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Rigid differentially closed fields

David Marker

Using ideas from geometric stability theory we construct differentially closed fields of characteristic 0 with no nontrivial automorphisms.

1. Introduction

Our goal is to construct countable differentially closed fields of characteristic 0 (DCF_0) with no nontrivial automorphisms. We refer to such fields as *rigid*. This answers a question posed by Russell Miller. I will say something about Miller's motivation in my closing remarks. This may at first seem surprising. One often, naively, thinks that differentially closed fields should behave like algebraically closed fields, where there are always many automorphisms. Also, differential closures of proper differential subfields always have nontrivial automorphisms. We sketch the proof of this using ideas from Shelah's proof [18] of the uniqueness of prime models for ω-stable theories (see [12, §6.4] or [21, §9.2]). This is a well-known construction.

Proposition 1.1. *Let k be a differential field with differential closure $K \supset k$. Then there are nontrivial automorphisms of K/k.*

Proof. First note that if $\boldsymbol{d} \in K^n$ and $k\langle \boldsymbol{d} \rangle$ is the differential field generated by $\boldsymbol{d}$ over k, then K is a differential closure of $k\langle \boldsymbol{d} \rangle$. This follows from the fact that in an ω-stable theory $\mathcal{M}$ is prime over $A \subset \mathcal{M}$ if and only if $\mathcal{M}$ is atomic over A and there are no uncountable sets of indiscernibles (see [21, Theorem 9.2.1]).

Let $a \in K \setminus k$. Since K is the differential closure of k, $\text{tp}(a/k)$ is isolated by some formula $\phi(v)$ with parameters from k. If a is the only element of K satisfying ϕ, then a is in $\text{dcl}(k) = k$, a contradiction. Thus there is $b \in K$ such that $a \neq b$ and $\phi(b)$.

Since a and b realize the same type over k, there are $L \models DCF_0$ with $k\langle b \rangle \subseteq L$ and $\sigma : K \to L$ an isomorphism such that $\sigma \mid k$ is the identity and $\sigma(a) = b$.

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K is a differential closure of both $k\langle a\rangle$ and $k\langle b\rangle$. Thus L is a differential closure of $k\langle b\rangle$ and, by uniqueness of differential closures, there is an isomorphism $\tau : L \to K$ that is the identity on $k\langle b\rangle$. Then $\tau\circ\sigma$ is an automorphism of K sending a to b. $\square$

Remarks. • This argument really shows that if T is an ω-stable theory, A is a definably closed substructure of a model of T that is not a model of T and $\mathcal{M}$ is a prime model extension of A, then there is a nontrivial automorphism of $\mathcal{M}$ fixing A pointwise.

• While this argument guarantees the existence of a nontrivial automorphism of K/k, it is possible that there is only one. If k is a model of Singer's theory of *closed ordered differential fields* [20], then $k^{\text{diff}} = k(i)$ and complex conjugation is the only nontrivial automorphism of k^{diff}/k.

Omar León Sánchez pointed out that the construction of rigid differentially closed fields gives the first known examples of differentially closed fields K such that $K \neq k(i)$ for any closed ordered differential field $k \subset K$.

• Proposition 1.1 tells us that the rigid differentially closed fields we construct are not the differential closure of any proper differential subfield.

Our construction of rigid differentially closed fields uses ideas from geometric stability theory and work on strongly minimal sets in differentially closed fields of Rosenlicht [17] and Hrushovski and Sokolović [9]. We describe the results we need in Section 2 and construct rigid differentially closed fields in Section 3. We begin Section 3 with a warm up constructing arbitrarily large rigid models and then give the more subtle construction of rigid countable models. We refer the reader to [15] for unexplained model theoretic concepts.

2. Preliminaries

We work in $\mathbb{K} \models \text{DCF}$, a monster model of the theory of differentially closed fields of characteristic zero with a single derivation. The constant field C is $\{x \in \mathbb{K} : x' = 0\}$. If k is a differential field and $X \subset \mathbb{K}^n$ is definable over k, we let $X(k)$ denote the k-points of X, i.e., $X(k) = k^n \cap X$. Of course, by quantifier elimination, X is quantifier-free definable over k

Our main tool will be the strongly minimal sets known as *Manin kernels* of elliptic curves. Manin kernels arose in Manin's proof [10] of the Mordell conjecture for function fields in characteristic zero and were central to both Buium's [2] and Hrushovski's [8] proofs of the Mordell–Lang conjecture for function fields in characteristic zero. The model theoretic importance of Manin kernels was developed in the beautiful unpublished preprint of Hrushovski and Sokolović [9]. Proofs of the results from [9] that we will need all appear in Pillay's survey [16], and [11] is another survey on the construction and some of the basic properties of Manin kernels.

For $a \in K$, let E_a be the elliptic curve $Y^2 = X(X-1)(X-a)$. Let $E_a^\sharp$ be the minimal definable differential subgroup of E. $E_a^\sharp$ is the closure of $\mathrm{Tor}(E_a)$ in the Kolchin topology.

Theorem 2.1 (Hrushovski–Sokolović). (i) *If $a' \neq 0$, then $E_a^\sharp$ is a nontrivial locally modular strongly minimal set.*

(ii) *The Manin kernels $E_a^\sharp$ and $E_b^\sharp$ are nonorthogonal if and only if E_a and E_b are isogenous. In particular, if a and b are algebraically independent over $\mathbb{Q}$ then $E_a^\sharp$ and $E_b^\sharp$ are orthogonal.*

In particular, Manin kernels are orthogonal to the field of constants $C = \{x : x' = 0\}$.

More generally, if A is a simple abelian variety that is not isomorphic to an abelian variety defined over the constants we can construct a Manin kernel $A^\sharp$ which is the Kolchin closure of the torsion of A and a minimal infinite definable subgroup of A. $A^\sharp$ is nontrivial locally modular strongly minimal and Hrushovski and Sokolović also showed that if X is any nontrivial locally modular strongly minimal subset of a differentially closed field, then $X \not\perp A^\sharp$ for some abelian variety A.

The other building blocks of our construction are strongly minimal sets introduced by Rosenlicht [17] in his proof that the differential closure of a differential field k need not be minimal.

Let $f(X) = X/(1+X)$. For $a \neq 0$, let $X_a = \{x : x' = af(x), x \neq 0\}$.

Theorem 2.2 (Rosenlicht). (i) *If $a \in k$ and $x \in X_a \setminus k$, then $C(k) = C(k\langle x\rangle)$.*

(ii) *Suppose $k \subset K$ are differential fields, with $C(K) \subseteq C(k)^{\mathrm{alg}}$. Suppose $a, b \in k^\times$, $x \in X_a(K)$, $y \in X_b(K)$ and x and y are algebraically dependent over k. Then x, y are algebraic over k or $x = y$. In particular, if $a \neq b$, then X_a and X_b are orthogonal.*

Part (i) follows from Proposition 2 of [17] while (ii) is a slight generalization of Proposition 1 of [17] and Gramain [5]. These results appear as Theorems 6.12 and 6.2 of [13].

Corollary 2.3. *Each X_a is a trivial strongly minimal set.*

Proof. By Theorem 2.2(i), X_a is orthogonal to the constants. If X_a were nontrivial, then $X_a \not\perp A^\sharp$, the Manin kernel of a simple abelian variety. But if $x \in X_a \setminus k^{\mathrm{alg}}$, then $k\langle x\rangle = k(x)$ is a transcendence degree 1 extension. But by results of Buium [2], Manin kernels, or anything nonorthogonal to one, give rise to extensions of transcendence degree at least 2. Thus X_a is trivial. □

3. Constructing rigid differentially closed fields

Warm up.

Proposition 3.1. *There are arbitrarily large rigid differentially closed fields.*

For this construction we only need Rosenlicht strongly minimal sets. Let κ be a cardinal with $\kappa = \aleph_\kappa$. We construct a differentially closed field K of cardinality κ such that $|X_a(K)| \neq |X_b(K)|$ for each nonzero $a \neq b$, guaranteeing there is no automorphism sending $a \mapsto b$.

We build a chain of differentially closed fields $K_0 \subset K_1 \subset \cdots \subset K_\alpha \subset \cdots$ for $\alpha < \kappa$ such that $|K_\alpha| = \aleph_\alpha$. We simultaneously build an injective enumeration $a_0, a_1, \ldots, a_\alpha, \ldots$ of $K^\times$, where $K = \bigcup K_\alpha$.

We construct K as follows.

(i) $K_0 = \mathbb{Q}^{\text{diff}}$.

(ii) Given K_α and $a_\alpha \in K_\alpha$, build $K_{\alpha+1}$ by adding $\aleph_{\alpha+1}$ new independent elements of X_{a_α} and taking the differential closure.

(iii) If α is a limit ordinal, let $K_\alpha = \bigcup_{\beta<\alpha} K_\beta$.[1]

Since $X_{a_\alpha} \perp X_{a_\beta}$ for $\alpha < \beta$, adding new elements to X_{a_β} and taking the differential closure adds no new elements to X_{a_α}. Thus $X_{a_\alpha}(K) = X_{a_\alpha}(K_{\alpha+1})$. In particular, $|X_{a_\alpha}(K)| = \aleph_{\alpha+1}$. Thus there is no automorphism of K with $a_\alpha \mapsto a_\beta$ for $\alpha \neq \beta$.

One might worry that we have contradicted Proposition 1.1. Let B_α be all of the independent realizations of X_{a_α} that we added at stage α. Then K is the differential closure of $k = \mathbb{Q}\langle B_\alpha : \alpha < \kappa\rangle$. But, if $b \in X_{a_\alpha}$, then $a_\alpha = b'(b+1)/b \in \mathbb{Q}\langle b\rangle$. Thus $k = K$.

The countable case. To construct a countable differentially closed field with no automorphisms, we need a more subtle mixture of Rosenlicht extensions with extensions of Manin kernels.

Suppose $b \notin C$. Let $\dim E_b^\sharp(k)$ be the number of independent realizations in k of the generic type of $E_b^\sharp$ over $\mathbb{Q}\langle b\rangle$. Manin kernels are useful to us as they can have any countable dimension. We build a countable $K \models \text{DCF}_0$ such that for each $a \neq 0$, there is a natural number

$$n_a = \max_{b \in X_a(K)} \dim E_b^\sharp(K)$$

such that $n_a \neq n_b$ for $a \neq b$. This guarantees that there is no automorphism with $a \mapsto b$.

[1]To build the desired enumeration, let $a_0, a_1, \ldots$ be an injective enumeration of K_0 and, at stage $\alpha + 1$, let $(a_\gamma : \omega_\alpha \leq \gamma < \omega_{\alpha+1})$ be an injective enumeration of $K_{\alpha+1} \setminus K_\alpha$.

Freitag and Scanlon [4], and more generally, Casale, Freitag and Nagloo [3], have given constructions of trivial strongly minimal sets which can take on any countable dimension. Presumably these could be used in an alternative construction.

We build a chain $K_0 \subset K_1 \subset \cdots \subset K_n \subset \cdots$, an injective enumeration $a_0, a_1, \ldots$ of $K^\times = \bigcup K_n^\times$ and a sequence of natural numbers $0 = n_0 < n_1 < \cdots$ such that

(1) $C(K_i) = C(K_0)$;

(2) $X_{a_i}(K) = X_{a_i}(K_{i+1})$;

(3) if $b \in X_{a_i}(K)$, then $E_b^\sharp(K) = E_b^\sharp(K_{i+1})$;

(4) $n_{i+1} = \max_{b \in X_{a_i}(K)} \dim E_b^\sharp$.[2]

If we can do that we will have guaranteed that there are no automorphisms of K.

Let $K_0 = \mathbb{Q}^{\text{diff}}$. At stage s we choose a new $a_s \in K_s$. Let b_s be an element of X_{a_s} generic over K_s, let $\boldsymbol{x}$ be $n_{s-1}+1$ independent realizations of the generic of $E_{b_s}^\sharp$ over $K_s\langle b_s\rangle = K_s(b_s)$ and let $K_{s+1} = K_s\langle b_s, \boldsymbol{x}\rangle^{\text{diff}}$.

By orthogonality considerations, it's clear that conditions (1)–(3) hold, as after stage $i+1$ we only add realizations of types orthogonal to X_{a_i} and $E_b^\sharp$, for $b \in X_{a_i}(K)$. To prove (4) we need to show that there is $n_s = \max_{d \in X_{a_s}} \dim E_d^\sharp(K_{s+1})$. We have arranged things so that if there is a bound n_s then $n_s > n_{s-1}$.

We need two preliminary lemmas.

Lemma 3.2. *If* $b' \neq 0$, *then* $\dim E_b^\sharp(\mathbb{Q}\langle b\rangle^{\text{diff}}) = 0$.

Proof. Suppose $x \in E_b^\sharp(\mathbb{Q}\langle b\rangle^{\text{diff}})$. All torsion points of E_b are in $\mathbb{Q}(b)^{\text{alg}}$, so we can suppose x is a nontorsion point. But x realizes an isolated type over $\mathbb{Q}\langle b\rangle$. Let ψ isolate the type of x over $\mathbb{Q}\langle b\rangle$. No torsion point can satisfy ψ. Thus by strong minimality ψ defines a finite set and $x \in \mathbb{Q}\langle b\rangle^{\text{alg}}$. □

Although we do not need it, we can say more in the special case that $\mathbb{Q}\langle b\rangle = \mathbb{Q}(b)$, such as if $b \in X_a$ for some $a \in \mathbb{Q}$. In this case Manin's theorem of the kernel [10] implies that $E_b^\sharp(\mathbb{Q}\langle b\rangle^{\text{alg}}) = \text{Tor}(E_b)$; see [1, Corollary K.3].

Lemma 3.3. *Suppose K is a differentially closed field, $b, d \in K$ and E_b and E_d are isogenous. Then* $\dim E_b^\sharp(K) = \dim E_d^\sharp(K)$.

Proof. If E_d and E_b are isogenous, then d and b are interalgebraic over $\mathbb{Q}$ and the isogeny f is defined over $\mathbb{Q}(d)^{\text{alg}} = \mathbb{Q}(b)^{\text{alg}}$. Since $f : \text{Tor}(E_d) \to \text{Tor}(E_b)$ is finite-to-one and the torsion is Kolchin dense in a Manin kernel, $f : E_d^\sharp \to E_b^\sharp$ is finite-to-one. It follows that $\dim E_d^\sharp(K) = \dim E_b^\sharp(K)$. □

The next lemma shows that we have the necessary bounds.

[2] Building the enumeration takes a bit more bookkeeping in this case. Let $d_{0,0}, d_{0,1}, \ldots$ be an injective enumeration of K_0 and let $d_{i,0}, d_{i,1}, \ldots$ be an injective enumeration of $K_i \setminus K_{i-1}$. Start our enumeration of K by letting $a_0 = d_{0,0}$. Suppose we start stage i with the partial enumeration $a_0, \ldots, a_M$. Then for $j = 0, \ldots, i$, let $a_{M+j+1} = d(i, i-j)$.

Lemma 3.4. *Suppose K is a differentially closed field constructed in a finite iteration $\mathbb{Q}^{\text{diff}} = k_0 \subset k_1 \subset \cdots \subset k_m = K$, where either*

(1) *$k_{i+1} = k_i\langle a\rangle^{\text{diff}}$, where a realizes a trivial type over k_i, or*

(2) *$k_{i+1} = k_i\langle \boldsymbol{x}_i\rangle^{\text{diff}}$, where $\boldsymbol{x}_i$ consists of n_i independent realizations of the generic type of a Manin kernel $E^\sharp_{b_i}$, where $b_i \in k_i$ and $E^\sharp_{b_i} \perp E^\sharp_{b_j}$ for $i \neq j$.*

If $d \in K \setminus C$, then $\dim E^\sharp_d(K) = n_i$ *for some i.*

Proof. We first argue that this is true for each $E^\sharp_{b_t}$. Define $l_0 \subseteq l_1 \subseteq \cdots \subseteq l_t$ such that $l_i = k_i\langle b_t\rangle^{\text{diff}}$. Note that $l_t = k_t$

By Lemma 3.2, $\dim E^\sharp_{b_t}(l_0) = 0$. As we construct $l_1, \ldots, l_t$ we are either doing nothing (if a_i or $\boldsymbol{x}_i \in l_{i-1}$) or adding realizations of types orthogonal to $E^\sharp_{b_t}$. Thus $\dim E^\sharp_{b_t}(k_t) = 0$ and $\dim E^\sharp_{b_t}(k_{t+1}) = n_t$. Since for $i > t$ all a_i and $\boldsymbol{x}_i$ realize types orthogonal to $E^\sharp_{b_t}$, $\dim E^\sharp_{b_t}(K) = n_t$.

Suppose $d \in K \setminus C$. If E_d is isogenous to some E_{b_i}, then, by Lemma 3.3, $\dim E^\sharp_d(K) = \dim E^\sharp_{b_i}(K) = n_i$. So we may assume $E^\sharp_d \perp E^\sharp_{b_i}$ for all i. We claim that in this case, $\dim E^\sharp_d(K) = 0$. For $i \leq m$, we let $l_i = k_i\langle d\rangle^{\text{diff}}$. By Lemma 3.2, $\dim E^\sharp_d(l_0) = 0$. As we continue the construction, as above, at each stage we either do nothing or realize types that are orthogonal to $E^\sharp_d$. Thus we add no new elements of $E^\sharp_d$ and $\dim E^\sharp_d(K) = 0$. □

We can interweave a many models construction. In [9] the authors noted that Manin kernels could be used to show that DCF_0 has eni-dop and concluded that there are $2^{\aleph_0}$ nonisomorphic countable differentially closed fields. An explicit version of this construction coding graphs into models is used in [14]. We can fold that coding into our construction of a rigid model.

Theorem 3.5. *There are $2^{\aleph_0}$ nonisomorphic countable rigid differentially closed fields. Each of these fields is not the differential closure of a proper differential subfield.*

Consider $X = X_1(\mathbb{Q}^{\text{diff}})$. This is an infinite set of algebraically independent elements. Let $G = (X, R)$ be a graph with vertex set X and edge relation R. Let $(\{u_i, v_i\} : i = 0, 1, \ldots)$ be an enumeration of two element subsets of X. We modify our construction such that at stage s we also add a generic element of $E^\sharp_{u_i+v_i}$ if and only if $(u_i, v_i) \in R$. We can still apply Lemma 3.4 and our construction will produce a rigid differentially closed field K. From K we can recover the graph in an $\mathcal{L}_{\omega_1,\omega}$-definable way. Thus nonisomorphic graphs give rise to nonisomorphic rigid differentially closed fields.

Similarly, we could interweave graph coding steps in the proof of Proposition 3.1 and build 2^κ nonisomorphic rigid differentially closed fields of cardinality κ when $\kappa = \aleph_\kappa$.

4. Remarks and Questions

In [6; 7] the authors introduce the notion of computable and Borel functors between classes of countable structures. For example, in Theorem 3.5, recovering the graph from the differentially closed field is a Borel functor from differentially closed fields to graphs. Miller wondered if there could be invertible functors between these classes. If there is an invertible functor F from graphs to differentially closed fields, then the authors show that the corresponding automorphism groups $\mathrm{Aut}(G)$ and $\mathrm{Aut}(F(G))$ would be isomorphic. Miller's original idea was that, since there are rigid graphs, one could show there was no such functor by showing that there are no rigid differentially closed fields. While our construction shows that this idea does not work, nevertheless, one can show there is no such functor by looking at possible automorphism groups. It is easy to construct a countable graph with an automorphism of order $n > 2$. But no differentially closed field can have an automorphism of order $n > 2$. Suppose K is differentially closed and σ is an automorphism of order $n > 2$. Let F be the fixed field of σ. Then K/F is an algebraic extension of order $n > 2$. By the Artin–Schreier theorem, this is impossible for K algebraically closed.

Question 1. Is there a differentially closed field K where $|\mathrm{Aut}(K)| = 2$? If so, is the fixed field a model of CODF? More generally, if K is a real closed differential field and $K(i)$ is differentially closed, must K be a model of CODF?

Question 2. Are there rigid differentially closed fields of cardinality $\aleph_1$?

The construction of such a model would require a new strategy. Perhaps it would help to assume the set theoretic principle $\diamondsuit$? Or perhaps one could use the methods of [19].

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I am pleased to submit this paper in honor of Udi Hrushovski's belated 60th birthday. The main result relies heavily on his work.

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