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SEMIINFINITE MOVING CRACK IN A SHEAR-FREE ORTHOTROPIC STRIP

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We have considered a semiinfinite crack moving with constant velocity in an orthotropic strip with shear-free boundaries. The crack is propagating subjected to constant normal displacements applied at the boundaries of the strip. The Fourier transformation is applied to convert the boundary value problem into the standard Wiener–Hopf equation. This equation has been solved to find out the analytical expressions for the stress intensity factor and crack-opening displacement. The graphs of stress intensity factor and crack-opening displacement have been plotted against various parameters such as crack velocity, strip width etc. to show the effects of these parameters and material orthotropy on stress intensity factor and crack-opening displacement.

1. Introduction

In recent times the study of the nature of elastic waves in the presence of cracks has gained momentum in many engineering applications like designing metal and polymer-forming processes, machining, etc. These types of research problems have great importance and versatility in potential applications in the fields of seismology and geophysics. The analytical study of cracks and inclusion is very important in civil, aerospace, nuclear and mechanical engineering, especially in civil and mechanical engineering where designing of load-bearing components of vehicles, power generation, reduction of cracks are important and challenging issues. The foremost motive in civil structure is to elude the progress of a crack initiated originally. Researchers found that the stress has a square root singularity at the tip of the crack. A nondimensional quantity called the stress intensity factor has been calculated to show the nature of stress at the tip of the crack. Many researchers did their work in this field to find the stress intensity factor and other expressions related to fracture. Initially researchers considered the problems involving static cracks only.

The moving Griffith crack model was introduced in [Yoffe 1951]. She considered that the crack propagates with a constant speed and without a change in length along the crack propagation axis. Knauss [1966] studied the problem of stresses in an infinitely long isotropic strip of finite width containing a straight semiinfinite crack for the case of displaced clamped boundaries normal to the crack. He applied the Wiener–Hopf technique to find the stress intensity factor and stresses. The correction to this work was made in [Rice 1967]. Nilsson [1972; 1973] explored a method to find the analytical expression for the stress intensity factor by the Wiener–Hopf technique. The same technique was later used in [Atkinson and Popelar 1979; Kousiounelos and Williams Jr. 1982; Georgiadis 1986]. Georgiadis and Papadopoulos [1987] analyzed the steady state solution for the stress intensity factor in a cracked plane orthotropic strip by the Wiener–Hopf technique. De and Patra [1990] considered the problem of a moving Griffith crack

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in a stressed orthotropic strip. The problem of four coplanar Griffith cracks moving in an infinitely long elastic strip under antiplane shear stress was solved in [Sarkar et al. 1996]. Lee [2000] obtained stress and displacement fields for the propagating crack along the interface of dissimilar orthotropic materials under dynamic mode I and mode II load. Wang et al. [2001] analyzed the problem of the dynamic stress intensity factor for a semiinfinite crack in orthotropic materials with concentrated shear impact loads. They found out the dynamic stress intensity factor by the Wiener–Hopf technique for orthotropic as well as isotropic media. The problem of the stress intensity factor around a moving Griffith crack in an infinite elastic layer between two elastic half-planes was studied in [Itou 2004]. Ma et al. [2005] considered the problem of the moving Griffith crack in the functionally graded orthotropic strip under plane loading. Bagheri et al. [2015] investigated the analytical solution of multiple moving cracks in functionally graded piezoelectric strip. Nourazar and Ayatollahi [2016] studied the problem of multiple moving interfacial cracks between two dissimilar piezoelectric layers under electromechanical loading.

This paper is a generalization of investigations done in [Nilsson 1972; 1973; Georgiadis and Papadopoulos 1987]. Nilsson obtained the expression for the stress intensity factor for an isotropic strip weakened by a semiinfinite moving crack. He considered both shear-free and clamped strip boundaries. Later, Georgiadis and Papadopoulos investigated the problems of an orthotropic strip weakened by a semiinfinite static crack. They first (1987) considered the problem where shear-free boundaries were considered and later (1988) they solved another problem where clamped boundaries were considered. The dynamical problem of the work of Georgiadis and Papadopoulos [1988] had been done in [Basak and Mandal 2017]. We have considered the dynamical problem of the work done in [Georgiadis and Papadopoulos 1987]. Here we have considered an orthotropic strip which contains a semiinfinite crack moving along the mid-plane of the strip with constant velocity. The crack surfaces and the strip boundaries have been assumed traction-free. The crack is subjected to constant normal displacements applied at the boundaries of the strip. In order to reduce the boundary value problem to the standard Wiener–Hopf equation, we have applied the Fourier transform technique. One can solve the Wiener–Hopf equation to obtain the necessary expression for the stresses and the displacements. Due to mathematical complexities, the explicit solution of the Wiener–Hopf equation has not been found; instead, asymptotic expressions for the stresses and displacements have been obtained for limited cases only. However, this asymptotic solution is sufficient to obtain the analytical expressions for the stress intensity factor and crack-opening displacement, which are the quantities of physical interest. One thing here to note that many of the earlier problems involving the Wiener–Hopf technique solved by many authors, did not follow the correct analysis of the method as pointed in [Nilsson 1973]. We have applied the analysis of the Wiener–Hopf method as revised and corrected in [Nilsson 1973]. Finally, the deductions of the solution have been made for isotropic and statical cases to compare and ensure the accuracy of the solution.

2. Formulation of the problem

Let us consider the problem of interaction of a semiinfinite crack situated at the interior of an orthotropic strip of width $2h$ moving along the plane with constant velocity c. The crack is propagating due to the actions of the constant normal displacements applied at the boundaries of the strip. Let X, Y, Z be the fixed cartesian coordinates which are the axes of symmetry of the orthotropic material. The strip is defined by $-\infty < X < \infty,\ -h < Y < h,\ -\infty < Z < \infty$. At any time t, position of the crack is assumed as $-\infty < X \le ct,\ Y = 0$, which is propagating along the positive X-axis with a constant velocity c. Due to symmetry we shall consider the two dimensional (X, Y) problem.

The nonvanishing displacements $U(X, Y, t)$ and $V(X, Y, t)$ along the X- and Y-axes satisfy the following Navier's equations:

$$C_{11}\frac{\partial^2 U}{\partial X^2} + C_{66}\frac{\partial^2 U}{\partial Y^2} + (C_{12} + C_{66})\frac{\partial^2 V}{\partial X\,\partial Y} = \rho\frac{\partial^2 U}{\partial t^2}, \tag{1}$$

$$C_{66}\frac{\partial^2 V}{\partial X^2} + C_{22}\frac{\partial^2 V}{\partial Y^2} + (C_{12} + C_{66})\frac{\partial^2 U}{\partial X\,\partial Y} = \rho\frac{\partial^2 V}{\partial t^2}, \tag{2}$$

where C_{66} is the shear modulus, ρ is the material density and C_{11}, C_{22}, C_{12} are material constants related to orthotropic elastic constants by

$$C_{11} = E_1/\big(1 - (E_2/E_1)\,\nu_{12}^2\big), \quad C_{22} = (E_2/E_1)\,C_{11}, \quad C_{12} = \nu_{12}C_{22} = \nu_{21}C_{11}$$

for generalized plane stress problems. Moreover E_1, E_2 are Young's Moduli and ν_{12}, ν_{21} are Poisson's ratios of the medium.

To make the crack stationary, the Galilean transformation $x = X - ct,\ y = Y,\ t = t$ is introduced so that the above displacement equations (1) and (2) reduce to

$$(C_{11} - \rho c^2)\frac{\partial^2 u}{\partial x^2} + C_{66}\frac{\partial^2 u}{\partial y^2} + (C_{12} + C_{66})\frac{\partial^2 v}{\partial x\,\partial y} = 0, \tag{3}$$

$$(C_{66} - \rho c^2)\frac{\partial^2 v}{\partial x^2} + C_{22}\frac{\partial^2 v}{\partial y^2} + (C_{12} + C_{66})\frac{\partial^2 u}{\partial x\,\partial y} = 0, \tag{4}$$

where $u(x, y) = U(X, Y, t)$ and $v(x, y) = V(X, Y, t)$ are the displacements in the moving coordinate system.

The required stresses can be obtained from the well-known relations

$$\tau_{xx} = C_{11}\frac{\partial u}{\partial x} + C_{12}\frac{\partial v}{\partial y}, \tag{5}$$

$$\tau_{yy} = C_{12}\frac{\partial u}{\partial x} + C_{22}\frac{\partial v}{\partial y}, \tag{6}$$

$$\tau_{xy} = C_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right). \tag{7}$$

The crack surfaces are assumed to be traction-free, so we consider the following boundary conditions (see Figure 1):

$$\tau_{yy}(x, 0) = 0, \qquad x < 0, \tag{8}$$

$$\tau_{xy}(x, 0) = 0, \qquad -\infty < x < \infty, \tag{9}$$

$$v(x, 0) = 0, \qquad x > 0, \tag{10}$$

$$\tau_{xy}(x, \pm h) = 0, \qquad -\infty < x < \infty, \tag{11}$$

$$v(x, \pm h) = \pm v_0, \qquad -\infty < x < \infty, \tag{12}$$

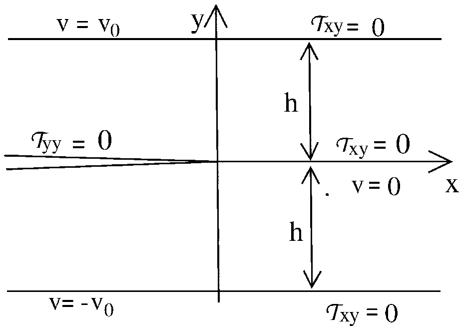

Figure 1. Geometry of the original problem.

where v_0 is a known constant.

We wish to solve the problem by using the Wiener–Hopf technique, but the above boundary conditions are not eligible for applying that technique. It can be shown that [Georgiadis and Papadopoulos 1987] the above set of boundary conditions can be converted to a more suitable form (see Figure 2):

$$\tau_{yy}(x,0)=\tau_0 \qquad x<0, \tag{13}$$

$$v(x,0)=0, \qquad x>0, \tag{14}$$

$$\tau_{xy}(x,0)=0, \quad -\infty<x<\infty, \tag{15}$$

$$\tau_{xy}(x,\pm h)=0, \quad -\infty<x<\infty, \tag{16}$$

$$v(x,\pm h)=0, \quad -\infty<x<\infty \tag{17}$$

by introducing a constant homogeneous load $\tau_{yy}=\tau_0$ in the system. Although the selection of the constant load τ_0 is not arbitrary, it satisfies the relation $\tau_0=-(C_{22}/h)v_0$ for the given problem.

Now we slightly change the boundary condition (13) as discussed in [Nilsson 1973] introducing a slightly variable load

$$\tau_{yy}(x,0)=\tau_0\, e^{\epsilon x}, \quad x<0, \tag{13$'$}$$

where ϵ is a very small positive quantity which can be assumed as tending to zero. Since the problem is symmetric with respect to the x-axis, it is sufficient to consider the half-strip $0\le y\le h$ only.

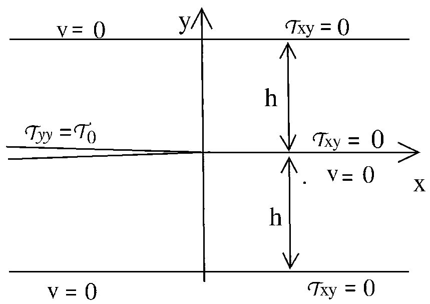

Figure 2. Geometry of the transformed problem.

The well-known Fourier transform is defined by

$$\bar{f}(\omega,\ y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x,y)\,e^{i\omega x}\,dx \tag{18}$$

with the inverse

$$f(x,y)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \bar{f}(\omega,y)\,e^{-i\omega x}\,d\omega, \tag{19}$$

where $\omega=\sigma+i\tau$ is the complex variable in the Fourier transform plane.

Now introducing the Fourier transform on (3) and (4), the solutions can be assumed as

$$\bar{u}(\omega,y)=P(\omega)\,e^{\gamma_1\omega y}+Q(\omega)\,e^{-\gamma_1\omega y}+R(\omega)\,e^{\gamma_2\omega y}+S(\omega)\,e^{-\gamma_2\omega y}, \tag{20}$$

$$\bar{v}(\omega,y)=i[\alpha_1P(\omega)\,e^{\gamma_1\omega y}-\alpha_1Q(\omega)\,e^{-\gamma_1\omega y}+\alpha_2R(\omega)\,e^{\gamma_2\omega y}-\alpha_2S(\omega)\,e^{-\gamma_2\omega y}], \tag{21}$$

where $\bar{u}(\omega,y)$ and $\bar{v}(\omega,y)$ are Fourier transforms of $u(x,y)$ and $v(x,y)$, respectively; α_j, $(j=1,2)$ are given by

$$\alpha_j=\frac{C_{11}-\rho c^2-C_{66}\,\gamma_j^2}{(C_{12}+C_{66})\,\gamma_j},\qquad j=1,2, \tag{22}$$

where $i=\sqrt{-1}$ and γ_1^2, γ_2^2 are the positive roots of the equation

$$C_{22}\,C_{66}\,\gamma^4+\{C_{12}^2+2C_{12}\,C_{66}-C_{11}\,C_{22}+(C_{66}+C_{22})\,\rho c^2\}\,\gamma^2+(C_{11}-\rho c^2)(C_{66}-\rho c^2)=0. \tag{23}$$

Moreover, $P(\omega)$, $Q(\omega)$, $R(\omega)$ and $S(\omega)$ are unknown functions of ω.

The expressions for the stresses can be obtained as

$$\frac{1}{C_{66}}\bar{\tau}_{xy}(\omega,y)=\omega(\gamma_1+\alpha_1)\big(e^{\gamma_1\omega y}P(\omega)-e^{-\gamma_1\omega y}Q(\omega)\big)+\omega(\gamma_2+\alpha_2)\big(e^{\gamma_2\omega y}R(\omega)-e^{-\gamma_2\omega y}S(\omega)\big), \tag{24}$$

$$\begin{aligned}\bar{\tau}_{yy}(\omega,y)=i\omega(C_{22}\,\alpha_1\gamma_1-C_{12})&\big(e^{\gamma_1\omega y}P(\omega)+e^{-\gamma_1\omega y}Q(\omega)\big)\\&+i\omega(C_{22}\,\alpha_2\gamma_2-C_{12})\big(e^{\gamma_2\omega y}R(\omega)+e^{-\gamma_2\omega y}S(\omega)\big),\end{aligned} \tag{25}$$

where $\bar{\tau}_{xy}(\omega,y)$ and $\bar{\tau}_{yy}(\omega,y)$ are the Fourier transforms of $\tau_{xy}(x,y)$ and $\tau_{yy}(x,y)$.

3. Method of solution

The normal stress $\tau_{yy}(x,0)$ and displacement $v(x,0)$ are unknown for $x>0$ and $x<0$. Hence we consider

$$\tau_{yy}(x,0)=f(x),\qquad x>0, \tag{26}$$

$$v(x,0)=g(x),\qquad x<0. \tag{27}$$

The objective of the method is to obtain the expression of the stress $f(x)$ at the tip of the crack. We define half-range Fourier transforms of the above functions as

$$\bar{f}_+(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^\infty f(x)\, e^{i\omega x}\, dx, \tag{28}$$

$$\bar{g}_-(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^0 g(x)\, e^{i\omega x}\, dx, \tag{29}$$

where the existence of the above transforms are still unclear but can be proved by the following assumptions that the stresses and displacements are bounded at the infinity, so the functions $f(x)$ and $g(x)$ should be bounded there. Therefore, without any loss of generality we may assume that

$$|f(x)| < F x^{-l_f}, \qquad \text{as } x \to \infty, \tag{30}$$

$$|g(x)| < G|x|^{-l_g}, \qquad \text{as } x \to -\infty \tag{31}$$

for some $l_f > 0$, $l_g > 0$ with F and G being finite positive numbers. The conditions (30) and (31) ensure the existence of the transforms (28) and (29); in fact the functions $\bar{f}_+(\omega)$ and $\bar{g}_-(\omega)$ are now analytic for $\tau \geq 0$ and $\tau \leq 0$.

Introducing Fourier transforms on the boundary conditions (15), (16) and (17) with the help of (21) and (24), the unknown functions $Q(\omega)$, $R(\omega)$ and $S(\omega)$ can be expressed in terms of $P(\omega)$ as

$$Q(\omega) = \frac{\Delta_1}{\Delta} P(\omega), \quad R(\omega) = \frac{\Delta_2}{\Delta} P(\omega), \quad S(\omega) = \frac{\Delta_3}{\Delta} P(\omega), \tag{32}$$

where

$$\Delta_1 = 2(\gamma_2 + \alpha_2)\, e^{\gamma_1 \omega h} \sinh(\gamma_2 \omega h)(\alpha_2 \gamma_1 - \alpha_1 \gamma_2), \tag{33}$$

$$\Delta_2 = 2(\gamma_1 + \alpha_1)\, e^{-\gamma_2 \omega h} \sinh(\gamma_1 \omega h)(\alpha_1 \gamma_2 - \alpha_2 \gamma_1), \tag{34}$$

$$\Delta_3 = 2(\gamma_1 + \alpha_1)\, e^{\gamma_2 \omega h} \sinh(\gamma_1 \omega h)(\alpha_1 \gamma_2 - \alpha_2 \gamma_1), \tag{35}$$

$$\Delta = 2(\gamma_2 + \alpha_2)\, e^{-\gamma_1 \omega h} \sinh(\gamma_2 \omega h)(\alpha_2 \gamma_1 - \alpha_1 \gamma_2). \tag{36}$$

Next, using the conditions (13′), (14) with the help of (28), (29) we get a pair of equations involving $\bar{f}_+(\omega)$ and $\bar{g}_-(\omega)$ as

$$\bar{\tau}_{yy}(\omega, 0) = \bar{f}_+(\omega) + \frac{\tau_0}{\sqrt{2\pi}(\epsilon + i\omega)}, \tag{37}$$

$$\bar{v}(\omega, 0) = \bar{g}_-(\omega). \tag{38}$$

Replacing the values of the stress and displacement from (21), (25) and after some manipulation using (32)–(36) we get the following equation of two unknown functions:

$$\bar{f}_+(\omega) = K(\omega)\, \bar{g}_-(\omega) - \frac{\tau_0}{\sqrt{2\pi}(\epsilon + i\omega)}, \tag{39}$$

where the kernel $K(\omega)$ is given by

$$K(\omega) = \omega K_1(\omega)/K_2(\omega), \tag{40}$$

with

$$K_1(\omega) = (C_{22}\,\alpha_1\gamma_1 - C_{12})(1+e^{2\gamma_1\omega h})\,\Delta + (C_{22}\,\alpha_2\gamma_2 - C_{12})(1+e^{2\gamma_2\omega h})\,\Delta_2, \tag{41}$$

$$K_2(\omega) = (\alpha_1 - \alpha_1\, e^{2\gamma_1\omega h})\,\Delta + (\alpha_2 - \alpha_2\, e^{2\gamma_2\omega h})\,\Delta_2. \tag{42}$$

Equation (39) is the standard Wiener–Hopf equation with the kernel given by (40).

The first and most important step of the Wiener–Hopf technique is the factorization of the kernel $K(\omega)$ in the following form:

$$K(\omega) = K_+(\omega)\,K_-(\omega) \tag{43}$$

such that the function $K_+(\omega)$ is analytic and nonzero in some upper half plane $\tau > \tau_1$ $(\tau_1 < 0)$ and $K_-(\omega)$ is analytic and nonzero in some lower half plane $\tau < \tau_2$ $(\tau_2 > 0)$.

Once the factorization (43) is done, (39) can be written as

$$\frac{\bar{f}_+(\omega)}{K_+(\omega)} = K_-(\omega)\,\bar{g}_-(\omega) - \frac{\tau_0}{\sqrt{2\pi}\,(\epsilon + i\omega)\,K_+(\omega)}. \tag{44}$$

The next task is the decomposition of the last part of (44) as

$$\frac{\tau_0}{\sqrt{2\pi}\,(\epsilon + i\omega)\,K_+(\omega)} = L(\omega) = L_+(\omega) + L_-(\omega), \tag{45}$$

where

$$L_+(\omega) = \frac{\tau_0}{\sqrt{2\pi}\,(\epsilon + i\omega)}\left[\frac{1}{K_+(\omega)} - \frac{1}{K_+(i\epsilon)}\right], \tag{46}$$

$$L_-(\omega) = \frac{\tau_0}{\sqrt{2\pi}\,(\epsilon + i\omega)\,K_+(i\epsilon)}. \tag{47}$$

It can be shown [Noble 1958] that $L_+(\omega)$ and $L_-(\omega)$ are analytic and nonzero in the regions $\tau > \tau_1$ and $\tau < \epsilon$.

Now utilizing (45), equation (44) can be written as

$$\frac{\bar{f}_+(\omega)}{K_+(\omega)} + L_+(\omega) = K_-(\omega)\,\bar{g}_-(\omega) - L_-(\omega), \tag{48}$$

It may be noted that the regions of analyticity of the functions $\bar{f}_+(\omega)$, $\bar{g}_-(\omega)$, $K_+(\omega)$, $K_-(\omega)$, $L_+(\omega)$ and $L_-(\omega)$ are $\tau \geq 0$, $\tau \leq 0$, $\tau > \tau_1(\tau_1 < 0)$, $\tau < \tau_2(\tau_2 > 0)$, $\tau > \tau_1(\tau_1 < 0)$ and $\tau < \epsilon$, respectively. Therefore, the left-hand side of (48) is analytic in the upper half plane $\tau \geq 0$ and the right-hand side is analytic in the lower half plane $\tau \leq 0$ for any arbitrary small positive values of ϵ. Since the regions of analyticity overlap and the line $\tau = 0$ is the common line of analyticity, hence by a well-known theorem on the analytic continuation the whole equation (48) is analytic and single valued throughout the complex ω-plane. We may now assume that both sides of (48) is equal to an entire function, say $J(\omega)$.

It may be easily verified that both $K_+(\omega)$ and $K_-(\omega)$ tend to $\omega^{1/2}$ for large ω. Furthermore, $\bar{f}_+(\omega)$ and $\bar{g}_-(\omega)$ are bounded for large ω. Therefore, the left-hand side of (48) and consequently $J(\omega)$ tends to $\omega^{-1/2}$ for large value of ω in the upper half plane $\tau \geq 0$; similarly, the right-hand side of (48) and consequently $J(\omega)$ tends to $\omega^{1/2}$ for large ω in the lower half plane $\tau \leq 0$. Hence, by the extended Liouville theorem it can be concluded that $J(\omega)$ is a constant, more specifically zero.

Therefore, from (46)–(48) we find

$$\bar{f}_+(\omega) = \frac{\tau_0}{\sqrt{2\pi}(\epsilon + i\omega)}\left[\frac{K_+(\omega)}{K_+(i\epsilon)} - 1\right], \quad \bar{g}_-(\omega) = \frac{\tau_0}{\sqrt{2\pi}(\epsilon + i\omega)}\frac{1}{K_+(i\epsilon)K_-(\omega)}.$$

Now for constant loading we may take $\epsilon \to 0$; consequently the above equations become

$$\bar{f}_+(\omega) = \frac{\tau_0}{\sqrt{2\pi}\,i\omega}\left[\frac{K_+(\omega)}{K_+(0)} - 1\right], \tag{49}$$

$$\bar{g}_-(\omega) = \frac{\tau_0}{\sqrt{2\pi}\,i\omega}\frac{1}{K_+(0)K_-(\omega)}. \tag{50}$$

The main difficulty in the Wiener–Hopf technique is the decomposition of the kernel $K(\omega)$. Due to mathematical complexities, factorization of the kernel $K(\omega)$ is not easy. Nilsson [1972] introduced a technique where behavior of normal stress at the crack-tip can be obtained by only knowing the values of $K(\omega)$ for very large ω and also for very small ω.

It can be easily verified that

$$\lim_{\omega\to\infty}\frac{K(\omega)}{\omega} = \theta, \tag{51}$$

where

$$\theta = \frac{(C_{22}\alpha_2\gamma_2 - C_{12})(\gamma_1+\alpha_1) - (C_{22}\alpha_1\gamma_1 - C_{12})(\gamma_2+\alpha_2)}{\alpha_1\gamma_2 - \alpha_2\gamma_1} \tag{52}$$

and

$$\lim_{\omega\to 0} K(\omega) = -\frac{C_{22}}{h} - \frac{C_{12}(\gamma_1^2 - \gamma_2^2 + \alpha_1\gamma_1 - \alpha_2\gamma_2)}{h\gamma_1\gamma_2(\alpha_1\gamma_2 - \alpha_2\gamma_1)} = \theta_0, \text{ say.} \tag{53}$$

Now for very large values of ω, equations (49) and (50) can be written as

$$\lim_{\omega\to\infty}\bar{f}_+(\omega) = \lim_{\omega\to\infty}\frac{\tau_0}{\sqrt{2\pi}\,i\,K_+(0)\,\omega^{1/2}}\frac{K_+(\omega)}{\omega^{1/2}} - \lim_{\omega\to\infty}\frac{\tau_0}{\sqrt{2\pi}\,i\omega} \tag{54}$$

and

$$\lim_{\omega\to\infty}\bar{g}_-(\omega) = \lim_{\omega\to\infty}\frac{\tau_0}{\sqrt{2\pi}\,i\,K_+(\omega)\,\omega^{3/2}}\frac{\omega^{1/2}}{K_-(\omega)}, \tag{55}$$

Taking Inverse Fourier transform on (54) and (55) and using asymptotic property with the help of equations (51)–(53) we get

$$\lim_{x\to 0^+} f(x) = -\tau_0\sqrt{\frac{2\theta}{\pi\theta_0}}\,x^{-1/2}, \tag{56}$$

and

$$\lim_{x\to 0^-} g(x) = -\tau_0\sqrt{\frac{1}{\pi\theta\theta_0}}\,(-x)^{1/2}. \tag{57}$$

Equation (56) shows the normal stress τ_{yy} just outside the crack-tip. It is found that the normal stress component τ_{yy} has a square root singularity at the tip of the crack which was expected. Moreover, equation (57) represents the displacement v just inside the crack-tip.

4. Quantities of physical interest

The stress intensity factor (SIF) that denotes the state of stress at the crack-tip is defined as

$$\mathrm{SIF} = \lim_{x \to 0^+} \sqrt{2\pi x}\, \tau_{yy}(x, 0) \tag{58}$$

and found to be

$$\mathrm{SIF} = -\tau_0 \sqrt{2\theta/\theta_0}, \tag{59}$$

where θ is given by (52). Therefore, the stress intensity factor of the original problem is given by

$$K_I = \sqrt{\frac{2\theta}{\theta_0}} \frac{C_{22} v_0}{h}. \tag{60}$$

Another quantity of physical interest is the crack-opening displacement (COD):

$$\mathrm{COD} = v(x, 0^+) - v(x, 0^-). \tag{61}$$

Since the problem is symmetric with respect to the x-axis, we can write

$$\mathrm{COD} = -2\tau_0 \sqrt{\frac{1}{\pi\theta\theta_0}} (-x)^{1/2}. \tag{62}$$

Therefore, COD of the original problem normalized with respect to v_0 is

$$\mathrm{COD} = 2\sqrt{\frac{-x}{\pi\theta\theta_0}} \frac{C_{22}}{h}. \tag{63}$$

Comparison of results. For isotropic media we write $C_{11} = C_{22} = \lambda + 2\mu$, $C_{12} = \lambda$ and $C_{66} = \mu$, where λ and μ are Lamé constants; so we get

$$\gamma_1^2 = 1 - c^2/c_1^2, \tag{64}$$

$$\gamma_2^2 = 1 - c^2/c_2^2, \tag{65}$$

$$\alpha_1 = \gamma_1^{-1}, \tag{66}$$

$$\alpha_2 = \gamma_2, \tag{67}$$

$$\theta = \frac{\mu\{(1+\gamma_1^2)^2 - 4\gamma_1\gamma_2\}}{\gamma_2(1-\gamma_1^2)}, \tag{68}$$

$$\theta_0 = \frac{\mu}{h} . \frac{(1+\gamma_1^2)^2 - 4\gamma_2^2}{\gamma_2^2(1-\gamma_1^2)}, \tag{69}$$

where $c_1 = \sqrt{\mu/\rho}$ and $c_2 = \sqrt{\lambda + 2\mu/\rho}$ are the velocities of shear waves and dilatational waves, respectively. Therefore, using (64)–(69), the expression (59) of the stress intensity factor becomes

$$\mathrm{SIF}_{\mathrm{ISO}} = -\tau_0 \sqrt{2h\gamma_2 \frac{(1+\gamma_1^2)^2 - 4\gamma_1\gamma_2}{(1+\gamma_1^2)^2 - 4\gamma_2^2}}, \tag{70}$$

which is exactly the expression Nilsson [1972] obtained for isotropic strip.

For statical problem ($c=0$) the expression for the stress intensity factor after some algebraic manipulation has been obtained as

$$K_I^{\text{stat}} = C_{22} v_0 \sqrt{\frac{2\bar{\gamma}_1\bar{\gamma}_2}{(\bar{\gamma}_1+\bar{\gamma}_2)h}}, \tag{71}$$

where $\bar{\gamma}_1^2$, $\bar{\gamma}_2^2$ are positive roots of the equation

$$C_{22}C_{66}\gamma^4 + (C_{12}^2 + 2C_{12}C_{66} - C_{11}C_{22})\gamma^2 + C_{11}C_{66} = 0. \tag{72}$$

Georgiadis and Papadopoulos [1987] obtained the same expression as that of (71) for statical problem of semiinfinite crack at the interior of orthotropic strip.

The above two results show the correctness of the solution of our problem.

5. Numerical results and discussions

In elasticity, two well-known wave velocities — velocities of shear waves and dilatational waves — are given by

$$c_1 = c_s = \sqrt{\mu_{12}/\rho} \quad \text{and} \quad c_2 = c_L = \sqrt{C_{11}/\rho}. \tag{73}$$

Moreover, the velocity of the Rayleigh surface waves is denoted by c_R. It is known that the value of c_R is slightly less than that of the shear waves. Also, Rayleigh surface wave velocity is the theoretical upper limit [Broberg 1999] of the crack propagation velocity, although practically the maximum crack propagation velocity is much less than the Rayleigh surface wave velocity. Hence, in this problem we assume that velocity of the crack $c < c_R < \min\{c_1, c_2\}$.

From the expressions of the stress intensity factor (SIF) and crack-opening displacement (COD) the numerical values of the SIF and COD have been plotted against various parameters. Material constants (in GPa) and densities (in g/cm^3) of some orthotropic materials are given [Rubio-Gonzalez and Lira-Vergara 2011] in Table 1.

The values of the stress intensity factor (SIF) normalized with respect to v_0 have been plotted against the velocity (cm/μs) of the crack for each of the two types of materials.

For type I material, velocity of Rayleigh surface wave is $c_R = 0.2138\,\text{cm}/\mu\text{s}$. From Figure 3, it is clear that the SIF depends on the crack propagation velocity c. For fixed crack width, the SIF decreases with the increasing value of c and tends to zero as c approaches c_R. Consequently, $K_I = 0$ when $c = c_R$. Practically our concern is for $c < c_R$, hence the case $K_I = 0$ is of no interest. The graph is not valid for super Rayleigh velocities ($c > c_R$) as c_R is the theoretical upper limit of the crack velocity. Moreover, for fixed crack propagation velocity the value of the SIF decreases as the width of the strip h increases which is obvious from the expression of the SIF in (60). This is also justified with the fact that as the strip becomes wider, the impact of the displacement v_0 on the crack surface is less. Similarly, for

		C_{11}	C_{22}	C_{12}	μ_{12}	ρ
type I	graphite epoxy	155.36	16.31	3.67	7.48	1.6
type II	E-glass epoxy	46.09	12.60	2.86	5.50	2.1

Table 1. Engineering elastic constants.

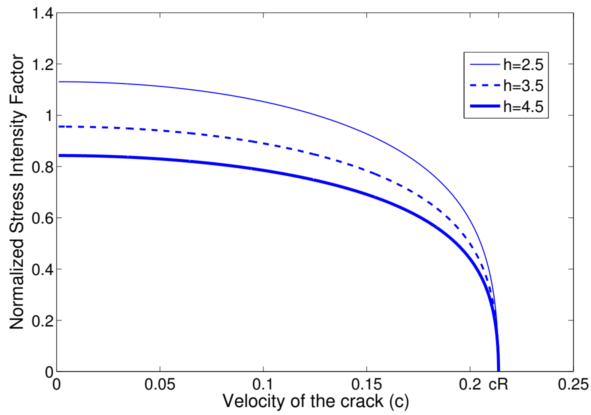

Figure 3. SIF against the crack velocity c for type I material.

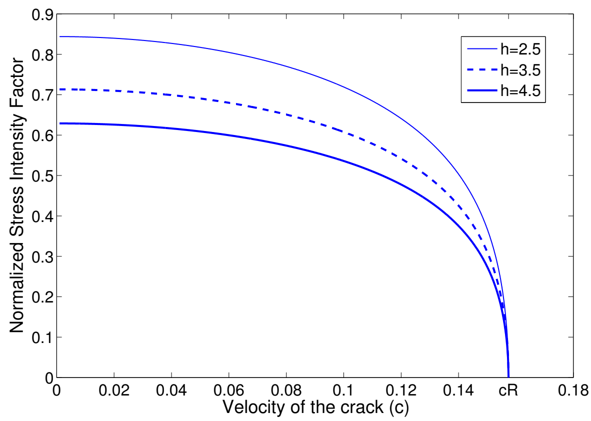

Figure 4. SIF against the crack velocity c for type II material.

type II materials the velocity of the Rayleigh waves is $0.1574\,\text{cm}/\mu\text{s}$ and the graph (Figure 4) is very much similar to Figure 3. These results agree with the results obtained in [Nilsson 1972] for isotropic materials.

Values of the COD has been plotted against negative x-axis for different parameters viz. crack velocity and strip width. From all the figures of the COD (Figures 5–8) it is clear that the values of the COD normalized with respect to v_0 decreases as we approach the crack-tip along the negative x-axis and finally vanishes at the crack-tip. This result is very much expected and agrees with the physical nature of the crack. Figures 5 and 7 show the effect of strip width h on the COD for fixed value of the crack velocity c and it shows that the COD decreases with the increasing value of h for fixed c. Again, Figures 6 and 8 show the effect of crack velocity c on the COD for fixed value of crack width h. It is clear that for subsonic propagation the value of the COD increases with the increasing value of the crack velocity subjected to same crack width.

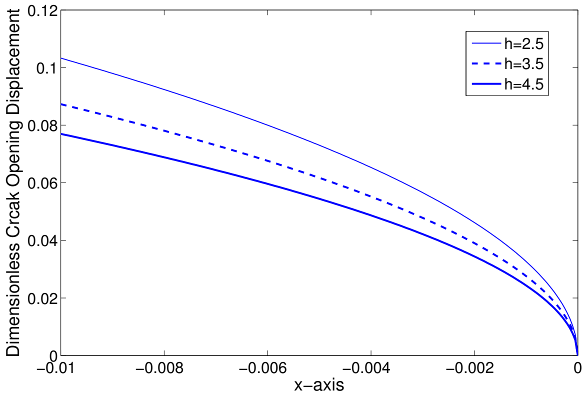

Figure 5. Crack-opening displacement against x for type I material and $c = 0.12$.

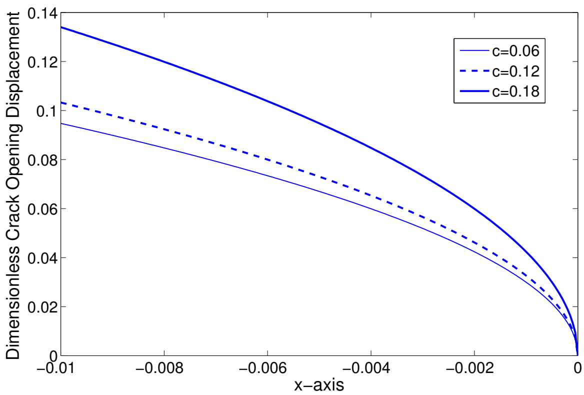

Figure 6. Crack-opening displacement against x for type I material and $h = 2.5$.

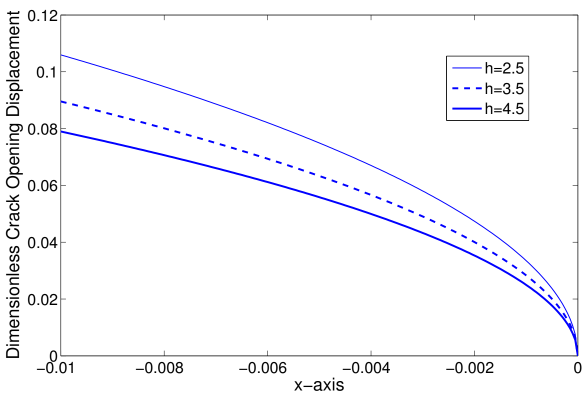

Figure 7. Crack-opening displacement against x for type II material and $c = 0.08$.

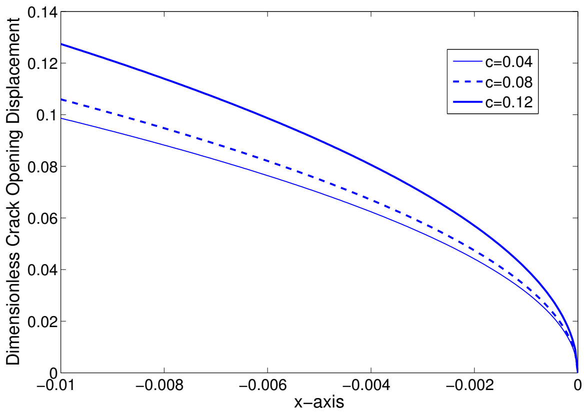

Figure 8. Crack-opening displacement against x for type II material and $h = 2.5$.

6. Conclusions

The diffraction problem of a semiinfinite moving crack in an orthotropic strip due to shear-free boundaries has been investigated. At first, the problem has been converted to the problem of a semiinfinite moving crack subjected to uniform normal stresses at the crack surfaces to make the boundary conditions suitable for the Wiener–Hopf technique. Then Fourier transform has been used to reduce the boundary value problem to the standard Wiener–Hopf equation which has been solved for asymptotic cases to obtain the expressions of the stress intensity factor and crack-opening displacement. The dependence of the stress intensity factor on the material constants, velocity and strip width have been shown with graphs. As the target of this work is to find an idea regarding how to arrest the propagation of the crack, it is necessary to keep the values of the stress intensity factor and crack-opening displacement within a certain limit. Propagation of crack depends on the SIF at the tip of the crack. From graphs it is observed that by controlling stress on the crack i.e., displacement on the surface of the strip, crack velocity and consequently propagation of crack can be made insignificant. The results given by (60) and (63) are applicable in fabrication process of large construction.

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References

[Atkinson and Popelar 1979] C. Atkinson and C. H. Popelar, "Antiplane dynamic crack propagation in a viscoelastic strip", *J. Mech. Phys. Solids* **27**:5-6 (1979), 431–439.

[Bagheri et al. 2015] R. Bagheri, M. Ayatollahi, and S. M. Mousavi, "Analytical solution of multiple moving cracks in functionally graded piezoelectric strip", *Appl. Math. Mech.* **36**:6 (2015), 777–792.

[Basak and Mandal 2017] P. Basak and S. C. Mandal, "Semi-infinite moving crack in an orthotropic strip", *Int. J. Solids Struct.* **128** (2017), 221–230.

[Broberg 1999] K. B. Broberg, *Cracks and fracture*, Academic Press, 1999.

[De and Patra 1990] J. De and B. Patra, "Moving Griffith crack in an orthotropic strip", *Int. J. Eng. Sci.* **28**:8 (1990), 809–819.

[Georgiadis 1986] H. G. Georgiadis, "Complex-variable and integral-transform methods for elastodynamic solutions of cracked orthotropic strips", *Eng. Fract. Mech.* **24**:5 (1986), 727–735.

[Georgiadis and Papadopoulos 1987] H. G. Georgiadis and G. A. Papadopoulos, "Determination of SIF in a cracked plane orthotropic strip by the Wiener–Hopf technique", *Int. J. Fract.* **34** (1987), 57–64.

[Georgiadis and Papadopoulos 1988] H. G. Georgiadis and G. A. Papadopoulos, "Cracked orthotropic strip with clamped boundaries", *J. Appl. Math. Phys.* **39** (1988), 573–578.

[Itou 2004] S. Itou, "Stress intensity factors around a moving Griffith crack in a non-homogeneous layer between two dissimilar elastic half-planes", *Acta Mech.* **167**:3 (2004), 213–232.

[Knauss 1966] W. H. Knauss, "Stresses in an infinite strip containing a semi-infinite crack", *J. Appl. Mech.* (*ASME*) **33**:2 (1966), 356–362.

[Kousiounelos and Williams Jr. 1982] P. N. Kousiounelos and J. H. Williams Jr., "Dynamic fracture of unidirectional graphite fiber composite strips", *Int. J. Fract.* **20** (1982), 47–63.

[Lee 2000] K. H. Lee, "Stress and displacement fields for propagating the crack along the interface of dissimilar orthotropic materials under dynamic mode I and II load", *J. Appl. Mech.* (*ASME*) **67**:1 (2000), 223–228.

[Ma et al. 2005] L. Ma, L. Wu, and L. Guo, "On the moving Griffith crack in a nonhomogeneous orthotropic strip", *Int. J. Fract.* **136**:1 (2005), 187–205.

[Nilsson 1972] F. Nilsson, "Dynamic stress-intensity factors for finite strip problems", *Int. J. Fract. Mech.* **8**:4 (1972), 403–411.

[Nilsson 1973] F. Nilsson, "Erratum to Dynamic stress-intensity factors for finite strip problems", *Int. J. Fract.* **9**:4 (1973), 477.

[Noble 1958] B. Noble, *Methods based on the Wiener–Hopf technique*, Pergamon Press, New York, 1958.

[Nourazar and Ayatollahi 2016] M. Nourazar and M. Ayatollahi, "Multiple moving interfacial cracks between two dissimilar piezoelectric layers under electromechanical loading", *Smart Mater. Struct.* **25**:7 (2016), 075011.

[Rice 1967] J. R. Rice, "Discussion: "Stresses in an infinite strip containing a semi-infinite crack" (Knauss, W. G., 1966, ASME J. Appl. Mech., 33, pp. 356–362)", *J. Appl. Mech.* (*ASME*) **34**:1 (1967), 248–249.

[Rubio-Gonzalez and Lira-Vergara 2011] C. Rubio-Gonzalez and E. Lira-Vergara, "Dynamic response of interfacial finite cracks in orthotropic materials subjected to concentrated loads", *Int. J. Fract.* **169**:2 (2011), 145–158.

[Sarkar et al. 1996] J. Sarkar, M. L. Ghosh, and S. C. Mandal, "Four coplanar Griffith cracks moving in an infinitely long elastic strip under antiplane shear stress", *Proc. Indian Acad. Sci.* (*Math. Sci.*) **106**:1 (1996), 91–103.

[Wang et al. 2001] C. Y. Wang, C. Rubio-Gonzalez, and J. Mason, "The dynamic stress intensity factor for a semi-infinite crack in orthotropic materials with concentrated shear impact loads", *Int. J. Solids Struct.* **38**:8 (2001), 1265–1280.

[Yoffe 1951] E. H. Yoffe, "The moving Griffith crack", *Philos. Mag.* **42**:330 (1951), 739–750.

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