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BUCKLING OF CIRCULAR CFDST SLENDER COLUMNS WITH COMPLIANT INTERFACES: EXACT SOLUTION

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The paper presents a new mathematical model and its exact solution for the buckling analysis of circular CFDST slender columns with compliant interfaces between the steel tubes and sandwiched concrete. The exact critical buckling loads are calculated for the first time. The results are compared to the experimental results and good agreement is obtained. A parametric study is also performed to investigate the effects of different parameters on buckling loads. It is shown that buckling loads can be significantly affected by the finite interface compliance.

1. Introduction

Concrete-filled double-skin steel tubular (CFDST) columns are structural members that are increasingly being used worldwide in new building developments. Owning to their good mechanical properties, fire, corrosion, blast and impact resistance and light weight cross-sections they are used as tall piers for bridges, electric transmission towers, underwater pressure vessels, columns in high-rise building, and so on. Nonetheless, when subjected to compressive loading, slender CFDST columns can be prone to buckling.

In the past, several series of test have been performed on short CFDST columns [Han et al. 2004; Tao et al. 2004; Hassanein et al. 2013; Hassanein and Kharoob 2014b; Huang et al. 2010; Li et al. 2012; Yang et al. 2012] and only a few studies exist on slender CFDST columns [Romero et al. 2015; Essopjee and Dundu 2015; Romero et al. 2017; Hassanein and Kharoob 2014a]. Romero et al. [2015; 2017] investigated slender double-tube ultra-high strength concrete-filled tubular columns under ambient temperature and fire. Similarly, Hassanein and Kharoob [2014a] investigated CFDST columns with external stainless steel tubes and Essopjee and Dundu [2015] experimentally and numerically analyzed circular CFDST columns in compression. However, as far as the authors' knowledge is concerned all these studies assumed perfect contact between the components of CFDST columns. In practice, full bonding of steel tubes and sandwiched concrete is impossible to obtain. As a result, always an incomplete or partial connection exists between the steel tubes and concrete core which can have a considerable effect on buckling behaviour of CFDST columns.

The present paper derives a mathematical model and its *exact* solution for investigation of slender CFDST columns with interface compliance between the steel tubes and sandwiched concrete. The theoretical basis used in the derivation of this new mathematical model is partially taken from [Schnabl et al. 2007; Kryžanowski et al. 2009], and from [Schnabl et al. 2015], where the concrete-filled steel tubular columns with compliant interfaces was analyzed analytically. The proposed mathematical model is validated to the test and adjusted code (see [Eurocode 2004; SANS 2011]) results obtained in [Essopjee and Dundu 2015]. After the validation, a parametric study is performed by which the effects of different

Keywords: buckling, double-skin, debonding, column, exact.

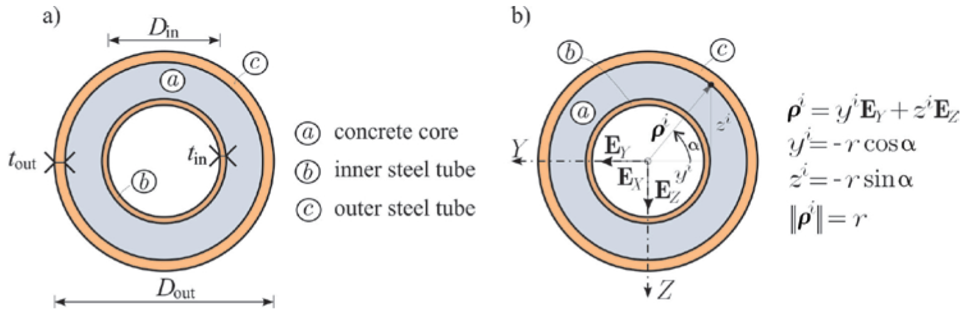

Figure 1. Cross-section of a circular CFDST slender column: details and dimensions (left) and coordinate system (right).

parameters like column slenderness, hollow and thickness ratio, and concrete elastic modulus on the buckling load of CFDST columns is investigated in detail.

2. Problem formulation

2.1. ***Preliminaries.*** We consider a straight, planar, geometrically perfect CFDST slender column shown in Figure 1. The CFDST slender column has an undeformed length L. It consists of sandwiched concrete core, a, concentric inner, b, and outer, c, steel tubes. All these three components are joined together by an interface adhesive layer of negligible thickness and finite stiffness.

The CFDST slender column is placed in the (X, Z) plane of a global Cartesian coordinate system with coordinates (X, Y, Z) and unit base vectors $\boldsymbol{E}_X$, $\boldsymbol{E}_Y$, and $\boldsymbol{E}_Z$. The reference axis of the CFDST slender column is common to all three components. It is parameterized by the undeformed arc-length x. The material particles of the sandwiched concrete core and inner and outer steel tubes are identified by material coordinates (x^i, y^i, z^i), $(i = a, b, c)$ in local coordinate system with coordinates (x, y, z). It is assumed that initially the local coordinate system coincides with the global coordinate system, and then follows the deformation of the CFDST slender column. The CFDST slender column is subjected axially to a conservative compressive load P in such a way, that homogeneous stress-strain state is achieved in the primary configuration of the column.

2.2. ***Preliminaries.*** The formulation of the governing equations of the mathematical model of a CFDST slender column is based on the following basic assumptions as well:

- The sandwiched concrete core, inner and outer steel tubes are prismatic, homogeneous, isotropic and linear elastic.
- A linearized Reissner planar shear-stiff beam theory [Reissner 1972] is used for each component.
- The components are continuously connected and contact stiffness is finite and constant.
- The components can deform relative to each other, i.e. generalized slips in axial, radial and circumferential direction can occur.
- Only the global type of instability can occur.
- The generalized slips are small.

2.3. ***Algebraic-differential equations of a CFDST column.*** The linearized Reissner algebraic-differential equations of the CFDST column are kinematic, equilibrium, constitutive and constraining equations along with the boundary conditions. In this paper, a compact comma notation $(\cdot)^i$ will be used, where superscript $i = (a,\ b,\ c)$ indicates to which components the quantity $(\cdot)$ belongs to. Hence, the governing equations of a CFDST column constitute a system of 34 algebraic and first order differential equations with constant coefficients for 34 unknown functions u^i, w^i, φ^i, ε^i, κ^i, R^i_X, R^i_Z, M^i_Y, $\Delta_{X,\text{in}}$, $\Delta_{X,\text{out}}$, $\Delta_{Z,\text{in}}$, $\Delta_{Z,\text{out}}$, $p_{X,\text{in}}$, $p_{X,\text{out}}$:

Kinematic equations

$$\frac{\mathrm{d}u^i}{\mathrm{d}x} - \varepsilon^i = 0, \tag{1}$$

$$\frac{\mathrm{d}w^i}{\mathrm{d}x} + \left(1 - \frac{P}{\sum_i E^i A^i}\right)\varphi^i = 0, \tag{2}$$

$$\frac{\mathrm{d}\varphi^i}{\mathrm{d}x} - \kappa^i = 0, \tag{3}$$

Equilibrium equations

$$\frac{\mathrm{d}R^a_X}{\mathrm{d}x} - p_{X,\text{out}} + p_{X,\text{in}} = 0, \tag{4}$$

$$\frac{\mathrm{d}R^b_X}{\mathrm{d}x} - p_{X,\text{in}} = 0, \tag{5}$$

$$\frac{\mathrm{d}R^c_X}{\mathrm{d}x} + p_{X,\text{out}} = 0, \tag{6}$$

$$\frac{\mathrm{d}R^a_Z}{\mathrm{d}x} - p_{Z,\text{out}} + p_{Z,\text{in}} = 0, \tag{7}$$

$$\frac{\mathrm{d}R^b_Z}{\mathrm{d}x} - p_{Z,\text{in}} = 0, \tag{8}$$

$$\frac{\mathrm{d}R^c_Z}{\mathrm{d}x} + p_{Z,\text{out}} = 0, \tag{9}$$

$$\frac{\mathrm{d}R^a_Z}{\mathrm{d}x} - p_{Z,\text{out}} + p_{Z,\text{in}} = 0, \tag{11}$$

$$\frac{\mathrm{d}R^b_Z}{\mathrm{d}x} - p_{Z,\text{in}} = 0, \tag{12}$$

$$\frac{\mathrm{d}M^i_Y}{\mathrm{d}x} - \left(\frac{E^i A^i}{\sum_i E^i A^i} P\right)\frac{\mathrm{d}w^i}{\mathrm{d}x} - \left(1 - \frac{P}{\sum_i E^i A^i}\right)R^i_Z = 0, \tag{13}$$

Constitutive equations

$$R^i_X - E^i A^i \varepsilon^i = 0, \tag{14}$$

$$M^i_Y - E^i J^i \kappa^i = 0, \tag{15}$$

Constraining equations

$$\Delta_{X,\text{in}} - u^b + u^a = 0, \tag{16}$$
$$\Delta_{X,\text{out}} - u^a + u^c = 0, \tag{17}$$
$$\Delta_{Z,\text{in}} - w^b + w^a = 0, \tag{18}$$
$$\Delta_{Z,\text{out}} - w^a + w^c = 0, \tag{19}$$
$$p_{X,\text{in}} - K_{\text{in}}\Delta_{X,\text{in}} = 0, \tag{20}$$
$$p_{X,\text{out}} - K_{\text{out}}\Delta_{X,\text{out}} = 0, \tag{21}$$
$$p_{Z,\text{in}} - C_{\text{in}}\Delta_{Z,\text{in}} = 0, \tag{22}$$
$$p_{Z,\text{out}} - C_{\text{out}}\Delta_{Z,\text{out}} = 0, \tag{23}$$

Boundary conditions
$x = 0$:

$$S_1^i + R_X^i(0) = 0 \quad \text{or} \quad u_1^i - u^i(0) = 0, \tag{24}$$
$$S_2^i + R_Z^i(0) = 0 \quad \text{or} \quad u_1^i - w^i(0) = 0, \tag{25}$$
$$S_3^i + M_Y^i(0) = 0 \quad \text{or} \quad u_3^i - \varphi^i(0) = 0. \tag{26}$$

$x = L$:

$$S_4^i - R_X^i(L) = 0 \quad \text{or} u_4^i - u^i(L) = 0, \tag{27}$$
$$S_5^i - R_Z^i(L) = 0 \quad \text{or} \quad u_5^i - w^i(L) = 0, \tag{28}$$
$$S_6^i - M_Y^i(L) = 0 \quad \text{or} \quad u_6^i - \varphi^i(L) = 0, \tag{29}$$

where u_k^i and S_k^i ($k = 1$-6) are the generalized boundary displacements and their complementary generalized forces at the edges of the components of the CFDST column.

2.4. ***Exact solution of buckling equations.*** In order to obtain the exact solution of the system of governing algebraic-differential equations (1)–(23) it is suitable to write the equations (1)–(23) and boundary conditions (24)–(29) in a compact form as a homogeneous system of 18 first order linear differential equations:

$$\frac{\mathrm{d}\boldsymbol{Y}}{\mathrm{d}x}(x) = \boldsymbol{A}\boldsymbol{Y}(x) \quad \text{and} \quad \boldsymbol{Y}(0) = Y_0, \tag{30}$$

where $\boldsymbol{Y}(x)$ is the eigenvector, $\boldsymbol{Y}(0)$ is the vector of unknown integration constants, i.e. generalized displacements and forces at the component's boundary, and $\boldsymbol{A}$ is the constant real 18×18 matrix. The analytical solution of (30) can be given as, see [Perko 1991]

$$\boldsymbol{Y}(x) = \exp^{Ax} \boldsymbol{Y}_0. \tag{31}$$

The vector of unknown integration constants $\boldsymbol{Y}(0)$ can be obtained from the boundary conditions (24)–(29). By inserting (31) into (24)–(29) one obtains a system of 18 homogeneous linear algebraic equations for the same number of unknown integration constants

$$\boldsymbol{K}\boldsymbol{Y}_0 = \boldsymbol{0}, \tag{32}$$

where $\boldsymbol{K}$ is the tangent stiffness matrix. A non-trivial solution of (32) is obtained from the condition of vanishing determinant of the tangent stiffness matrix $\boldsymbol{K}$. For the known geometric parameters, the only unknown is the applied load P. As a result, the critical buckling load P_{cr} is determined from the characteristic equation

$$|\boldsymbol{K}| = 0, \tag{33}$$

and is equal to the lowest eigenvalue of (31). In this case, the analytical expressions are obtained using Mathematica. However, these analytical expressions are too extensive to be shown in the paper in a closed-form, except for the two limiting cases, namely for perfectly bonded and perfectly debonded case. Thus, for the CFDST column with perfectly bonded interface between the sandwiched concrete core and the steel tubes, the critical buckling load is

$$P_{\text{cr}} = \sum_i P_{\text{cr}}^i = P_{\text{cr}}^a + P_{\text{cr}}^b + P_{\text{cr}}^c = \frac{\pi^2 \sum_i E^i J^i}{(1+\varepsilon)L^2} = \frac{\pi^2 (E^a J^a + E^b J^b + E^c J^c)}{(1+\varepsilon)L^2}, \tag{34}$$

where P_{cr}^a, P_{cr}^b, and P_{cr}^c are the buckling loads of the concrete core, inner and outer steel tubes, respectively. On the other hand, in the limiting case when the interface is fully debonded the critical buckling loads is

$$P_{\text{cr}} = P^a + P_{\text{cr}}^b + P^c = (E^a A^a + E^c A^c)\varepsilon + \frac{\pi^2 E^b J^b}{(1+\varepsilon)L^2}, \tag{35}$$

where P^a and P^c are the corresponding axial loads carried by the concrete core and outer steel tube, respectively.

3. Results and discussion

3.1. *Validation.* The principal aim of this section is to check and demonstrate the validity of this mathematical model and its exact solution in comparison to the available experimental and design results in the literature. Therefore, the validation of the critical buckling loads of CFDST slender columns uses the four available specimens in the literature experimentally tested in [Essopjee and Dundu 2015] and whose geometric and material properties are given in Table 1. Further, the exact critical buckling loads are compared to the adjusted (see [Eurocode 2004; SANS 2011]) design predictions obtained in [Essopjee and Dundu 2015] for the same four specimens. The results are given in Table 2 and Figure 2.

Specimen	L[cm]	D_{out}[cm]	t_{out}[mm]	D_{in}[cm]	t_{in}[mm]	E_a[kN/cm^2]	E_b, E_c[kN/cm^2]
S139.2-2.5	250	13.92	3.0	7.6	2.0	2821	20323.3
S152.4-2.5	250	15.24	3.0	7.6	2.0	2821	20622.0
S165.1-2.5	250	16.51	3.0	7.6	2.0	2821	20353.0
S193.7-2.5	250	19.37	3.5	7.6	2.0	2821	20796.2

Table 1. Material and geometric properties of CFDST columns.

Specimen	N_{TEST}[kN]	N_{EC4}[kN]	N_{SANS}[kN]	P^*_{cr}[kN]	P^{**}_{cr}[kN]	$C(P_{\text{cr}} = N_{\text{TEST}})$[kN/cm^2]
S139.2-2.5	732.1	692	679	672.1	1674.2	$6.15\ 10^{-4}$
S152.4-2.5	949.0	920	882	792.4	2314.5	$2.80\ 10^{-4}$
S165.1-2.5	1036.5	1011	1028	907.7	3044.6	$1.86\ 10^{-4}$
S193.7-2.5	1458.7	1272	1467	1270.5	5793.8	$2.04\ 10^{-4}$
					Average	$3.21\ 10^{-4}$

Table 2. Comparison of exact, test and adjusted EC4 and SANS code predictions. P^*_{cr} and P^{**}_{cr} are the buckling loads of, respectively, a fully debonded and a fully bonded CFDST column.

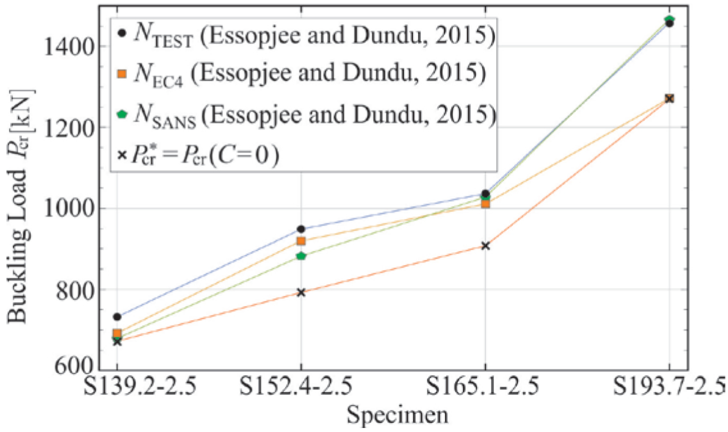

Figure 2. Comparison of exact, test and adjusted code predictions of buckling loads of circular CFDST slender columns.

It can be seen from Table 2 and Figure 2 that the code buckling loads are in general smaller than test results. The test buckling loads of all four specimens are between the results for fully debonded ($P^*_{\text{cr}} = P_{\text{cr}}(C = 0)$) and fully bonded ($P^{**}_{\text{cr}} = P_{\text{cr}}(C = \infty)$) CFDST slender columns. In fact, in these particular cases, the test results are closer to the exact buckling loads of fully debonded CFDST columns. In addition, with the inverse analysis, one can calculate the corresponding contact stiffness K and C from the equality of test and analytical buckling loads, i.e. $P_{\text{cr}} = N_{\text{TEST}}$. Nevertheless, it is found out that the critical buckling load of a CFDST slender column is independent of K. Thus, only C that corresponds to the test results obtained in [Essopjee and Dundu 2015] are given in Table 2. The exact buckling loads are in good agreement with the test results if adequate contact stiffness is used in the calculations.

The exact buckling loads are also compared to the numerical results obtained by finite element (FE) analysis in [Hassanein and Kharoob 2014a]. Table 3 shows a summary of the comparison for different columns and various C. The geometric and material data of each column are given in [Hassanein and Kharoob 2014a]. It is clear from Table 3 that P_{cr} is decreasing with increasing the P_{cr} and decreasing C.

For intermediate slender column $C5$, the FE buckling load P_{FE} is always smaller than P_{cr} irrespective to C. In case of other columns, e.i. $C6$–$C17$, the FE buckling load P_{FE} is always between the two limits, i.e. fully debonded ($C = 0$) and perfectly bonded ($C = \infty$) interface between the concrete and steel.

			P_{cr}[kN]				
Column	λ	P_{FE}[kN]	$C=0$	$C=10^{-6}$	$C=10^{-5}$	$C=10^{-4}$	$C=\infty$
C5	51	9440	12711	12733	12926	14775	35992
C6	62	8431	8621	8652	8936	11512	24410
C7	73	7358	6228	6273	6661	9891	17636
C8	84	6591	4709	4768	5273	8910	13335
C9	94	5803	3686	3759	4391	8039	10435
C10	105	5014	2963	3054	3816	7109	8388
C11	116	4393	2433	2544	3433	6186	6889
C12	127	3951	2033	2166	3169	5354	5759
C13	138	3140	1725	1881	2974	4641	4885
C14	149	2827	1482	1661	2812	4043	4197
C15	160	2455	1287	1491	2659	3544	3644
C16	171	2156	1128	1358	2501	3127	3194
C17	182	1971	997	1253	2336	2776	2822

Table 3. Comparison of analytical buckling loads with the FE buckling loads from [Hassanein and Kharoob 2014a].

3.2. ***Parametric study.*** The derived mathematical model and its exact solution are used to analyse the effects of different parameters like column slenderness ratio (λ), hollow ratio (χ), concrete elastic modulus (E_a), and thickness ratio (τ) on buckling loads (P_{cr}) of CFDST slender column, in this case the specimen S139.2-2.5 is used in the analysis. The results are given in Figures 3 and 4.

3.2.1. *Effect of column slenderness ratio.* The column slenderness ratio is generally one of the most important parameters that influence the buckling behaviour of circular CFDST slender columns. Here, the column slenderness ratio is defined as $\lambda = L\sqrt{A}/\sqrt{I}$, where L is the effective buckling length, A is the cross-sectional area and I is the moment of inertia of the CFDST column. The effect of λ on P_{cr} is illustrated for different contact stiffness (fully bonded ($C=\infty\,\text{kN/cm}^2$) and fully debonded ($C=0\,\text{kN/cm}^2$), and all in between) in Figure 3, left.

It can be seen that P_{cr} decreases as λ increases. Notice also that the buckling load $\bar{P}_{cr} = P_{cr}(C = 3.21\cdot 10^{-4}\,\text{kN/cm}^2)$ that belongs to the average contact stiffness of the selected tested four specimens in [Essopjee and Dundu 2015] is for $\lambda \le 100$ closer to $P^{*}_{cr} = P_{cr}(C=0)$, while for $\lambda > 100$ is closer to $P^{**}_{cr} = P_{cr}(C=\infty)$. Thus, it is seen that for very slender CFDST columns the effect of C on P_{cr} is insignificant and can be neglected.

3.2.2. *Effect of hollow ratio.* The effect of hollow ratio ($\chi = D_{in}/(D_{out}-2t_{out})$) on P_{cr} is shown in Figure 3, middle. This effect is investigated by varying the external tube thickness t_{out}. It can be seen from the graph that increasing χ, which reduces the cross-sectional area of the sandwiched concrete, increases P_{cr}, especially for nearly fully debonded columns, while on the other hand, for fully bonded columns, this effect is negligible. Again, it could be seen that $\bar{P}_{cr}$ is closer to fully debonded CFDST columns.

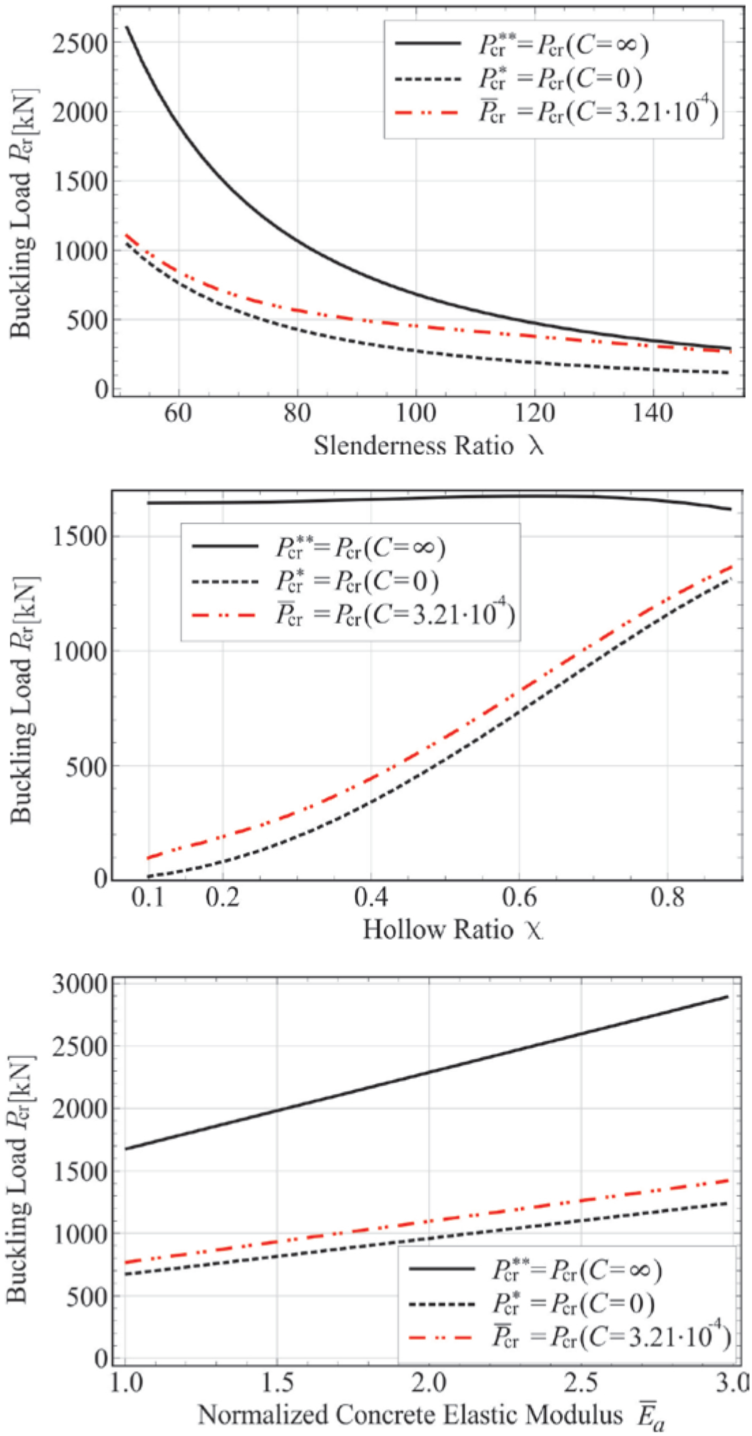

Figure 3. Effect of slenderness ratio λ (top), hollow ratio χ (middle) and elastic modulus (E_a) of sandwiched concrete (bottom) on the buckling load (P_{cr}) of a circular CFDST slender column.

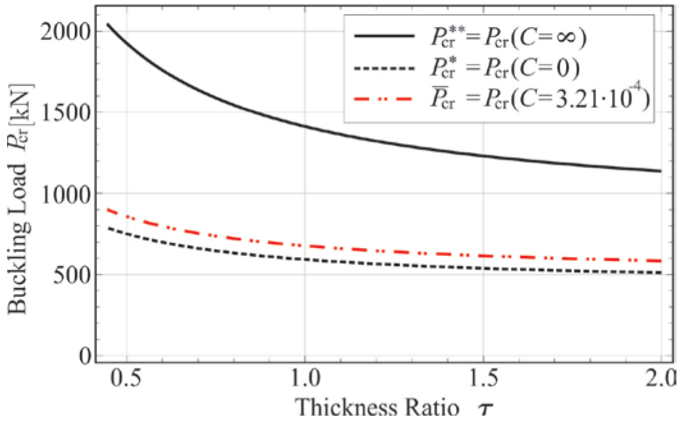

Figure 4. Effect of thickness ratio (τ) on buckling load (P_{cr}) of circular CFDST slender column.

3.2.3. *Effect of concrete elastic modulus.* The effect of concrete elastic modulus on P_{cr} is investigated next. Figure 3, bottom, shows P_{cr} versus normalized concrete elastic modulus, i.e., $\bar{E}_a = E_a/2821$.

As can been seen, the elastic modulus of sandwiched concrete E_a has a considerable effect on P_{cr}. The increasing of E_a increases linearly P_{cr}. This effect in more pronounced for CFDST columns with highly bonded components. Similarly as above, for the circular CFDST slender column whose contact stiffness corresponds to the test results, the buckling capacity is again closer to one with almost fully debonded components.

3.2.4. *Effect of thickness ratio.* The effect of thickness ratio ($\tau = t_{in}/t_{out}$) on P_{cr} of CFDST column is also investigated. This effect is investigated by varying the outer tube thickness (t_{out}). Figure 4. shows P_{cr} versus τ for values of contact stiffness between the one of fully bonded components to the one of fully debonded components. Note that τ increases by decreasing the outer tube thickness. As would be expected, by increasing of τ, the buckling load of CFDST column decreases. This effect is more pronounced for smaller values of τ while for higher is almost negligible. Again, $\bar{P}_{cr}$ is nearly the same as P_{cr}^{*}.

4. Conclusions

The paper presented a new mathematical model and its exact solution for the buckling analysis of circular CFDST columns with compliant interfaces between the steel and sandwiched concrete. After the validation of the results, the exact critical buckling loads were calculated using the proposed exact model and a parametric study was also performed by which the effects of different parameters on buckling behaviour of CFDST columns were investigated. Based on the results obtained, the following conclusions can be made:

1. The present mathematical model and its exact solution are simple, efficient and derived for the first time.
2. The exact results agree well with the experimental results obtained in [Essopjee and Dundu 2015].
3. The results also agree well with the adjusted design predictions [Eurocode 2004; SANS 2011].

4. The exact results can agree completely with the numerical results of [Hassanein and Kharoob 2014a] if calibrated values of C are used in the calculations.
5. The buckling load is independent on axial contact stiffness K.
6. The buckling load of CFDST column decreases with increasing the column slenderness and thickness ratio, while increases with increasing the elastic modulus of sandwiched concrete and hollow ratio.
7. The exact results can be used as a benchmark solution.

Conflict of Interests. The authors declare that they have no conflict of interests regarding the publication of this paper.

Data Availability. No data were used to support this study.

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Notation. The following symbols are used in this paper:

A = cross-sectional area (cm^2)
C = contact modulus in Z direction (kN/cm^2)
D = tube diameter (cm)
E = elastic modulus (kN/cm^2)
J = moment of inertia (cm^4)
K = contact modulus in X direction (kN/cm^2)
L = column length (cm)
M_Y = cross-sectional bending moment (kNcm)
P = centrally applied point load (kN)
P_{cr} = critical bucklig load (kN)
p_X, p_Z = contact tractions in X and Z directions (kN/cm^2)
R_X, R_Z = X and Z components of the cross-sectional equilibrium force (kN)
t = tube thickness (cm)
u = axial displacement (cm)
w = deflection (cm)
Δ = generalized slip (cm)
ε = extensional strain
κ = presudocurvature (rad/cm)
φ = rotation (rad)

Subcripts

in = inner
out = outer

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