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We examine some of Zilber's early theorems through the lens of the "model-theoretic sublime".

1. Introduction

A recent email exchange between myself and Boris Zilber, stimulated by a lecture he gave in Helsinki a few years prior, began by discussing his various moves to generalize the syntax/semantics distinction.[1] Boris's repurposing of the syntax/semantics distinction — a distinction taken more or less for granted in foundational practice — has always been interesting; but in our exchange Boris also broke out *philosophically*:

> BZ: These are, I guess, two ways of how we perceive the world: the intellectual, words-based way, and the intuitive, sensory way. In mathematics, the first way requires you to write down a full proof of the fact (the ultimate explanation). The second, semantical way, is to see a picture, mental or graphical, that talks to your experience of the world. It is also what is responsible [for the] division of mathematics into Algebra and Geometry. Michael Atiyah (in his millennium lecture?) says that Geometry-Algebra is like Space-Time pairing: In geometry you see the whole at once, no time needed. In algebra you need time to read it letter-by-letter, but not space.

The words-based way and the semantical way, to wit: the mathematician is tethered to the *sign*, to formal correctness and to the "letter-by-letter" of proof; while on the other hand there is *insight* and *experience*, *meaning* and *seeing the whole picture*. Two poles pulling away from each other, and the mathematician caught somewhere in between.

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[1] The email exchange took place in May of 2023.

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In this note I would like to think about the way Boris pulls the curtain back on this binary practice of the mathematician, in his rich remark to me, so full of philosophical moves. One is struck by the phrase "seeing the whole at once, no time needed" — a move toward the *sublime*, I suggest, an aesthetic category important in 18th century British philosophy and of renewed interest today in the form of, for example, the environmental sublime.[2]

Thinking through Boris's beautiful remark in the context of the sublime helps us to place his remark, and beyond that his mathematical work, *philosophically*. Generally speaking, the philosophical content of a foundational attitude often has to do with its (so-called) existential or metaphysical commitments — or its lack thereof: how entangled with set theory it is, its putative second-order content, the theory's constructive content, and so on. I would like to think about Boris's work, though, by drawing on ideas coming from somewhat outside the foundations of mathematics culture. One is, of course, not *against* foundations of mathematics; for, to paraphrase Emily Apter [2013, p. 2], if one is against foundations, as a logician, what could one possibly be for? It is just that the interest here is in developing novel interpretive strategies.

2. The sublime

Boris's phrase "seeing the whole at once, no time needed" reminded me of the remark of the 18th century aesthetic theorist Alexander Gerard, that sublimity is the state in which "the mind . . . imagines itself present in every part of the scene it contemplates" [Gerard 1759, p. 14].[3]

More commonly[4] the sublime is thought of in the terms Kant laid out for it, namely in terms of a physical immensity, usually in nature — think of standing at the precipice of an enormous crevasse — that pitches the subject into a kind of vertigo; "doing violence to the imagination", as Kant put it; leaving the subject's cognitive apparatus undone. As Emily Brady writes, on the Kantian sublime:

> The sources of the sublime response are linked to the physical properties of magnitude or power in nature but importantly also to the failure of imagination, without which it could not occur. Imagination's activity in the sublime, in contrast to the beautiful, is "serious", where some object is "contrapurposive for our power of judgment, unsuitable for our faculty

[2] See, e.g., [Brady 2013]. Kant discusses the sublime mainly in the so-called third critique, the *Critique of the power of judgment* [Kant 2000, Sections 25–28].

[3] Gerard continues: ". . . and from the sense of this immensity, feels a noble pride, and entertains a lofty conception of its own capacity."

[4] More commonly in the philosophical aesthetics literature at least. As a referee has helpfully pointed out, the *everyday* meaning of the term "sublime", evoking properties such as "calmness" or "beauty", differs markedly from its meaning in philosophy.

> of presentation, and as it were doing violence to our imagination, but is nevertheless judged all the more sublime for that."[5]

This "astonishment bordering on terror", as Kant rather hyperbolically called it, involves anxiety, then, bordering on fear, but also, somehow, pleasure: the pleasure of being in the vicinity of danger, while at the same time being out of it; the pleasure of being in awe of something. *Negative* pleasure was Kant's term for this, while positive pleasure is pleasure in the beautiful, which "brings with it a promotion of life" [Kant 2000, p. 128].[6] Interestingly enough, because pleasure is involved, sublimity is theorized by philosophers an *aesthetic* category. The sublime response, in other words, is an *aesthetic* response.

Kant distinguishes the dynamic sublime, in which the subject is undone, so to speak, by a natural scene, from the mathematical sublime, in which the subject experiences a failure of the imagination, not in the face of a natural immensity but in the face of an infinite number sequence. In the encounter with the mathematical sublime the subject is thrown into confusion once again, for not having a grip on the contours of the thing at hand; but also being inexorably compelled. One might call this mixture of attraction and unease the *mathematician's* negative pleasure.

Kant's observation was that although the senses fail to deliver a conceptual unity on their own, the sequence can nevertheless "be *completely* comprehended *under* one concept", and this is due to a faculty of "suprasensible" reason:

> And what is most important is that to be able only to think it [the infinite: JK] as a *whole* indicates a faculty of mind which surpasses every standard of sense .... Nevertheless, *the bare capability of thinking* this infinite without contradiction requires in the human mind a faculty itself suprasensible.[7]

In other words, Kant gives us what the (classical) mathematician would say is the correct outcome. Reason meets the imagination at its point of collapse, delivering the infinite object as a conceptual unity — just as reason delivers a phenomenal unity in the case of overwhelming natural phenomena. The important point here is that this faculty of reason is *suprasensible*, so going beyond (sense) experience.

Sublimity, then, is not an experience of defeat, or not wholly; one is able to move out of it with the help of the mind's ability to synthesize the atomized scene, to structure chaos as nonchaos[8] — by means of a conception.

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[5] See [Brady 2013, p. 57]. The interior quote is from Kant's *Critique of the power of judgment* [Kant 2000, p. 129].

[6] See also [Brady 2013, p. 57].

[7] [Kant 2000, Section 26, p. 94]. See also [Ginsborg 2005].

[8] The expression is due to the artist Eva Hesse [Bourdon 1970].

As an aside, the sublime has a moral dimension, putting us in touch, as Brady [Brady 2013, p. 59] writes, with our moral capacities. The sublime tutors us in "[loving] something, even nature, without interest . . . even contrary to our (sensible) interest" [Kant 2000, p. 151]. Witnessing the failure of the imagination, the failure of her imagination to comprehend the scene, the subject remains "undemeaned", as Kant put it, even so, and even has a feeling of superiority over nature, or in our case, the mathematical field, while at the same time "the human being must submit to that dominion" [Kant 2000, pp. 261–262]. A century later Leo Marx would coin the term "the *technological* sublime" to describe the conflict arising from holding the romantic (sublime) conception of the American landscape of the late 19th century, seeing that terrain as a kind of virginal paradise, while employing the rhetoric of industrial progress [Marx 1964, p. 7]. And just a few years after that Hilbert would lace his oft-cited 1930 "ignorabimus" address with the language of human supremacy, expressed in terms of the technological optimism typical of the period.

Sublimity, in other words, is always connected to *power*. In the wake of the various emergencies, climatic and otherwise, besetting human beings in the 21st century, it is not surprising that there is a renewed philosophical interest in the sublime!

Later, post-Kantian and post-Gerardian passes at the sublime by writers such as F. R. Ankersmit would untether sublimity from awe and the idealization of nature that was characteristic of the earlier theories, so that the sublime could now be deployed in other domains, such as history, or psychoanalysis.[9] Ankersmit in particular took a *melancholic* view of sublimity, emphasizing the static quality of

[9]From [Ankersmit 2005, p. 335]:

> The traumatic experience is too terrible to be admitted to consciousness: The experience exceeds, so to speak, our capacities to make sense of experience. Whereas normally the powers of association enable us to integrate experience into the story of our lives, the traumatic experience remains dissociated from our life's narrative since these powers of association are helpless and characteristically insufficient in the case of trauma. And there is one more resemblance between trauma and the sublime that is of relevance in the present context. Characteristic of trauma is the incapacity to actually suffer from the traumatic experience itself . . . . The subject of a traumatic experience is peculiarly numbed by it; he is, so to speak, put at a distance from what caused it. The traumatic experience is dissociated from one's "normal" experience of the world . . . . Now, much the same can be observed for the sublime. When Burke speaks about this "tranquility tinged with terror," this tranquility is possible (as Burke emphasizes) thanks to our awareness that we are not really in danger. Hence, we have distanced ourselves from a situation of real danger — and in this way, we have dissociated ourselves from the object of experience. The sublime thus provokes a movement of derealization by which reality is robbed of its threatening potentialities. As such Burke's description of the sublime is less the pleasant thrill that is often associated, with it than a preemptive strike against the terrible.

the sublime response, the idea that the subject is locked into a back-and-forth cycle of attraction and repulsion. Sublimity, in other words, is a site of conflict:

> Now, aesthetics provides us with the category of the sublime for conceptualizing such a conflict of schemes without reconciliation or transcendence. Thus the Kantian sublime is not a transcendence of reason and understanding and the entry to a new and higher order reality, but can only be defined in terms of the inadequacy of both reason and understanding .... Similarly, it is only by way of the positive numbers that we can get access to the realm of negative numbers; and gaining this access does not in the least imply the abolition or transcendence of the realm of the positive numbers, but a continuous awareness of their existence as well.[10]

Kant's account of the role of intuition and reason in delivering conceptual coherence within sublimity is embedded in a complex theory of the mind, one drawing on specific conceptualizations of the faculties of imagination and reason. Kant's theory of the mathematical sublime is about our mathematical capacities überhaupt, and as such it slots easily into the contemporary conversation in the foundations of mathematics, the debates about the nature of finitary intuition, or what constitutes a genuinely constructive proof.

What rather holds my interest in thinking through Boris's beautiful remark though, are not the foundational issues per se, but the way his remark reveals that logic too is a site of conflict: a conflict that gets read into the syntax/semantics distinction, a conflict that renders logic so alive philosophically. It is astonishing that logic can even take the exact mathematical measure of that conflict, that is to say drawing out deep theorems from it, limitative results such as the incompleteness theorems due to Gödel, or the undefinability of truth due to Tarski.

In Ankersmit's writings the mathematical sublime is domesticated, as it were, so that mathematical sublimity now signifies anything in the way of a mathematical unknown:

> Think of the equation $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 12x$. Differential calculus shows that this function will have a local maximum for $x = -4$ and a local minimum for $x = 3$. In this way differential calculus can be said to perform what, analogously, could not possibly be performed for the relationship between narrative and experience. So one might say that historical writing is in much the same situation as mathematics was before the discovery of differential calculus by Newton and Leibniz. Before this discovery there was something "sublime" about the question of where the equation $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 12x$ would attain its local optimum and minimum: One could only hit on it experimentally (that is, by simply

[10] [Ankersmit 2002, p. 207].

> trying out different values for x), but no adequate explanation could be given for this. It has been Newton's and Leibniz's feat of genius to reduce what was "sublime" to what could be figured out, or to reduce what was incommensurable to what could be made commensurable thanks to the magic of differential calculus.[11]

Ankersmit is thinking about sublime historical experience in this passage, but we can draw the moral from it that Kant's notion of the mathematical sublime (which applied only to extended objects) was too narrow. It is not just that the imagination cannot take in infinite totalities; the mathematical field is full of concepts and ideas that cause the mathematician to lose his footing. There is the concept of a model class, for example — or how to get a foothold there? In logic one has the space of all first-order theories — how to find a way through that morass? Set theory also, with its large cardinal hierarchy, is threaded with sublimity through and through.

3. Categoricity and classification

Let us now turn to Boris's work, in particular its synthetic aspect within what one might call the *model-theoretic sublime*. Let us take "synthesis" to refer to an act of (mathematical) reason that structures some heretofore unstructured part of the mathematical field — unstructured in the sense of being untheorized, or unclassified, or simply formless.

The suggestion here is that both categoricity and classification can be viewed as devices imposing structure on the mathematical field, albeit in different ways: categoricity, a notion occupying a central place in Boris's mathematical work, by collapsing the space of all possible models (of a fixed cardinality) of an uncountably categorical theory to a single point (up to isomorphism);[12] classifiability in virtue of being an organizing principle, a kind of scaffolding structure for the space of first-order theories.[13]

Categorical theories are "logically perfect", in Boris's terminology, where logical perfection means the following: "...a mathematical object of a certain 'size' is logically perfect if in a certain formal language it allows a 'concise' description fully determining the object" [Cruz Morales et al. 2021, p. 2]. Precisely:

> The amazing conclusion derived from the research is that among the huge diversity of mathematical structures there are very few which satisfy the (slightly narrower) definition of categoricity, and those can be classified.

[11] [Ankersmit 2005, p. 175]. Ankersmit's reading of the historical details with regard to commensurability may be regarded by some as contentious.

[12] A theory T is said to be "categorical" if T has a unique model, up to isomorphism. T is said to be "categorical in power" if for all cardinals κ, T has a unique model of size κ, up to isomorphism.

[13] For an example of classifiability, see the below discussion of the main gap theorem.

> These certainly seem to corresponding to an ideal of logical perfection, in the following sense: categorical structures M determine a first-order theory $\mathrm{Th}(M)$ (the set of all sentences that are true in M) and then comes the reason why we call them "logically perfect": all other structures that satisfy the theory $\mathrm{Th}(M)$ and are of the same cardinality as M are isomorphic to M. In other words, uncountably categorical structures are inextricably linked to their logical description; the description $T = \mathrm{Th}(M)$ completely determines the structure M.[14]

The search for categorical axiomatisations of canonical mathematical theories is a philosophical project, fundamentally, albeit one pursued entirely within mathematics (or, precisely, within mathematical logic). If our canonical mathematical theories have a unique interpretation, referential indiscernibility is eliminated — which is just simply to say that in mathematics, or at the very least in the case at hand, we really do "mean what we say". For that reason, perhaps, categoricity represents, for Boris, the apotheosis of logical perfection. He has even conjectured, boldly, that "Categoricity is bound to play the role that analyticity played for number theory, but for physics" (see [Villaveces 2022]).

Andrés Villaveces writes eloquently about the epistemological aspect of categoricity, its evidentiary force, in a remark that seems, somehow, to gesture at sublimity:

> Al enfrentarnos a ciertas descripciones o afirmaciones nuestra reacción natural de incredulidad puede ser vista como una de las raíces de la búsqueda de atrapar, aprehender, mediante el lenguaje, la descripción de un fenómeno, de un objeto matemático o de un evento. Al vernos enfrentados a una afirmación (matemática o no), la primera reacción natural en muchas circunstancias suele ser de incredulidad. Ante la duda, intentamos buscar confirmación a como dé lugar. Dejando de lado búsquedas de verificación por autoridad, podemos señalar dos grandes tipos de confirmación: por verificación directa, por una buena descripción de la teoría que sustenta la afirmación en cuestión.[15]

[14] [Cruz Morales et al. 2021, p. 6].

[15] [Villaveces 2022]. In translation:

> When faced with certain descriptions or statements, our natural reaction of disbelief can be seen as one of the roots of the search to capture, apprehend, through language, the description of a phenomenon, of a mathematical object. or an event. When faced with a statement (mathematical or not), the first natural reaction in many circumstances is usually disbelief. When in doubt, we try to seek confirmation no matter wherefrom. Leaving aside verification by authority, we can point out two main types of confirmation: by direct verification, or by a good [i.e., categorical: JK] description of the theory that supports the statement in question.

Categorical theories are "logically perfect", in Boris's terminology, not only because they provide a compact description of a seemingly intractable field of concepts, but for enabling the possibility of regarding space as a coherent way of pasting localized versions of itself — a perfection realized, for Boris and coauthors in [Cruz Morales et al. 2021], by the notion of an affine scheme due to Grothendieck.

Synthesis emerges in model theory also through classification. Instead of a heterogeneous collection of theories (so theories this time, instead of models), and the mathematician having to creep from one theory to the next, to paraphrase Gerard,[16] Boris offered up the trichotomy conjecture,[17] which turned out to hold of the (very ample) Zariski structures:

Conjecture. If X is a strongly minimal set, then exactly one of the following is true about X.

(1) X is trivial in the sense that algebraic closure (on a saturated model of the theory of X) defines a degenerate pregeometry (for any set $A \subseteq X$ one has $\mathrm{acl}(A) = \bigcup\{\mathrm{acl}(\{a\}) \mid a \in A\}$).

(2) X is essentially a vector space. That is, possibly after adding some constant symbols to the language of X, there is an infinite group space G bi-interpretable with X for which every definable subset of any Cartesian power of G is a finite Boolean combination of cosets of definable subgroups.

(3) X is bi-interpretable with an algebraically closed field.[18]

Classification theorems in mathematics, then, serve as a move toward synthesis: *resisting* or *dissolving* sublimity, structuring the heretofore unstructured mathematical field as nonchaos, providing a scaffolding.

Together with Boris's work on trichotomy one should mention Shelah's main gap theorem [Shelah 1990], which is another masterpiece in the genre of classification theorems. The theorem states that the class of all first-order theories falls into two categories: the tame or classifiable, and the nonclassifiable. The former have "few" models and admit a dimension-like set of geometric invariants; the latter have the

[16]Gerard [1759] remarks:

> Objects cannot possess that largeness, which is necessary for inspiring a sensation of the sublime, without simplicity. Where this is wanting, the mind contemplates, not one large, but many small objects: it is pained with the labour requisite to creep from one to another; and is disgusted with the imperfection of the idea, with which, even after all this toil, it must remain contented. But we take in, with ease, one entire conception of a simple object, however large: in consequence of this facility, we naturally account it one . . . the view of any single part suggests the whole, and enables fancy to extend and enlarge it to infinity, that it may fill the capacity of the mind.

[17]See [Zilber 1984]. For Hrushovski's counterexample, see [Hrushovski 1993]. For the trichotomy theorem, see [Hrushovski and Zilber 1993].

[18]For a survey of recent work in the area see [Baldwin and Villaveces 2024] in this volume.

maximum number of models possible, and are entangled with each other in a way that makes it difficult to tell some of them apart.[19] The main gap theorem almost seems to be written in the language of the sublime!

4. Geometry as place

Returning to Boris's philosophical remark, if "in geometry you see the whole at once, no time needed", one may ask, what is this *"whole"* that Boris sees at once, no time needed? I would like to touch down here, albeit lightly, in the notion of *place*. Perhaps what geometry allows one to see is a kind of *place* — not in any literal sense but in the sense that the architectural theorist Juhani Pallasmaa means in his writings about placeness: a site of experiential cohesion, one resonating "with the inner qualities of placeness in our minds . . . a constitutive condition for anything to exist in human consciousness" [Pallasmaa 2023]. Pallasmaa states:

> The experience of placeness can . . . arise from countless characteristics and features, but fundamentally it is a consequence of experiential cohesion, spatial or formal singularity, communal agreement, or meaningfulness of a distinct entity in the physical world . . . . Through constructions, both material and mental, useful and poetic, practical and metaphysical, we create places, existential footholds in the otherwise meaningless world.

The thought here is that through geometry and its suggestion of place, through thinking of geometry as creating the conditions for the notion of place, the mathematician is led toward the possibility of concretizing, structuring, contextualizing and internalizing mathematical ideas. Note that we take places in at once, no time needed. As Pallasmaa puts it, "We 'understand' qualities of places unconsciously before we have had any chance for intellectual evaluation or understanding."

If architecture, for Pallasmaa, is engaged with the lived meaning of space, "[projecting] predictable order and meaning into human existence", "[mediating] between the threatening immensity of the world, the infinity and anonymity of space, as well as the endlessness of time", here geometry stands in for, in the sense of functioning as, architecture, in grounding the mathematician in the mathematical field, in enabling the possibility of lived mathematical experience.

[19] More precisely, Shelah's main gap theorem divides all countable, complete first-order theories into two categories: in the classifiable case, there is a bound on the number of models (up to isomorphism), and they can be characterized by a tree of geometric invariants, like the dimension of a vector space, while at the same time in the nonclassifiable case, there is a precise sense in which no notion of dimension can be extracted, and the case is chaotic in the sense that the structures are hard to tell apart.

There is also the ontological question: is anything real in mathematics, that is *not* related to geometry? "Nothing is that is not placed", as Plato has reportedly said.[20]

5. Conclusion

Amid the debates in philosophical aesthetics, such as whether aesthetic properties reside in the subject or in the object, or whether aesthetic experience involves cognition or not, the sublime persists as a central irritant. In the hands of contemporary philosophers the sublime has been extended well beyond the categories Kant envisaged, as we saw, so that we now have the romantic sublime, the technological sublime, the environmental sublime that Emily Brady writes about so eloquently, the historical sublime, the moral sublime, and so on. There is a substantial philosophical literature, by now, on the sublime; let us add to it the category of the *model-theoretic sublime*.

In this brief note I have strayed into philosophical territory; but in fact the correspondence between Boris and I ended with Boris going to ground philosophically:

JK: In your own work though, how is it helpful to think of the syntax/semantics distinction in the way you do?

BZ: ...here is one of my talks on the topic, attached. It is what resulted from my attempts to understand what "non-commutative geometry" is and how it originated in Heisenberg's physics. In more detail, you can download a couple of papers from my web-page, like "The geometric semantics of algebraic quantum mechanics".

Boris's mathematical work stages a beautiful encounter with the mathematical sublime. It is essential that we recognize it as a logician's encounter with the mathematical sublime, that is to say, one occurring within *logic*. This is because the display of power here originates exactly in the logician's gift, unique to him among all mathematicians, namely his sensitivity to language — utilizing and directing that power, as Boris does, onto mathematics and physics.

In writing model theory in the language of geometry, a hallmark of Boris's mathematical practice, the conflicted aspect of sublimity, the idea of stasis and being locked into a cycle, is set aside, and the conditions for a *rapprochement* between the words-based way and the semantical way are laid out, because of geometry. It is a road that opens out into freedom for the logician; it is a road that delivers the logician into the mathematical arena. And while coming to grips with Boris's work involves a great deal of negative pleasure — because of the toil

[20] Jeff Malpas in his lecture at the *Understanding and Designing Place* Symposium at the Tampere University on 3 April, 2017.

involved but also being, as we are, in awe of what he has done — then if positive pleasure is pleasure in the *beautiful*, simply and for itself, Boris's work gives us that too — straight to the heart.

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