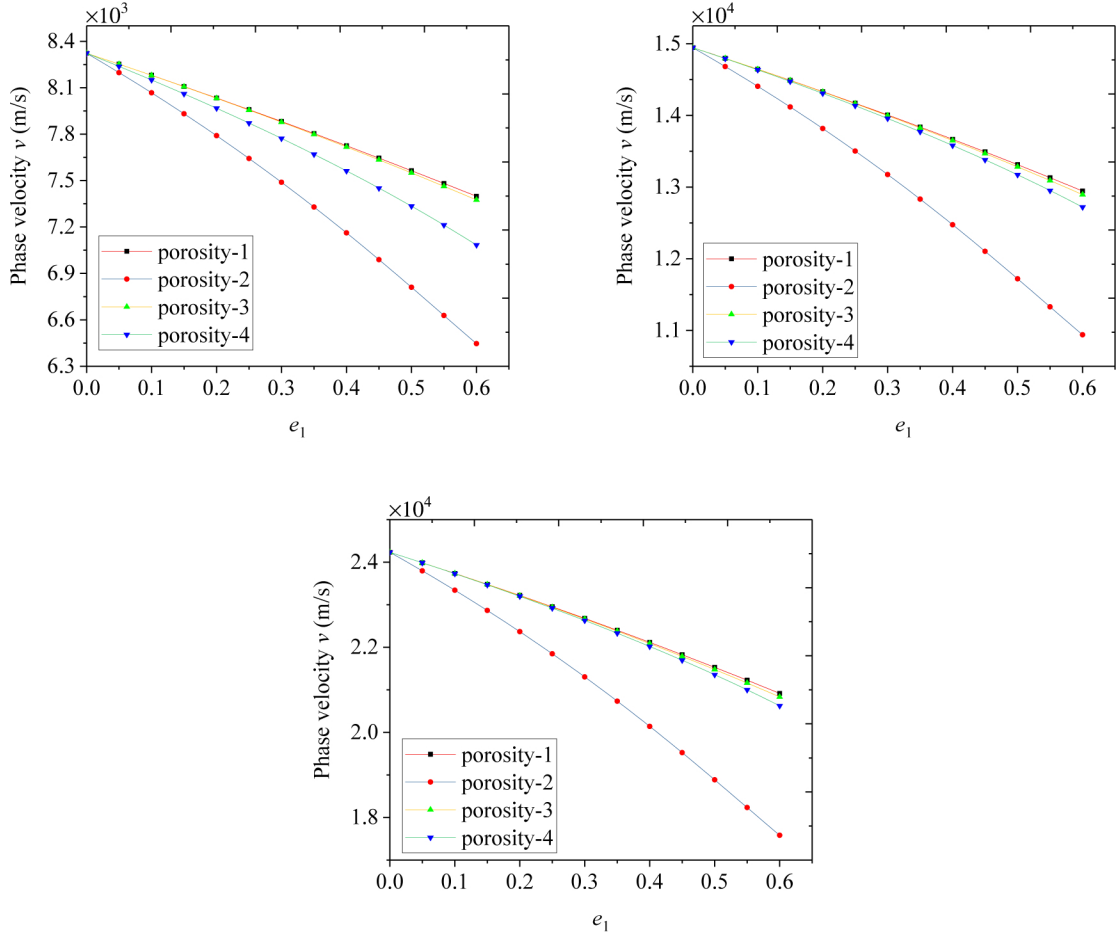
**Figure 6.** Phase velocity versus longitudinal wave number  $k$  of 3D-GA shell ( $e_1 = 0.5$ ,  $n = 1$ ,  $N_p = 0 \text{ N}$ ,  $k_w = 0 \text{ N/m}^3$ ,  $k_p = 0 \text{ N/m}$ ): porosity-1 (top left), porosity-2 (top right), porosity-3 (bottom left) and porosity-4 (bottom right).

In Figure 7, the dispersion relations between the phase velocity and circumferential wave number  $n$  for different porosity distributions are shown. It is found that for the first L-R-C mode, the phase velocity decreases initially and then increases with the circumferential wave number. Moreover, the lowest phase velocity occurs at  $n = 2$ . As for the second and third L-R-C modes, the phase velocities exhibit an increasing-trend variation with increasing circumferential wave number.

Figure 8 depicts the effect of porosity coefficient  $e_1$  on the dispersion relations for different porosity distributions. One can find that the phase velocities in the 3D-GA shell decrease with the increasing porosity coefficient. Among all types of porosity distribution, the porosity-1 shell has the largest phase velocity and the porosity-2 shell has the smallest phase velocity. Moreover, the phase velocity in the porosity-4 shell is smaller than that in the porosity-3 shell.



**Figure 7.** Phase velocity versus circumferential wave number  $n$  of 3D-GA shell ( $e_1 = 0.5$ ,  $k = 8 \text{ m}^{-1}$ ,  $N_p = 0 \text{ N}$ ,  $k_w = 0 \text{ N/m}^3$ ,  $k_p = 0 \text{ N/m}$ ): porosity-1 (top left), porosity-2 (top right), porosity-3 (bottom left) and porosity-4 (bottom right).

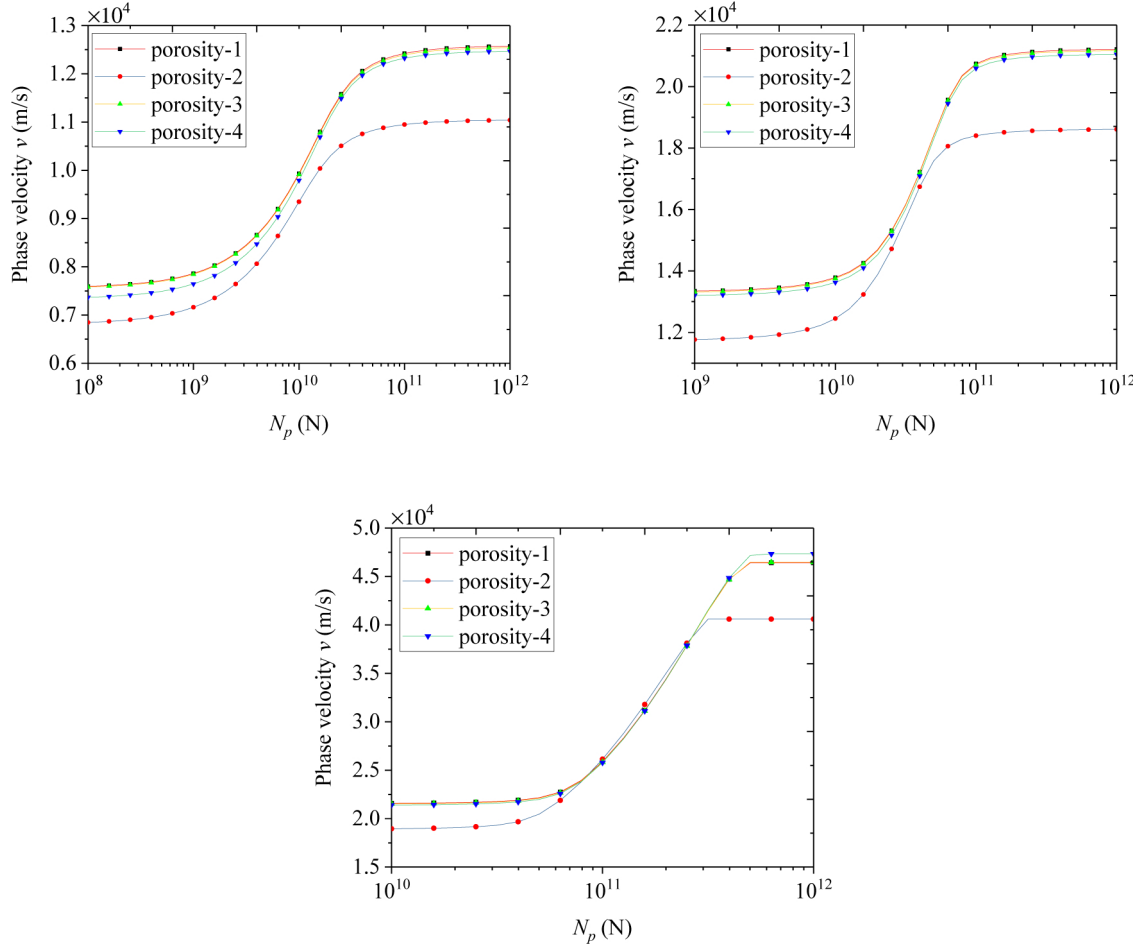


**Figure 8.** Phase velocity versus porosity coefficient  $e_1$  of 3D-GA shell ( $n = 1$ ,  $k = 8 \text{ m}^{-1}$ ,  $N_p = 0 \text{ N}$ ,  $k_w = 0 \text{ N/m}^3$ ,  $k_p = 0 \text{ N/m}$ ): the first L-R-C mode (top left), the second L-R-C mode (top right), and the third L-R-C mode (bottom).

The influence of applied force  $N_p$  on the dispersion relations of the 3D-GA shell is plotted in Figure 9. For all the porosity distributions, the phase velocities initially increase slightly, then increase rapidly, and finally approach to constant. This interesting phenomenon indicates that the applied force has significant effect on the dispersion relation in certain range; beyond this range, however, it affects slightly the dispersion relation.

Figure 10 illustrates the effect of spring constant  $k_w$  on the dispersion relations of 3D-GA shells with different porosity distributions. It is interesting that a similar variation tendency of phase velocity can be observed with the comparison of Figure 9. In certain range, spring constant has significant influence on the phase velocity but it is insignificant for phase velocity beyond this range. For example, the phase velocity for the first L-R-C mode changes rapidly when  $10^{11} \text{ N/m}^3 \leq k_w \leq 10^{13} \text{ N/m}^3$ , while it almost remains constant if  $k_w < 10^{11} \text{ N/m}^3$  or  $k_w > 10^{13} \text{ N/m}^3$ .

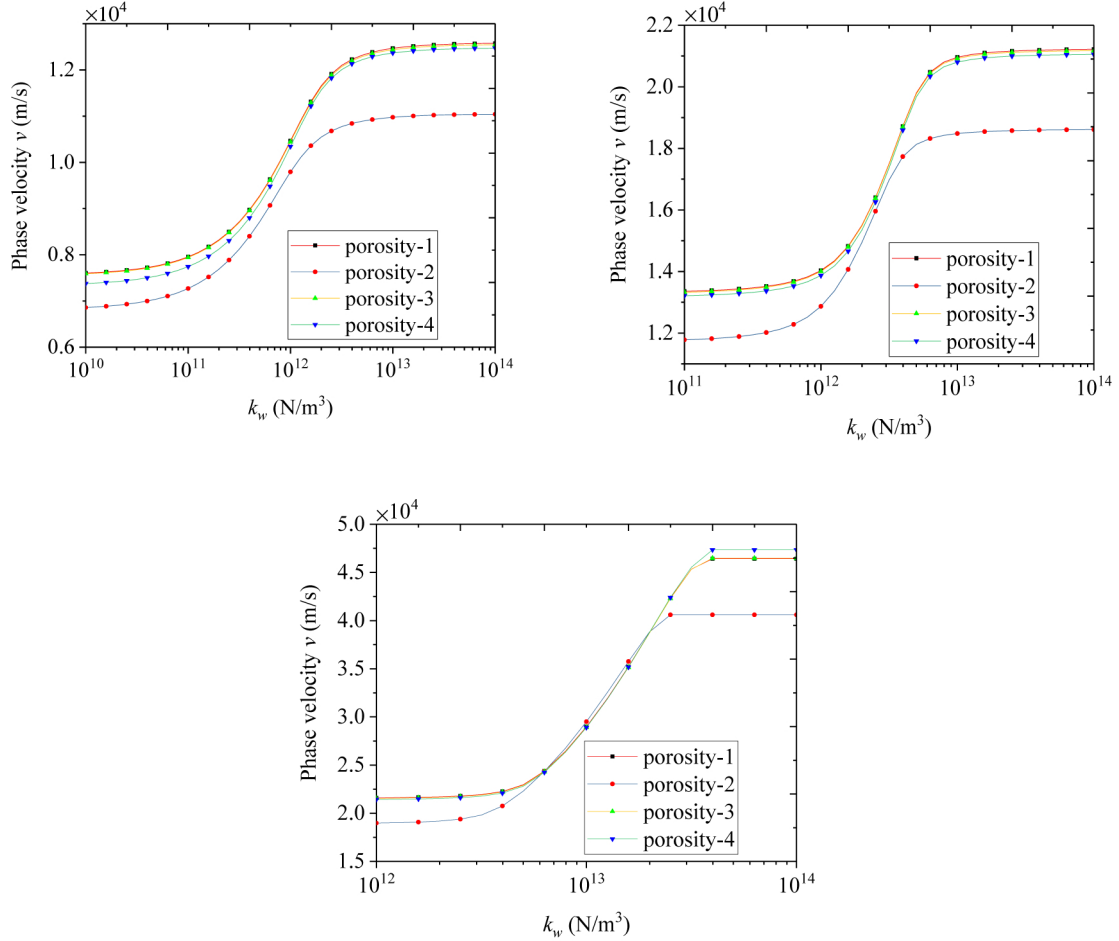




**Figure 9.** Phase velocity versus applied force  $N_p$  of 3D-GA shell ( $e_1 = 0.5$ ,  $n = 1$ ,  $k = 8 \text{ m}^{-1}$ ,  $k_w = 0 \text{ N/m}^3$ ,  $k_p = 0 \text{ N/m}$ ): the first L-R-C mode (top left), the second L-R-C mode (top right), and the third L-R-C mode (bottom).

Figure 11 examines the effect of shear constant  $k_p$  on the dispersion relations of the 3D-GA shell. When the shear constant is in the certain range, it has conspicuous effect on the phase velocity of the 3D-GA shell. From Figures 9–11, it is concluded that the applied force, spring constant and shear constant have similar influence on the phase velocity of 3D-GA shells. Such observations are significant for guiding the structural design of 3D-GA shells by adjusting the corresponding parameters.

In Figure 12, phase velocity versus radius-to-thickness ratio  $r/h$  of the 3D-GA shell is depicted. It is found that the phase velocities for the L-R-C modes initially decrease rapidly and then tend to be constant. In particular, the phase velocities for the second and third L-R-C modes decrease faster than that for the first L-R-C mode. Additionally, it is worth mentioning that when  $r/h > 15$ , the phase velocities are insensitive to the radius-to-thickness ratio.



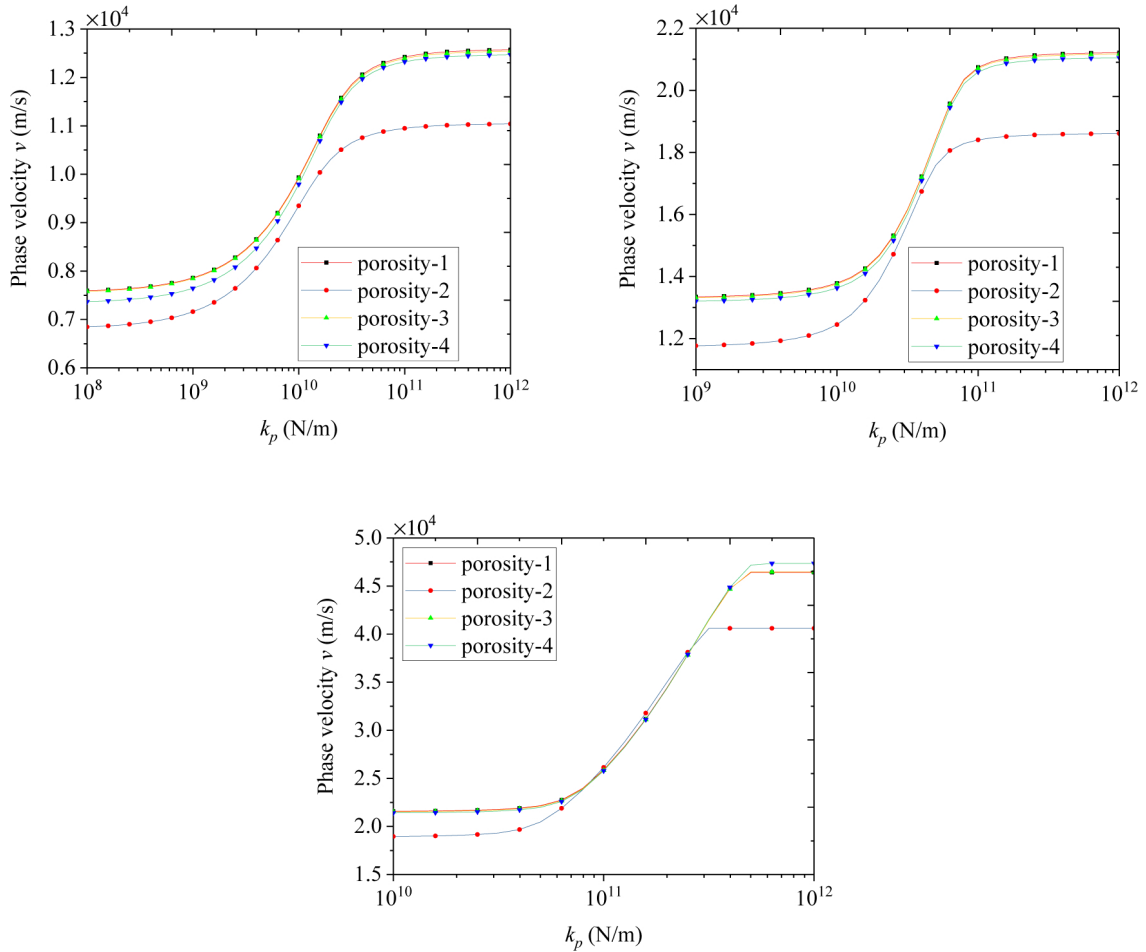
**Figure 10.** Phase velocity versus spring constant  $k_w$  of 3D-GA shell ( $e_1 = 0.5$ ,  $n = 1$ ,  $k = 8 \text{ m}^{-1}$ ,  $N_p = 0 \text{ N}$ ,  $k_p = 0 \text{ N/m}$ ): the first L-R-C mode (top left), the second L-R-C mode (top right), and the third L-R-C mode (bottom).

## 5. Concluding remarks

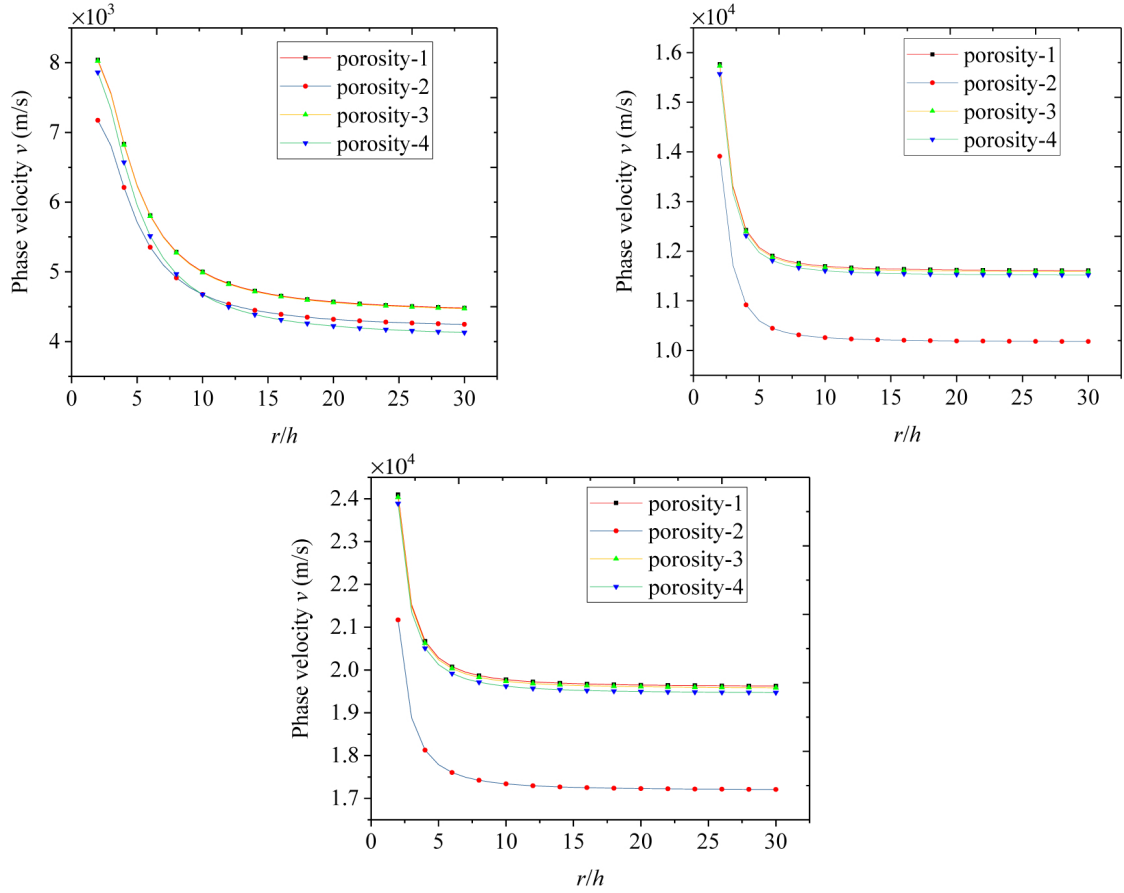
Wave propagation characteristics in 3D-GA cylindrical shells are studied in the framework of the FSD shell theory. Hamilton's principle is utilized to derive the governing equations. The effects of longitudinal and circumferential wave numbers, the porosity distribution, the porosity coefficient, the applied forces, the Winkler–Pasternak elastic foundation and the radius-to-thickness ratio on wave dispersion relations of 3D-GA shells are investigated. The main conclusions are summarized as follows:

- (1) An increasing porosity coefficient leads to a decrease in the phase velocities of 3D-GA shells. Among different types of porosity distribution, the porosity-1 shell has the largest phase velocity whereas the porosity-2 shell has the smallest phase velocity.

- (2) For the L-R-C modes of 3D-GA shells, the phase velocities show nonlinear variation with the change of longitudinal and circumferential wave numbers. It is found that phase velocities of the three L-R-C modes approach to constant and are close to each other when the longitudinal wave number is large.
- (3) As the applied forces, the spring constant and the shear constant increase, the phase velocities of 3D-GA shells initially increase slightly, then increase rapidly, and finally approach to constant.
- (4) With the increase in radius-to-thickness ratio, the phase velocities of 3D-GA shells initially decrease rapidly but finally are insensitive to the radius-to-thickness ratio.



**Figure 11.** Phase velocity versus shear constant  $k_p$  of 3D-GA shell ( $e_1 = 0.5$ ,  $n = 1$ ,  $k = 8 \text{ m}^{-1}$ ,  $N_p = 0 \text{ N}$ ,  $k_w = 0 \text{ N/m}^3$ ): the first L-R-C mode (top left), the second L-R-C mode (top right), and the third L-R-C mode (bottom).



**Figure 12.** Phase velocity versus radius-to-thickness ratio  $r/h$  of 3D-GA shell ( $e_1 = 0.5$ ,  $n = 1$ ,  $k = 8 \text{ m}^{-1}$ ,  $N_p = 0 \text{ N}$ ,  $k_w = 0 \text{ N/m}^3$ ,  $k_p = 0 \text{ N/m}$ ): the first L-R-C mode (top left), the second L-R-C mode (top right), and the third L-R-C mode (bottom).

### Appendix A

$$\begin{aligned}
 A_{11} &= \int_{-h/2}^{h/2} \frac{E(z)}{1 - \mu^2} dz & A_{12} &= \int_{-h/2}^{h/2} \frac{E(z)}{1 - \mu^2} z dz & A_{13} &= \int_{-h/2}^{h/2} \frac{\mu E(z)}{1 - \mu^2} dz & A_{14} &= \int_{-h/2}^{h/2} \frac{\mu E(z)}{1 - \mu^2} z dz \\
 B_{11} &= \int_{-h/2}^{h/2} G(z) dz & B_{12} &= \int_{-h/2}^{h/2} G(z) z dz & C_{11} &= A_{12} & C_{12} &= \int_{-h/2}^{h/2} \frac{E(z)}{1 - \mu^2} z^2 dz \\
 C_{13} &= A_{14} & C_{14} &= \int_{-h/2}^{h/2} \frac{\mu E(z)}{1 - \mu^2} z^2 dz & D_{11} &= B_{12} & D_{12} &= \int_{-h/2}^{h/2} G(z) z^2 dz
 \end{aligned}$$

### Appendix B

$$\begin{aligned}
 L_{11} &= A_{11} k^2 r^2 + B_{11} n^2 & L_{12} &= (A_{13} + B_{11}) knr & L_{13} &= -i A_{13} kr \\
 L_{14} &= A_{12} k^2 r^2 + B_{12} n^2 & L_{15} &= (A_{14} + B_{12}) knr & L_{21} &= (A_{13} + B_{11}) knr
 \end{aligned}$$



$$\begin{aligned}
L_{22} &= A_{11}n^2 + B_{11}k^2r^2 + \kappa_s B_{11} & L_{23} &= -in(A_{11} + \kappa_s B_{11}) & L_{24} &= (A_{14} + B_{12}) knr \\
L_{25} &= A_{12}n^2 - \kappa_s B_{11}r + B_{12}k^2r^2 & L_{31} &= iA_{13}kr & L_{32} &= in(A_{11} + \kappa_s B_{11}) \\
L_{33} &= A_{11} + \kappa_s B_{11}(n^2 + k^2r^2) + N_{px}k^2r^2 + N_{p\theta}n^2 + k_w r^2 + k_p(n^2 + k^2r^2) \\
L_{34} &= i(A_{14} - \kappa_s B_{11}r) kr & L_{35} &= in(A_{12} - \kappa_s B_{11}r) & L_{41} &= C_{11}k^2r^2 + D_{11}n^2 \\
L_{42} &= (C_{13} + D_{11}) knr & L_{43} &= i(\kappa_s B_{11}r - C_{13}) kr & L_{44} &= \kappa_s B_{11}r^2 + C_{12}k^2r^2 + D_{12}n^2 \\
L_{45} &= (C_{14} + D_{12}) knr & L_{51} &= (C_{13} + D_{11}) knr & L_{52} &= -\kappa_s B_{11}r + C_{11}n^2 + D_{11}k^2r^2 \\
L_{53} &= in(\kappa_s B_{11}r - C_{11}) & L_{54} &= (C_{14} + D_{12}) knr & L_{55} &= \kappa_s B_{11}r^2 + C_{12}n^2 + D_{12}k^2r^2 \\
H_{11} &= I_1r^2 & H_{14} &= I_2r^2 & H_{12} &= H_{13} = H_{15} = 0 & H_{21} &= H_{23} = H_{24} = 0 \\
H_{22} &= I_1r^2 & H_{25} &= I_2r^2 & H_{31} &= H_{32} = H_{34} = H_{35} = 0 & H_{33} &= I_1r^2 \\
H_{41} &= I_2r^2 & H_{44} &= I_3r^2 & H_{42} &= H_{43} = H_{45} = 0 & H_{51} &= H_{53} = H_{54} = 0 \\
H_{52} &= I_2r^2 & H_{55} &= I_3r^2
\end{aligned}$$

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
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