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NONLINEAR DEFLECTION EXPERIMENTS: WRINKLING OF PLATES PRESSED ONTO FOUNDATIONS

NICHOLAS J. SALAMON AND PEGGY B. SALAMON

Experiments are done on elastic plates set unilaterally on foundations or substrates and driven into large deflections by a centrally located load. Previous and related work that report wrinkling in such problems, especially in thin layers on substrates, is reviewed. Deflection contours and tabulated data are given for one wrinkled state. Photographs of others illustrate wrinkling in plates of various geometry. It is found that for certain plate-foundation relative stiffnesses, a characteristic, periodic wrinkling shape evolves, but on the brink of instability that presents a challenge for both analytical solution and numerical simulation.

1. Introduction

The Problem is that of an elastic plate, set unilaterally on a foundation or elastic substrate, undergoing large deflections under a centrally located load. The issue is that such plates may naturally tend to wrinkle. To illustrate transitions from linear to nonlinear behavior, we presented in [Salamon 1984] photographs of The Problem's response for both thick and thin plates, the former deflect into smooth shapes, the latter wrinkle, Figure 1. To relate such intriguing behavior with beam contact problems, Pawlak et al. [1985] drew upon the work of [Dundurs and Stippes 1970; Dundurs 1975], especially the counter-intuitive phenomena that in linear receding contact situations, beams maintain a constant contact length independent of nonzero load. Concurrently, Yu and Stronge [1985] applied an energy method to solve for large deformations in a plate fixed vertically to a circumferential ring yet free to move radially when pressed by a rigid, spherical punch; the plate wrinkled in regions of compressive circumferential stress and they found criteria for its occurrence.

Research on related problems inspired renewed interest in The Problem: wrinkles appear in thin surface layers attached to elastic substrates that undergo compression; for examples, see [Sun et al. 2012; Cerda and Mahadevan 2003; Huang et al. 2005]. Terminology varies (buckling, folding, blistering) as in [Dong et al. 2016; Chen and Hutchinson 2004; Ortiz and Gioia 1994]; "snapping" appears in [Holmes 2009] and "bifurcation" in [Masters and Salamon 1994].

Applications of The Problem further fire interest: examples include [Trejo et al. 2013] on bacteria, [DUMAIS 2007] on plant leaves, [Srinivasa and Ross 2005] on designing shapes, [Wang et al. 2019] on Van der Waals materials, [Semler et al. 2014] on nanotube-polymer bilayers. Pertinent applications involve the mechanics of exoskeletons on soft substrates; e.g., [Martini and Barthelat 2016] on hard plates both bonded and not bonded to soft substrates.

Several works employ experiments to address The Problem, but do not report wrinkling: [Kaiser 1936] measures large deflections in a thin square plate, simply supported, but "free-standing", and does finite

Keywords: plate, nonlinear, deflection, foundation, unilateral, contact, wrinkling.

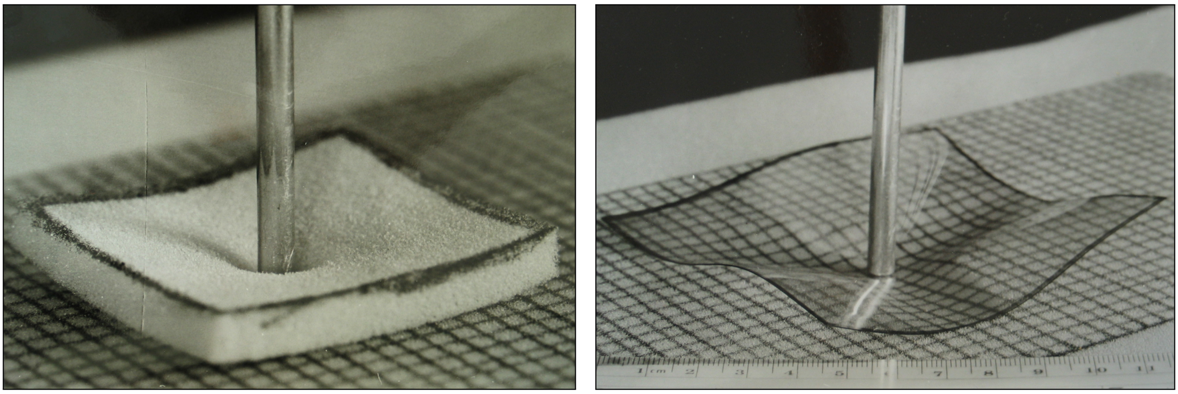

Figure 1. Plate deflection: linear smooth (left) and nonlinear wrinkled (right). From [Salamon 1984].

difference calculations over a declared "symmetric" triangular region that disagree with experiments only where, notably, the in-plane forces go compressive; [Laermann 1981a] combines photoelasticity and moiré methods with difference analysis to solve nonlinear differential equations for centrally loaded plates resting on elastomeric substrates; [Laermann 1981b] extends experiments to include strain gage methods; [Klučka et al. 2014] employs a proprietary apparatus to measure deflection in an edge-loaded circular plate resting on an elastic foundation; the plate may be too stiff or insufficiently loaded to wrinkle. The role of in-plane compression is paramount: indeed it is seen in [Timoshenko 1940, p. 332] and [Timoshenko and Woinowsky-Krieger 1959, p. 399] that nonlinear bending of plates generates in-plane compressive forces.

But in what may be the first report of wrinkling behavior, Biot [1959] finds surface wrinkling in an analytical solution of a semi-infinite viscoelastic medium under compression. Although a half-space deviates from The Problem, visco-effects are very much of interest.

This work presents both laboratory and desktop experiments. One deflection state is graphically shown and quantitatively tabulated. Observations of similar experiments are described. Photographs of desktop experiments are presented for plates of various geometry resting on flat soft foundations and on a stiff ring. Stability of and bifurcation in plate response and behavior of the foam foundation material are discussed.

2. Laboratory experiments

The objective of the laboratory experiments is to observe the response of a plate as it develops and measure a developed deflection state while holding it constant on laboratory models of The Problem. The experiments are done using a Tinius Olsen Universal Testing Machine to apply downward load through a 25.4 mm diameter, flat-bottomed rod centered on a model, each of which comprise a flat plate freely resting upon a foam foundation that sets on the tabletop of the machine; see Figure 2. Observations were made as the load on the model was increased until a desired end-state was reached, then the position of the machine crosshead was held constant while measurements of this deflection state were taken. These were obtained by traversing, magnetically clamping and reading an analog dial indicator, 0 to 25.4 mm

Figure 2. Plate on foam under load.

range (±0.01 mm accuracy), attached through extension rods to a magnetic base, at all grid intersections that could be reached; five to six grid points near the loading rod that could not be reached and several at two corners that lifted out of range of the instruments were determined by a two-dimensional quadratic curve fit using surrounding data; the measured descent of the loading rod provided a center point datum. Experiments were run on several rectangular plates. It should be noted that the Tinius Olsen machine, over-sized by far for these experiments, maintained crosshead position, although load read-out oscillated.

2.1. ***Properties of a square plate.*** The plate, of titanium stock, 305 mm square, nominally flat, 0.289 mm average thickness (maximum deviations: +0.019 mm, −0.022 mm) has a Young's modulus of 111 GPa as measured by an uniaxial tensile test run on a MTS 810 Material Test System. Poisson's ratio was not measured, nominally it's 0.34.

2.2. ***Properties of the foam foundation.*** The foundation material, polyurethane packing foam, density 27.9 kg/m^3, dimensions 305 mm square, 48.6 mm thick, is used in all testing machine experiments. Properties of the foam, complicated by its cellular structure, were determined through five compression tests run on the MTS 810 machine using two samples, each 108 mm diameter, 48.6 mm thick. In order to closely conform with conditions in the plate deflection experiments, test samples were oriented exactly the same as in the experiments and compressed between nonporous steel platens (hence deviating from the strict ASTM D3574 procedure) at an average load rate of 0.5 N/s. Both Stress and Poisson's ratio versus strain were slightly nonlinear over the full range of strain [0, −64)%, but (1) for the strain range (−0.2%, −12.7%), where 12.7% is the maximum strain in the plate bending experiments, the Young's modulus, averaged over four of the tests (one outlier dropped), is 35.3 kPa (maximum deviations +2.7, −1.7 kPa); (2) over the full strain range in one test, six measurements of circumference were made and converted for calculation of transverse strain; after dropping an outlier at low load, a curve fit to the remaining data gives Poisson's ratio as $0.1602 \times \text{strain} + 0.0988$. Single measurements in three other tests corroborate this equation. At large compressive strains the ratio goes negative as reported in the literature, overall this result tallies with [Pierron 2010, Figures 19 and 25] and with [Widdle et al. 2008]. It should

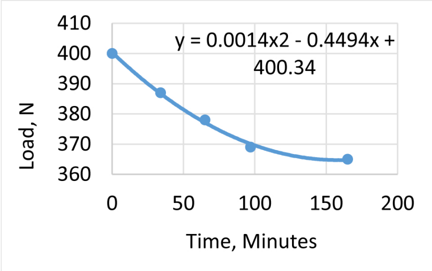

Figure 3. Load decrease with time while holding center deflection constant. Deflection contours in millimeters for the model in Section 2 show a trilobate shape.

be noted that all foam samples visually displayed full recovery after loading, hence the material appears to be elastic or, as indicated Figure 3, viscoelastic which shows the load decrease while the machine crosshead holds the center deflection of the square titanium plate constant.

2.3. ***The square plate deflections.*** Deflections, measured at points on a 12×12 grid drawn onto the top surface of the plate, required 2.75 hours to complete each of two passes. The first pass measured flatness of the model while it was held just snug by the loading rod; either model or plate could be slid horizontally by hand and the Tinius Olsen displayed an oscillating load too small to be reliable. Out-of-flatness ranged from -0.12 mm to $+0.56$ mm at one corner. The machine crosshead was then lowered slowly until the plate wrinkled into an end-state, Figure 2, center deflection 6.15 mm, load 400 N. As the load passed through 267 N, one quadrant, say, snapped into a lobate shape. (A shape characterized by radial valleys between radial ridges that resemble lobes.) The second pass measured deflections at points on the 12×12 grid while this center deflection was held constant. As readings were taken, the load to hold the center deflection constant decreased quadratically, Figure 3, over the 2.75 hours to 365 N, indicating time-dependent viscoelastic behavior of the foam. After unloading, both plate and foam fully recovered with no permanent set visible. Deflection readings minus out-of-flatness readings were processed using Mathematica 12.0 to obtain the plate deflection state shown in Figure 2 and as contours in Figure 4 with respective numerical values in Table 1, where values in red were obtained by a curve fit through neighboring values and those in blue are measured, but lie between grid points. These special values result from loading rod interference with the instruments.

3. Other model experiments

For tests run on several rectangular plates resting upon the same foam material, observations confirm the above deformation state and show an effect of aspect ratio, level of load and foundation to plate relative stiffness.

Two sister plates, same dimensions and material as in Section 2, evolved under load into trilobate shapes, similar in magnitude and pattern: one smoothly; the other snapped into position as did the plate in Section 2.3. The loading rod left a slightly visible permanent set in one plate. The foam recovered completely to the eye.

Two plates, much thinner, cut from 0.089 mm thick stainless steel roll stock, generated mixed results, perhaps in part due to their curvature. One, 305 mm square, lifted-off the foundation at all corners,

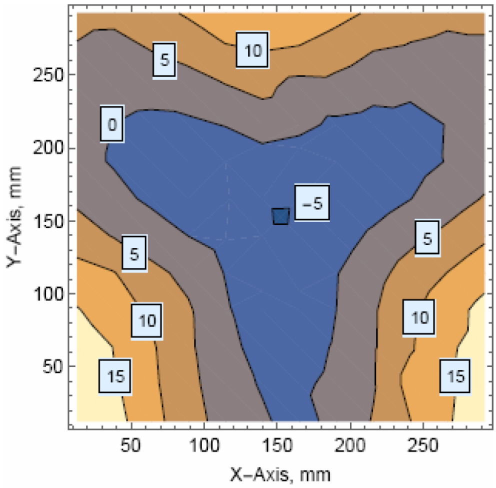

Figure 4. Deflection contours in millimeters for the model in Section 2 show a trilobate shape.

22.40	22.81	25.88	15.32	11.76	7.44	4.01	1.35	1.32	2.54	4.19	7.39
16.81	15.77	14.53	12.17	10.16	4.67	1.35	–0.38	–0.05	1.83	2.08	7.16
11.71	11.05	10.67	9.22	6.38	2.31	–0.81	–1.88	–1.09	1.60	5.05	8.26
7.06	6.30	5.79	5.64	3.61	–0.03	–2.92	–3.35	–0.89	2.72	6.71	10.57
3.12	2.36	1.55	0.66	–0.20	–1.52	–4.32	–3.63	0.20	4.60	9.22	13.46
0.76	–0.46	–1.88	–3.18	–3.94	–3.07	–3.63	0.05	2.11	6.32	10.08	14.35
–2.21	–0.86	–2.24	–3.53	–4.17	–3.18	–3.73	–1.40	1.30	2.97	9.45	13.61
1.96	1.14	0.51	–0.03	–0.25	–2.43	–4.75	–3.94	–0.94	3.28	7.42	11.53
5.28	4.62	4.50	4.47	3.30	0.89	–2.51	–3.78	–1.63	1.55	5.05	9.19
9.91	11.89	9.47	9.04	7.01	3.68	–0.10	–1.75	–1.30	0.84	4.29	7.49
14.76	14.58	14.35	12.90	10.06	6.22	2.59	0.20	0.13	1.24	3.51	6.32
16.51	17.05	16.68	16.51	13.13	9.12	5.33	2.64	1.83	2.06	4.04	6.32

Table 1. Deflection values in millimeters for the model in Section 2. Curve-fit values are in red, those measured between grids are blue. The center deflection (drop of machine crosshead) is -6.15 mm.

one diagonally opposite pair significantly higher than the other, denoting twist. The second plate, 178×305 mm, deformed into a near-trilobate shape, then upon subsequent reload, into a clear trilobate shape. Afterward, permanent set under the loading rod was evident in both plates.

An aluminum plate, 305 mm square, 0.946 mm thick, deformed smoothly into an unsymmetric bending mode, lifting off along two opposite edges and at all corners under central loads up to 800 N that left a permanent set under the loading rod.

Another aluminum plate, $254 \times 305 \times 1.52$ mm, deformed into a slightly unsymmetric, beam-like bending mode about the short dimension with slight liftoff on one end of the long dimension, the remainder in contact.

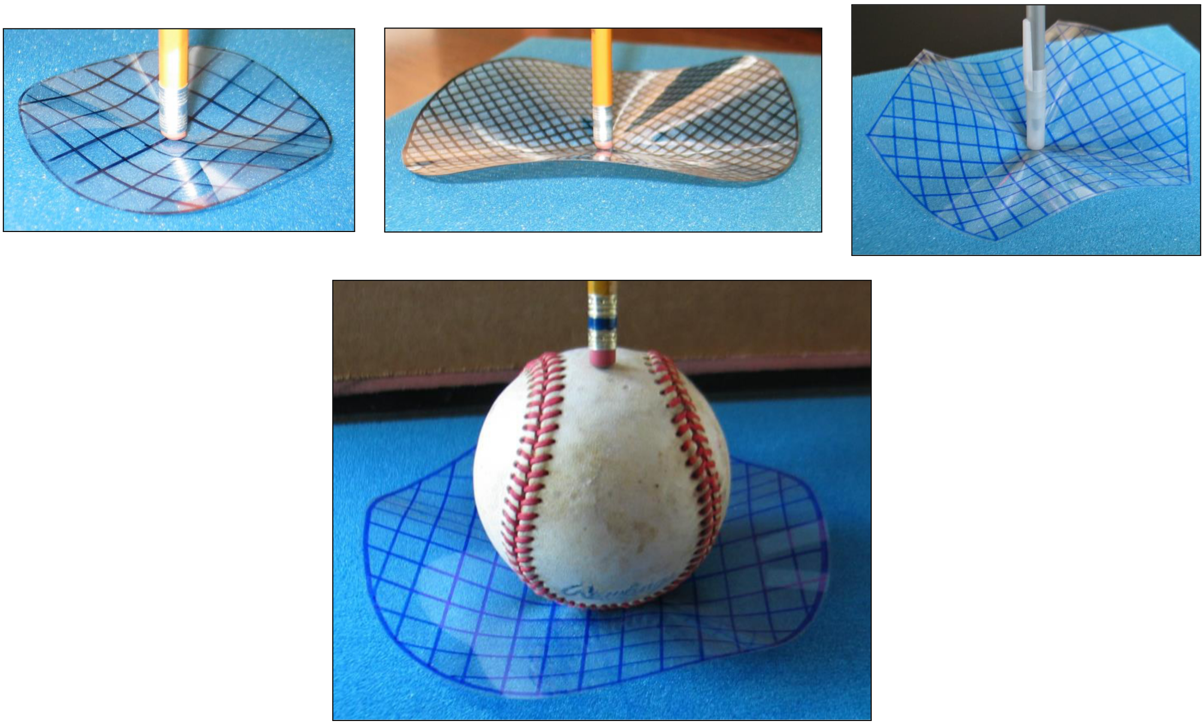

Figure 5. Top row: circular (left), elliptic (middle) and hexagonal (right) plates on foam, showing trilobate wrinkles. Below: Circular plate on foam, small punch, four lobes.

4. Desktop experiments

The desktop experiments are done with plastic plates set on two types of foundations, flat cushioning foam, as in Figures 5, and a stiff plastic jar lid (diameter 11.3 cm), as in Figures 6. On the foam, a concentrated force in the form of a pencil or pen is applied to circular, elliptical, and hexagonal plates (Figure 5, top row), and a punch-like load, a small ball, is applied to a circular plate (Figure 5, bottom). Similarly, on the jar lid, a circular plate is depressed by a punch-like load, a large ball (Figure 6, left) and a concentrated force (Figure 6, right). The plates are cut from 3M transparency film, PP2200, consisting

Figure 6. Circular plate on lid. Left: large punch, five lobes. Right: concentrated load, trilobate wrinkles.

of polyethylene terephthalate plastic, 0.1 mm thick. In Figures 5 and 6, the maximum number of wrinkles each model is capable of generating is displayed: three lobes under a concentrated load, four under the small-sized ball, five under the large one. It is noted that the jar lid limited the depth of deflection at the center to approximately 1.5 cm enabling that point to be held steady at maximum depression.

5. Discussion of results

When driven into large deflections, thin plates unilaterally set on foundations may wrinkle, forming a characteristic shape described herein as lobate. In the present experiments, under a central, concentrated increasing load, this shape terminates as trilobate. Under an increasing punch-like load, wrinkling may progress through more than three lobes. Experience with the experiments demonstrates that this behavior may not be smooth, indeed, for stiff plates on soft foundations, wrinkling may not occur at all, as witnessed in pressing the aluminum plates into the foam foundation. In general, with increasing load, in-plane compression develops, plates become inherently unstable and their response may bifurcate along different paths. Yet when unloaded, all plates studied (as well as foam foundations) fully recovered their flatness, the exception being some metal plates that yielded under the loading rod. Hence, to the eye, wrinkling in plates and compression of the foam foundation appears to be reversibly elastic.

5.1. ***Instability under concentrated load.*** For the metal plates machine loaded at a slow rate, of those that wrinkled, only one plate smoothly formed a trilobate shape; for all others, the third lobe would snap to complete the trilobate shape. This behavior may be subtle, neither visibly obvious, nor distinctly audible.

For the plastic plates, hand loading provided freedom to play. Under a rapid concentrated load, sometimes plates on the foam foundation developed a trilobate end-state directly. Most times, particularly under light load or a slow load rate, lobes tend to form stepwise: first one or two smoothly, then for the third and final lobe, perturbation might be required, i.e., in the form of flicking the plate with a finger.

5.2. ***Bifurcation under concentrated load.*** Sometimes a plate, metal or plastic, will develop one or two lobes that monotonically grow with increasing load to reach a stable end-state without launching additional lobes. At that point, perturbation has no effect. One example is the hexagonal plastic plate under a slowly increasing concentrated load: (1) sometimes an edge lifts off smoothly followed by a second opposite edge, both now proceed to a stable end-state — flicking does not trigger the third lobe; (2) other times, one corner lifts off smoothly followed by the snap of an opposite edge, then under increasing load, a second corner lifts off to form a stable trilobate shape. These two different responses denote bifurcation.

5.3. ***Response under punch-like loads.*** Intrigued by Yu and Stronge's problem [Yu and Stronge 1985], circular plastic plates were set on the jar lid. When subjected to punch-like loads, they wrinkled, but in a manner different from the analytical predictions in [Yu and Stronge 1985]. In Figure 6, wrinkles emanate from the center and extend radially, reaching an apex at a plate's periphery whereas in their analysis, wrinkles terminate there because of the imposition of zero vertical displacement around the plate edge. Furthermore, they report criterion for as many as eight wrinkles; the jar-lid experiments reveal a maximum of five wrinkles. (Not shown: a circular plastic plate set upon the open glass jar itself, which allows a deeper center depression, generates up to seven wrinkles with perturbation.) Hence the problems differ: their boundary conditions constrain the response, apparently enabling solution of a stable, well-posed problem; the unilateral conditions herein allow freedoms that permit instability.

Under increasing punch-like loads applied by either ball to circular, hexagonal and rectangular plastic plates set on the foam foundation, as in Figure 5, bottom, wrinkles beyond the first snapped into place, but did not require perturbation. The maximum number of wrinkles generated were five for the circular and hexagonal plates under the large ball, four for all others (not shown).

5.4. *Relative stiffness ratio.* In an attempt to predict wrinkling, a plate-to-foundation stiffness ratio for rectangular plates, $\mathrm{SR} = k_p/k_f$, is defined where $k_p = E_p t^3/L^2$ is the plate stiffness derived from plate flexure [Timoshenko and Woinowsky-Krieger 1959] and its associated rigidity modulus and where $k_f = E_f wL/d$ is the foundation stiffness derived from uniaxial deformation. For the plate, $w < L$ are its width and length, respectively, t is its thickness and E_p is the elastic modulus. For the foundation, d is the actual or a characteristic depth of the foundation (the foam in this case) and E_f is an elastic modulus in compression. To orders of magnitude, for the four metal plates above that wrinkled, $\mathrm{SR} \leq 10^{-4}$, for four that did not wrinkle, $\mathrm{SR} \geq 10^{-3}$.

But a contradiction arose: $\mathrm{SR} = 2.16 \cdot 10^{-05}$ for the square stainless steel plate, yet it did not wrinkle in a lobular fashion, it bent into a twist; ironically, the rectangular stainless steel plate with $\mathrm{SR} = 3.71 \cdot 10^{-05}$, did wrinkle. Could these two very thin, rather flimsy sheets lack sufficient structural integrity to act as plates? . . .Making their behavior a toss-up? With more certainty, the three titanium plates, $\mathrm{SR} = 4.26 \cdot 10^{-04}$, did wrinkle, and the two aluminum plates, $\mathrm{SR} = 4.76 \cdot 10^{-02}$ and $9.38 \cdot 10^{-03}$, did not wrinkle. Hence, stiffness ratio, while an indicator of wrinkling, is clearly not absolute. For nonrectangular plates, one would have to seek characteristic dimensions.

6. Conclusion

Plates have been well studied — Naruoka [1981] lists 12,717 references, yet deflection-induced wrinkling shown herein does not appear there. The Problem, fundamental in the class of problems dealing with plates on foundations, has come to the fore through a manifold of applications dealing with films or coatings on substrates, particularly in situations where bonding is weak. It is shown that the wrinkling of plates provides important lessons in mechanics. In analysis, such wrinkling demonstrates that the assumption of symmetry, common in linear mechanics, does not carry over to nonlinear mechanics: indeed, the misapplication of symmetry was done by one reference cited herein. In education, this seemingly simple problem vividly illustrates that the transition from linear to nonlinear behavior concomitant with the onset of instability and bifurcation in response, escalates its complexity and presents a challenge to both analytical solution and numerical simulation. The wrinkling of plates into such a characteristic shape denotes protean behavior (after Proteus, the shape-shifting Greek God); however, students bestowed upon it a more memorable description: “the flying carpet problem”.

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