

Pacific Journal of Mathematics

REMARK ON THE USE OF FORMS IN VARIATIONAL CALCULATIONS

LOUIS AUSLANDER

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In [1] we introduced in equation (2.2) the form

$$\omega = \sum \frac{\partial F}{\partial q'_i} dq_i - \left(\sum q'_i \frac{\partial F}{\partial q'_i} - F \right) dt.$$

The purpose of this note is to explain the reason for introducing precisely this form.

We considered the integral

$$I = \int_a^b F(q_1, \dots, q_n; q'_1, \dots, q'_n; t) dt.$$

Now let S be the set of all forms η such that if X denotes the vector field to a curve C with equations

$$q_i = q_i(t)$$

$$q'_i = \frac{dq_i}{dt}$$

$$t = t$$

Then $\langle X, \eta \rangle = F$ or $I = \int_a^b \langle X, \eta \rangle dt$. The set S is certainly not void since $F(q_1, \dots, q_n; q'_1, \dots, q'_n; t) dt$ and ω are contained in it. We will prove the following.

THEOREM. *There exists one and only one form ω in S such that along every curve of the above type ω and $d\omega$ give rise to forms in the space (q_1, \dots, q_n, t) .*

Proof. The hypotheses of this theorem are equivalent to the following two analytic conditions:

1. $\langle \partial/\partial q'_i, \omega \rangle = 0$
2. $\langle \partial/\partial q'_i \wedge \partial/\partial x, d\omega \rangle = 0$ when $q'_i dt = dq_i$ where x is any of the coordinates (q_i, q'_i, t) , $i=1, \dots, n$.

Condition 1. implies that $\omega = \sum a_i dq_i + b dt$. Now since $\omega \in S$ we must have

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$$\sum a_i \frac{dq_i}{dt} + b = F \quad \text{or} \quad \omega = \sum a_i dq_i + (F - \sum a_i q_i) dt .$$

By computing $d\omega$ and replacing $q_i dt$ by dq_i we get

$$\frac{\partial F}{\partial q'_i} = a_i .$$

This proves that $\omega = \sum \frac{F}{q'_i} dq_i - \left(\sum \frac{F}{q'_i} q'_i - F \right) dt$ is the only form which satisfies the theorem.

REFERENCE

1. Auslander, L., *The use of forms in variational calculations*, Pacific J. Math., **5** (1955), 853-859.

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