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Introduction

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Introduction

This volume comprises a mix of new research, retrospective reviews, and informal histories, all concerning or inspired by the contributions to model theory of our friend and ex-supervisor Boris Zilber, and dedicated to him in celebration of his 75th birthday. The varied subjects of the articles reflect aspects of Boris's mathematical focus at different stages of his career, which we can conveniently group around a number of influential conjectures. Boris's long-sighted vision is made nowhere clearer than in his formulation of conjectures which summarise our ignorance and point a way out of it. Each has inspired massive efforts, but none can yet be considered fully resolved.

A persistent theme of Boris's work has been the study of categorical structures. In the 1970s he developed a fine structure theory for uncountably and totally categorical structures. Along with such key results as the weak trichotomy theorem and the ladder theorem, this resulted in a pair of conjectures which are key to the first two themes of this volume. The *trichotomy conjecture* attempted a classification of the combinatorial geometries of strongly minimal sets, requiring a field structure to be responsible for any failure of (local) modularity. Although false, this conjecture remains fruitful. The articles of Castle and Hasson and of Chatzidakis and Hrushovski in this volume are directly concerned with situations in which the conjecture holds, and Moosa's article introduces a new theory in which it is strongly expected to hold. Kennedy's article is a philosophical take on the inspiration around the conjecture and Zilber's mathematics more generally. Related aspects of combinatorial geometries are discussed in Cameron's article. In recent decades, model theorists have become increasingly comfortable with extending ideas gestated in the categorical or ω-stable context to wider classes of theories, and this is on show in the articles of Ammer and Tent on strictly stable theories of open generalised polygons, of Hrushovski on approximate equivalence relations and their stabilisers, and of Malliaris and Shelah on new simple theories obtained from hypergraphs.

Groups interpretable in uncountably categorical structures, which are necessarily of finite Morley rank, play an important role in Zilber's structure theory. He obtained some of the foundational results on such groups, and formulated a form of what has become known as the *algebraicity conjecture*, or the *Cherlin–Zilber conjecture*. Poizat's article in this volume discusses its origins in detail, starting with Zilber's

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1977 article on categorical groups and rings, and leading to its commonly accepted modern formulation that any simple group of finite Morley rank is an algebraic group over an algebraically closed field. This has been the impetus for a substantial body of work. Along with Poizat's history of the problem, Cherlin's article is devoted to the case of odd type, where much is known, while Borovik's and Deloro's articles concern related results on linearisation of group actions in finite Morley rank.

Later, Boris returned to categoricity in a setting where it is hard to imagine anyone else daring to look for it: the complex exponential field. He developed his *quasiminimality conjecture* that every definable subset is countable or cocountable, and his *pseudoexponentiation conjecture*, which would extend this to a clear account of the coarse geometry of the structure. These, along with the related *conjecture on intersections with tori*, now known in generalised form as the *Zilber–Pink conjecture*, formed the main focus of Boris and his research group in Oxford for many years. The articles of Aslanyan and of Baldwin and Villaveces review various aspects of this programme and its offshoots. Meanwhile, Pila's article proves Zilber–Pink for raising to the power i, Kowalski's examines positive characteristic analogues of the Ax–Schanuel theorem, and Wilkie's article sets out his strategy for an analytic proof of the quasiminimality conjecture.

The above account skips over much of Boris's work. In particular, his thoughts in recent decades have increasingly turned towards applying model-theoretic methods to elucidate some difficulties in mathematical physics. This started with the noncommutative algebras arising in Zariski geometries, and continues through various incarnations of pseudofiniteness and notions of approximation. This work is proving to be a challenge for the model-theoretic community to absorb, perhaps reflected in the fact that, unfortunately, none of the contributions published in this volume ended up being on this subject.

Breaking up these scientific texts, we have included some more historical articles by Hodges, Makowsky, and Yomdin, relating their personal connections to Boris, and their view of the influence of his mathematics. In a similar vein, anecdotes from many people are collected in a piece on Logic Tea in Oxford. Apart from these articles of a biographical or anecdotal character, all articles were refereed to the journal's high standards.

A word on how this volume came to be. When we began thinking how to mark Boris's 75th birthday, the world had closed in response to the Covid-19 pandemic and bore no promise of reopening any time soon. With no clear prospect of an international conference being feasible in 2024, we settled for a "festlose Schrift", a Festschrift without the Fest. As it transpired, we have subsequently been able to arrange a conference, so at the time of writing we are looking forward to presenting the completed volume to Boris in Oxford in September 2024.

We would like to thank all the contributors to this volume for their wonderful articles, and also for being cooperative in making the editorial process a pleasurable task for us. We also thank the journal Model Theory for agreeing to publish this volume, and Martin Hils for guiding the process in his role as chief editor. Finally, we would like to thank Boris himself for his tremendous inspiration and his positive influence on our mathematics, our careers, and on our lives in general.

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