

# Model Theory

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*MAT*

Introduction



## Introduction

This special issue of *Model Theory* is in honour of Ehud Hrushovski (Oxford University). A conference to celebrate Hrushovski's 60th birthday, entitled "Model theory: from geometric stability to tame geometry", was originally scheduled for 8–12 June 2020 at the CIRM, Luminy, France. Owing to the Covid-19 pandemic, it was postponed until 13–17 December 2021, where it was held as a hybrid workshop of the same title at the Fields Institute in Toronto, as part of the Fields Institute *Thematic Program on Trends in Pure and Applied Model Theory*. This special issue arose from that workshop, with all workshop speakers invited to contribute an article.

Over almost 40 years, Hrushovski's influence on model theory, and on its relationships to other parts of mathematics, has been phenomenal. We cannot here do justice to this, but pick out a few highlights where his highly original contributions have astonished our community and spawned riches for many. We do not touch on his most recent work, which has comparable potential but is less widely absorbed.

Throughout his career, starting with his Berkeley Ph.D. thesis, geometric stability theory (definable groups, minimal types, internality and binding groups, orthogonality, canonical bases, imaginaries, often in unstable contexts) has been a guiding theme; an example is his 2000 paper with Hart and Laskowski *The uncountable spectra of countable theories*, a culmination of classification theory over countable languages. "Hrushovski constructions" first appeared in his talks in 1988 to give counterexamples to conjectures of Lachlan and Zilber but the ideas since then have yielded countless other important examples, as well as Zilber's pseudo-exponential field. His 1996 paper *Zariski geometries* with Zilber exhibits a natural context where Zilber's trichotomy conjecture holds (so is a counterpart to the Hrushovski constructions), and was a key ingredient to his subsequent work on diophantine geometry. Early drafts of his monograph with Cherlin, *Finite structures with few types*, as well as work on PAC structures, gave versions of the independence theorem which underpins simple theories.

On the more applied side of model theory, Hrushovski's work with Chatzidakis and Peterzil on ACFA opened up difference algebra as an area of model-theoretic applications, reinforced by his manuscript on the nonstandard Frobenius. Hrushovski startled not just the model theory community when he found applications of large

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chunks of geometric stability theory in diophantine geometry: he obtained for example a model-theoretic proof of the geometric Mordell–Lang conjecture, in all characteristics (a new result for function fields in characteristic  $p$ ), and a new proof of the Manin–Mumford conjecture, with explicit bounds. A series of joint papers with Pillay in the late 2000s (one paper also with Peterzil, another also with Simon) yielded new ways of thinking about NIP theories model-theoretically, proved the Pillay conjecture on definable groups in o-minimal expansions of ordered fields, exhibited the significance of Keisler measures, and found model-theoretic applications of Vapnik–Chervonenkis theory; it has yielded many further developments in definable dynamics by Hrushovski and others. His 2009 paper on approximate subgroups led to the Breuillard–Green–Tao classification of finite approximate subgroups and to many other riches. Over the last 25 years, the model theory of valued fields has been a major theme of his work, with ideas from stability theory feeding into our understanding of algebraically closed valued fields, with applications in motivic integration (Hrushovski and Kazhdan), nonarchimedean tame topology and Berkovich space (Hrushovski and Loeser), and zeta functions for groups (Hrushovski, Martin, and Rideau-Kikuchi).

We do not here comment on all the articles in this issue, but note how several of them reflect or grew out of Hrushovski’s work.

The paper *Residue field domination in some henselian valued fields* by C. Ealy, D. Haskell, and P. Simon builds directly on the monograph *Stable domination and independence in algebraically closed valued fields* by Haskell, Hrushovski, and Macpherson, which developed “stable domination” as an abstraction of the way, in algebraically closed valued fields, certain types (e.g., the generic type of the valuation ring) are governed by their trace in the residue field. The analogue to Berkovich analytification developed by Hrushovski and Loeser was the space of stably dominated types concentrating on a variety. The Ealy–Haskell–Simon paper develops an analogue of stable domination for other henselian valued fields.

Kamensky’s *Higher internal covers* grew from Hrushovski’s influential paper *Groupoids, imaginaries, and internal covers*.

*Remarks around the nonexistence of difference-closure* by Chatzidakis shows that difference fields do not in general have a difference closure, but develops a stronger notion of  $\kappa$ -closure which, under extra hypotheses, exists and is unique up to isomorphism. This paper is a natural development of the body of work initiated by Chatzidakis and Hrushovski (one paper also with Peterzil) on the theory ACFA, the model companion of the theory of difference fields.

Breuillard’s *An exposition of Jordan’s original proof of this theorem on finite subgroups of  $\mathrm{GL}_n(\mathbb{C})$*  is in part historical. It has connections to a number of important themes in Hrushovski’s work: finite approximate subgroups, pseudofinite dimension (originating in a paper of Hrushovski and Wagner), and a model-theoretic

approach to the Larsen–Pink strengthening of Jordan’s theorem, using the “dimension comparison lemma” of Hrushovski–Wagner.

In *Rigid differentially closed fields*, Marker constructs differentially closed fields of characteristic 0 with no nontrivial automorphisms. The construction makes essential use of the Hrushovski–Sokolovic analysis of strongly minimal sets in differentially closed fields, itself a key ingredient for his work on Mordell–Lang.

*Higher amalgamation properties in measured structures* by Evans builds on several themes in Hrushovski’s work. Higher amalgamation is central in Hrushovski’s “Groupoids” paper, and this paper by Evans focuses on  $\omega$ -categorical Hrushovski constructions as a source of examples. Measures in model theory are also a persistent theme in Hrushovski’s work, for example in his paper on approximate subgroups, in the above-mentioned NIP papers, and in much more recent work.

Measures in model theory are also central to *Definable convolution and idempotent measures, II* by Chernikov and Gannon. This article develops many ideas from the NIP papers of Hrushovski and Pillay, also involving Peterzil and Simon.

We believe the articles in this special issue are both important in their own right, and a fitting tribute to Hrushovski’s impact on model theory and its applications.

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