

Pacific Journal of Mathematics

**ON MAPPINGS FROM THE FAMILY OF WELL ORDERED
SUBSETS OF A SET**

SEYMOUR GINSBURG

ON MAPPINGS FROM THE FAMILY OF WELL ORDERED SUBSETS OF A SET

SEYMOUR GINSBURG

A simply ordered set E is called a k -set if there exists a simply ordered extension of the family of nonempty well ordered subsets of E , ordered by initial segments, into E . If E is not a k -set then it is called a k' -set. Kurepa [1;2] first discussed these sets. He showed that if E is a subset of the reals and if the smallest ordinal number α such that E does not contain a subset of order type α is ω_1 , then E is a k' -set. In particular the rationals and the reals, denoted by R and R^+ respectively, are both k' -sets. In this paper the existence of k -sets and k' -sets is discussed further. Theorem 7 states that each simply ordered set E is a terminal segment of some k -set $F(E)$. It is not true, however, that each simply ordered set E is similar to an initial section of some k -set $F(E)$ (Theorem 2). Finally, in Theorem 10 it is shown that each infinite simply ordered group is a k' -set.

Following the symbolism in [1;2] let E be a simply ordered set and ωE the family of all nonempty well ordered subsets of E , partially ordered as follows: For A and B in ωE , $A <_k B$ if and only if A is a proper initial segment of B .¹

Definition. A function f from ωE to E is called a k -function on E , if $A <_k B$ implies that $f(A) < f(B)$.

If there exists a k -function on E , that is, from ωE to E , then E is called a k -set. If not, then E is called a k' -set.

THEOREM 1. If f is a k -function on E , then for each nonempty well ordered subset W of E , there exists an element x in W such that $f(W) \leq x$.

Proof. Suppose that the theorem is false, that is, suppose that there exists an element W_1 in ωE with the property that $x < f(W_1)$ for each x in W_1 . Let $W_2 = W_1 \cup f(W_1)$. It is easily seen that W_2 is well ordered, $W_1 <_k W_2$, $x < f(W_2)$ for each element x in W_2 , and the order type of W_2 is ≥ 2 . Suppose that for each $0 < \xi < \alpha$, W_ξ is an element

Received October 17, 1955. Presented to the American Mathematical Society November, 1955.

¹ A is a (proper) initial segment of B if A is a (proper) subset of B and if, for each element z in A , $\{x | x \leq z, x \in B\}$ is a subset of A . A is a terminal segment of B if A is a subset of B and if, for each element z in A , $\{x | z \leq x, x \in B\}$ is a subset of A .

of ωE such that

(1) $x < f(W_\xi)$ for each x in W_ξ ,

(2) $W_\xi <_k W_\nu$ for $\xi < \nu < \alpha$,

and (3) the order type of W_ξ is $\geq \xi$.

Two possibilities arise.

(a) If $\alpha = \beta + 1$ let $W_\alpha = W_\beta \cup f(W_\beta)$. By (1) and the fact that W_β is well ordered, it follows that W_α is well ordered. Clearly $W_\beta <_k W_\alpha$. Thus $f(W_\beta) < f(W_\alpha)$. It is now easy to verify that (1), (2), and (3) are satisfied for $\xi \leq \alpha$.

(b) Suppose that α is a limit number. Let $W_\alpha = \bigcup_{\xi < \alpha} W_\xi$. Since $W_\xi <_k W_\nu$ for $\xi < \nu$, W_α is well ordered. It is obvious that (2) and (3) are satisfied for $\xi \leq \alpha$. Let x be any element of W_α . Then x is in W_ξ for some $\xi < \alpha$, thus $x < f(W_\xi) < f(W_\alpha)$. Hence (1) is also satisfied.

In this way W_ξ becomes defined for each ordinal number ξ . Thus W_δ is defined, where δ is the smallest ordinal number such that E contains no subset of order type δ . This is a contradiction since W_δ is of order type $\geq \delta$.

We conclude that no such set W_1 exists, that is, the theorem is true.

Suppose that E is a k' -set and that the ordered sum² $E + F$ is a k -set for some simply ordered set F . Let f be a k -function on $E + F$. Since E is a k' -set, for some well ordered subset W of E , $f(W)$ is not in E , thus is in F . Then $f(W) \leq x$ for some x in W is false. By Theorem 1, therefore, f is not a k -function on $E + F$. Hence we have

THEOREM 2. *If E is a k' -set then so is $E + F$ for every simply ordered set F .*

The simplest example of a k' -set E is any infinite well ordered set. This is an immediate consequence of the following observation, whose proof is by a straightforward application of transfinite induction.

'The initial segments of an infinite well ordered set of order type α form a set of order type $\alpha + 1$ '.

Another consequence of this observation is the following: For any infinite k -set E , the smallest ordinal number δ having the property that E contains no subset of order type δ , is a limit number.

Suppose that E is a k -set and has an initial segment of n -elements, say $x_0 < x_1 < \dots < x_{n-1}$. Letting $A_j = \{x_i \mid i < j\}$, by a simple application of Theorem 1, it is easily seen that $f(A_j) = x_{j-1}$ for each k -function f on E . In other words, there is no element x of A_j such that $f(A_j) < x$.

² The ordered sum $\sum_v E_v$, or $\dots + E_{v_1} + \dots + E_{v_2} + \dots$, of a family of pairwise disjoint simply ordered sets is the set $E = \bigcup_v E_v$ ordered as follows: If x and y are in the same E_v , then $x < y$ or $y < x$ according as $x < y$ or $y < x$ in E_v . If x is in E_v and y is in E_ν and $v < \nu$ in V , then $x < y$.

This result cannot occur if E has no first element. To be precise we have:

THEOREM 3. *If E is a k -set without a first element, then there exists a k -function g such that $g(W) < x$ for each element W in ωE and for some element x in W .*

Proof. Let f be a k -function on E . Well order the elements of ωE into the sequence $\{W_\xi\}$, $\xi < \delta$. Suppose that g is already defined for each W_ξ , $\xi < \theta$ (possibly other W_ξ also) such that

- (1) $g(W_\lambda) \leq f(W_\lambda)$ for each W_λ for which g is defined;
- (2) g is not defined for W_θ ;
- (3) if g is defined for W_γ , then g is also defined for each initial segment of W_γ ;
- (4) if $W_\sigma <_k W_\tau$ and g is defined for W_σ and W_τ , then $g(W_\sigma) < g(W_\tau)$;
- (5) if g is defined for W_ξ , then $g(W_\xi) < x_\xi$ for some element x_ξ in W_ξ .

Let $W_\theta = \{x_{\theta,\nu} | \nu < \alpha(\theta)\}$ and $W_{\theta,\xi} = \{x_{\theta,\nu} | \nu < \xi\}$ for $0 < \xi \leq \alpha(\theta)$. Let $W_{\theta,\gamma}$ be the first $W_{\theta,\xi}$ for which g is not defined: If $\gamma = 1$, that is, $W_{\theta,\gamma} = \{x_{\theta,0}\}$ let $g(W_{\theta,1})$ be some element of E which is $< \min [x_{\theta,0}, f(x_{\theta,0})]$. Such an element exists since E has no first element. Suppose that $\gamma = \beta + 1$, where $\beta > 0$. By induction, $g(W_{\theta,\beta}) < x_{\theta,\beta}$ for some element $x_{\theta,\beta}$ in $W_{\theta,\beta}$. Let $g(W_{\theta,\beta+1}) = \min [x_{\theta,\beta}, f(W_{\theta,\beta+1})]$. Since $W_{\theta,\beta} < W_{\theta,\beta+1}$, $x_{\theta,\beta}$ is not the last element in $W_{\theta,\beta+1}$. Thus $g(W_{\theta,\beta+1}) < x_{\theta,\beta+1}$ for some element $x_{\theta,\beta+1}$ in $W_{\theta,\beta+1}$. Suppose that $W_\sigma <_k W_{\theta,\beta+1}$. If $g(W_{\theta,\beta+1}) = x_{\theta,\beta}$, then $g(W_\sigma) \leq g(W_{\theta,\beta}) < x_{\theta,\beta} = g(W_{\theta,\beta})$. If $g(W_{\theta,\beta+1}) = f(W_{\theta,\beta+1})$, then

$$g(W_\sigma) \leq g(W_{\theta,\beta}) \leq f(W_{\theta,\beta}) < f(W_{\theta,\beta+1}) = g(W_{\theta,\beta+1}).$$

Suppose that γ is a limit number. Then $W_{\theta,\gamma}$ has no last element. It follows from Theorem 1 that there exists an element $x_{\theta,\gamma}$ in $W_{\theta,\gamma}$ so that $f(W_{\theta,\gamma}) < x_{\theta,\gamma}$. Let $g(W_{\theta,\gamma}) = f(W_{\theta,\gamma})$. If $W_\sigma <_k W_{\theta,\gamma}$, then

$$g(W_\sigma) \leq f(W_\sigma) < f(W_{\theta,\gamma}) = g(W_{\theta,\gamma}).$$

By transfinite induction g becomes defined for each $W_{\theta,\xi}$, thus for W_θ so as to satisfy (1), (3), (4), and (5). Thus g becomes defined for every W_ξ . From the manner of construction, that is (4), g is a k -function. By (5) g has the property that for each element W in ωE , $g(W) < x$ for some element x in W .

THEOREM 4. *If $\bar{A} \equiv \bar{B}^3$ and A is a k -set, then so is B . Equivalently, if $\bar{A} \equiv \bar{B}$ and A is a k' -set, then so is B .*

³ E being a simply ordered set, \bar{E} denotes the order type of E . $\bar{A} \equiv \bar{B}$ if there exists a similarity transformation of A into B and a similarity transformation of B into A .

Proof. Let g be a similarity transformation of A into B and h a similarity transformation of B into A . Suppose that f is a k -function of ωA into A . For each well ordered subset E of B , $h(E)$ is a well ordered subset of A which is similar to E . Let f^* be the function of ωB into B which is defined by $f^*(E) = gfh(E)$. Clearly $gfh(C) < gfh(D)$ if $C <_k D$. Thus f^* is a k -function, so that B is a k -set.

Turning to the construction of k -sets we have

THEOREM 5. *If $\{E_v | v \in V\}$ is a family of pairwise disjoint k -sets, and V is the dual⁴ of a well ordered set, then the ordered sum ΣE_v is a k -set.*

Proof. Let f_v be a k -function from ωE_v to E_v . Now let A be a nonempty well ordered subset of ΣE_v . Denote by w the largest element v in V such that $A \cap E_v$ is nonempty. Since V is the dual of a well ordered set, w exists. Let h be the function which is defined by $h(A) = f_w(A \cap E_w)$. There is no trouble verifying that h is a k -function from $\omega \Sigma E_v$ to ΣE_v .

COROLLARY. *The dual of a well ordered set is a k -set. One particular k -function is the mapping which takes a well ordered subset into its largest element.*

Another method of obtaining k -sets is to use the next result.

THEOREM 6. *Let $\{A_v | v \in V\}$ be a family of pairwise disjoint simply ordered sets where V is the dual of a well ordered set of order type α , α being a limit number. Furthermore suppose that for each element w in V , there exists a simply ordered extension f_w of $A^w = \omega \sum_{v > w} A_v$ into A_w ⁵. Then $A = \sum_{v \in V} A_v$ is a k -set.*

Proof. Let X be any nonempty well ordered subset of A . Let x_0 be the first element in X . x_0 is in one of the sets A_v , say A_r . Since α is a limit number, r has an immediate predecessor in V , say r^- . By hypothesis there exists a simply ordered extension f_{r^-} of $\omega A^{r^-} = \omega \sum_{v > r^-} A_v$ into A_{r^-} . Let $f(X) = f_{r^-}(X)$. Thus f is a well defined function from ωA into A .

Suppose that $Y <_k Z$ in ωA . The first element in Y , say y_0 , is also the first element in Z . If y_0 is in A_s , then $f(Y) = f_{s^-}(Y) < f_{s^-}(Z) = f(Z)$. Thus f is a k -function and A is a k -set.

Now let E_0 be any simply ordered set. It is known that each

⁴ $(\rho, <')$ is the dual of $(\rho, <)$ if $x <' y$ if and only if $x > y$, for every x and y in ρ .

⁵ f is a simply ordered extension of the partially ordered set B into the simply ordered set A if f maps B into A in such a manner that whenever $x < y$ in B , $f(x) < f(y)$ in A .

partially ordered set has a simply ordered extension [3]. Let f_0 be a simply ordered extension of ωE_0 into some set, say F_0 . Let E_1 be a simply ordered set such that $\bar{E}_1 = \bar{F}_0 + \bar{E}_0$. Continuing by induction we obtain for each ordinal number ν , a simply ordered extension f_ν of ωG_ν , where $\bar{G}_\nu = \dots + \bar{E}_\xi + \dots + \bar{E}_1 + \bar{E}_0$ ($\xi < \nu$), into a simply ordered set F_ν . Let E_ν be a simply ordered set such that $\bar{E}_\nu = \bar{F}_\nu + \bar{G}_\nu$. In particular, by Theorem 6, G_ω is a k -set. Thus we have

THEOREM 7. *Each simply ordered set E is a terminal segment¹ of some k -set $F(E)$.*

REMARK. Theorem 2 shows that there exist simply ordered sets E such that for no k -set $F(E)$ is E similar to an initial segment of $F(E)$.

We now consider products of simply ordered sets, ordered by last differences.

THEOREM 8. *If E and F are k -sets, then so is $E \times F$.*

Proof. Let f and g be k -functions for E and F respectively, and z a definite element of E . Let A be any well ordered subset of $E \times F$. Define A_τ to be the set $\{v \mid \text{for some } u, (u, v) \text{ is in } A\}$. Obviously A_τ is a well ordered subset of F . If A_τ has a last element, say w , let $A_\sigma = \{u \mid (u, w) \text{ is in } A\}$ and let $h(A) = (f(A_\sigma), g(A_\tau))$. If A_τ has no last element, let $h(A) = (z, g(A_\tau))$. To see that h is a k -function let $A <_k B$ in $\omega E \times F$. Since A is a proper initial segment of B , either A_τ is a proper initial segment of B_τ , or else $A_\tau = B_\tau$. If the former holds, then since $g(A_\tau) < g(B_\tau)$, $h(A) < h(B)$. Suppose that the latter holds. Since $A <_k B$, there exists an element (x, y) in B which is not in A . Thus $A \subseteq \{(u, v) \mid (u, v) < (x, y), (u, v) \text{ in } B\}$. Since $A_\tau = B_\tau$, it follows that y must be the last element of B_τ , thus also of A_τ . Therefore A_σ and B_σ exist. Since A is a proper initial segment of B , $A_\sigma <_k B_\sigma$. As f is a k -function, $f(A_\sigma) < f(B_\sigma)$. Hence

$$h(A) = [f(A_\sigma), g(A_\tau)] < [f(B_\sigma), g(A_\tau)] = h(B).$$

REMARKS. (1) Theorem 8 is no longer true if one of the sets, either A or B is a k' -set. This is seen by two examples.

(a) Let E be a set of one element and F a set order type ω . Then $E \times F$ is of order type ω , thus a k' -set.

(b) Interchange E and F in (a).

(2) The conclusion of Theorem 8 may be true if one of the sets is a k -set and the other is not. For example

(a) Let $\bar{E} = \omega^{\omega^*}$ and $\bar{F} = \omega$. Then $\bar{E} \times \bar{F} = \bar{E}$, and as easily seen, E

is a k -set. It is also easy to show that for each ordinal number α and each limit number δ , $A_\alpha \times B_\delta$ is a k -set, where $\overline{A}_\alpha = \alpha$ and $\overline{B}_\delta = \delta^*$. If $\alpha \geq \omega$, then $B_\delta \times A_\alpha$ is a k' -set.

(b) Let $A_0 = R$, f_1 be a simply ordered extension of wA_0 into B_1 , and $A_{-1} = (A_0 \times B_1)$. In general, let f_n be a simply ordered extension of $\omega(\sum_{i < n} A_{-i})$ into B_n , and $A_{-n} = (A_0 \times B_n)$. Let $F = \sum_{n < \omega} A_{-n}$. By Theorem 6, F is a k -set. Then $\overline{A_0 \times F} = \sum (\overline{A_0 \times A_{-n}}) = \sum \overline{A_{-n}} = \overline{F}$. Thus $A_0 \times F$ is a k -set. It is known [1; 2] that A_0 is a k' -set.

(3) Theorem 8 is no longer true if we have a product of an infinite number of k -sets. For example, for each negative integer ν let $E_\nu = \{0, 1\}$. Then $\prod E_\nu$ is the set of all zero-one sequences of order type ω^* , ordered by last differences. But $\overline{\prod E_\nu} = \lambda$, where $\lambda = \overline{R^+}$. R^+ is a k' -set [2]. By Theorem 4, $\prod E_\nu$ is a k' -set.

Question. Do there exist two k' -sets E and F such that $E \times F$ is a k -set?

THEOREM 9. If E is a k' -set and F is a simply ordered set with a first element, then $E \times F$ is a k' -set.

Proof. Let x_0 be the first element of F and $G = F - \{x_0\}$. Then $E \times F = E \times [\{x_0\} + G] = E \times \{x_0\} + E \times G$. Since $E \times \{x_0\}$ is a k' -set, by Theorem 2 so is $E \times \{x_0\} + E \times G$. Hence the result.

Since $\lambda = 1 + \lambda$ and $\eta = 1 + \eta$, where $\eta = \overline{R}$, it follows from Theorem 4 and Theorem 9 that for any k' -set A , $A \times R$ and $A \times R^+$ are k' -sets. In particular, Euclidean n -space, ordered by last differences of the coordinates of the points, is a k' -set.

THEOREM 10. Each infinite simply ordered group is a k' -set. If E is an ordered field, then there is no k -function from the bounded elements of ωE to E .

Proof. First suppose that E is an ordered field. Let 1 be the multiplicative identity. For $1 < x$ let $h(x) = 2 - 1/x$ where $2 = 1 + 1$. For $0 \leq x \leq 1$ let $h(x) = x$. For $x < 0$ let $h(x) = -h(-x)$. Then h is a similarity transformation of E onto $(-2, 2)$.

Suppose that f is a k -function from the bounded elements of ωE to E . Let $x_0 = z_0 = 0$, $z_1 = 1$, $x_1 = h(1)$, and $A_j = \{x_i | i < j\}$ for $j = 1, 2$. Let $y_1 = f(A_1)$ and $y_2 = f(A_2)$. Clearly $y_1 < y_2$. Let $z_2 = z_1 + (y_2 - y_1)$. Thus $z_2 - z_1 = y_2 - y_1$. Let $x_2 = h(z_2)$. In general suppose that for $1 < \xi < \alpha$, z_ξ , $x_\xi = h(z_\xi)$, $A_\xi = \{x_\nu | \nu < \xi\}$, and $y_\xi = f(A_\xi)$ are defined. Furthermore, suppose that $\{z_\xi\}$ and $\{y_\xi\}$ are strictly increasing and that $z_\xi - z_1 = y_\xi - y_1$ for

$1 < \xi$. Since E is a group, z_ξ and x_ξ are elements of E . Observe that $-2 < x_\xi < 2$, that is $\{x_\xi\}$ is a bounded sequence.

(1) Suppose that $\alpha = \beta + 1$. Let $A_\alpha = \{x_\xi | \xi < \alpha\}$, $y_\alpha = f(A_\alpha)$, $z_\alpha = z_\beta + (y_\alpha - y_\beta)$, and $x_\alpha = h(z_\alpha)$. Since $A_\beta <_k A_\alpha$, $y_\beta < y_\alpha$. Thus $z_\beta < z_\alpha$ and $x_\beta < x_\alpha$. Since $z_\alpha - z_\beta = y_\alpha - y_\beta$ and $z_\beta - z_1 = y_\beta - y_1$, we get $z_\alpha - z_1 = y_\alpha - y_1$.

(2) Suppose that α is a limit number. Let $A_\alpha = \{x_\xi | \xi < \alpha\}$ and $y_\alpha = f(A_\alpha)$. Since $A_\xi <_k A_\alpha$, for $\xi < \alpha$, $y_\xi < y_\alpha$. Let $z_\alpha = z_1 + (y_\alpha - y_1)$ and $x_\alpha = h(z_\alpha)$. Since $A_\xi <_k A_\alpha$ for $\xi < \alpha$, $y_\xi < y_\alpha$ and thus $z_\xi < z_\alpha$ and $x_\xi < x_\alpha$. Note that $z_\alpha - z_1 = y_\alpha - y_1$.

In this way, for each ξ we get an x_ξ . Let δ be the smallest ordinal number such that E contains no subset of order type δ . The elements of the set $\{x_\xi | \xi < \delta\}$ form a strictly increasing sequence of order type δ . From this contradiction we see that no such function f exists.

Now suppose that E is an infinite simply ordered group. Let $z_0 = 0$ and $z_1 > 0$. Let $A_j = \{z_i | i < j\}$ for $j = 1, 2$. Let $y_1 = f(A_1)$ and $y_2 = f(A_2)$. Repeat the procedure given above, defining y_ξ and z_ξ for each ξ , with $A_\nu = \{z_\xi | \xi < \nu\}$. We obtain a strictly increasing sequence of elements $\{z_\xi\}$, $\xi < \delta$, where δ has the same significance as above. Again we arrive at a contradiction.

REMARK. The second statement in Theorem 10 cannot be extended to hold for a group. For example, let E be the group consisting of all the integers, positive, negative, and zero. The bounded, well ordered subsets of E consist of the finite subsets of E . For this family there does exist a k -function, namely the function which maps each set into its maximal element.

REFERENCES

1. Kurepa, G. *Sur les fonctions réelles dans la famille des ensembles bien ordonnés de nombres rationnels*, Bull. Internat. Acad. Yougoslave. Cl. Sci. Math. Phys. Tech., **12** (1954), 35-42.
2. ———. *Fonctions croissantes dans la famille des ensembles bien ordonnés lineaires*; Bulletin Scientifique Yougoslavie, **2**, (1954), 9.
3. Szpilrajn, E. *Sur l'extension de l'ordre partiel*, Fund. Math., **16**, (1930), 386-389.

NORTHROP AIRCRAFT CORPORATION AND
UNIVERSITY OF SOUTHERN CALIFORNIA

PACIFIC JOURNAL OF MATHEMATICS

EDITORS

H. L. ROYDEN
Stanford University
Stanford, California

R. P. DILWORTH
California Institute of Technology
Pasadena 4, California

E. HEWITT
University of Washington
Seattle 5, Washington

E. G. STRAUS
University of California
Los Angeles 24, California

ASSOCIATE EDITORS

E. F. BECKENBACH
C. E. BURGESS
H. BUSEMANN
H. FEDERER

M. HALL
P. R. HALMOS
V. GANAPATHY IYER
R. D. JAMES

M. S. KNEBELMAN
I. NIVEN
T. G. OSTROM
M. M. SCHIFFER

J. J. STOKER
G. SZEKERES
F. WOLF
K. YOSIDA

SUPPORTING INSTITUTIONS

UNIVERSITY OF BRITISH COLUMBIA
CALIFORNIA INSTITUTE OF TECHNOLOGY
UNIVERSITY OF CALIFORNIA
MONTANA STATE UNIVERSITY
UNIVERSITY OF NEVADA
OREGON STATE COLLEGE
UNIVERSITY OF OREGON
UNIVERSITY OF SOUTHERN CALIFORNIA

STANFORD UNIVERSITY
UNIVERSITY OF UTAH
WASHINGTON STATE COLLEGE
UNIVERSITY OF WASHINGTON
* * *
AMERICAN MATHEMATICAL SOCIETY
CALIFORNIA RESEARCH CORPORATION
HUGHES AIRCRAFT COMPANY

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be typewritten (double spaced), and the author should keep a complete copy. Manuscripts may be sent to any of the editors. Manuscripts intended for the outgoing editors should be sent to their successors. All other communications to the editors should be addressed to the managing editor, E. G. Straus at the University of California, Los Angeles 24, California.

50 reprints of each article are furnished free of charge; additional copies may be obtained at cost in multiples of 50.

The *Pacific Journal of Mathematics* is published quarterly, in March, June, September, and December. The price per volume (4 numbers) is \$12.00; single issues, \$3.50. Back numbers are available. Special price to individual faculty members of supporting institutions and to individual members of the American Mathematical Society: \$4.00 per volume; single issues, \$1.25.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 2120 Oxford Street, Berkeley 4, California.

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), No. 10, 1-chome, Fujimi-cho, Chiyoda-ku, Tokyo, Japan.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies.

Pacific Journal of Mathematics

Vol. 6, No. 4

, 1956

Seymour Ginsburg, <i>On mappings from the family of well ordered subsets of a set</i>	583
Leon Ehrenpreis, <i>Some properties of distributions on Lie groups</i>	591
Marion K. Fort, Jr., <i>A geometric problem of Sherman Stein</i>	607
Paul R. Garabedian, <i>Calculation of axially symmetric cavities and jets</i>	611
Walter Mossman Gilbert, <i>Completely monotonic functions on cones</i>	685
William L. Hart and T. S. Motzkin, <i>A composite Newton-Raphson gradient method for the solution of systems of equations</i>	691
C. W. Mendel and I. A. Barnett, <i>A functional independence theorem for square matrices</i>	709
Howard Ashley Osborn, <i>The problem of continuous programs</i>	721
William T. Reid, <i>Oscillation criteria for linear differential systems with complex coefficients</i>	733
Irma Reiner, <i>On the two-adic density of representations by quadratic forms</i>	753
Shoichiro Sakai, <i>A characterization of W^*-algebras</i>	763
Robert Steinberg, <i>Note on a theorem of Hadwiger</i>	775
J. Eldon Whitesitt, <i>Construction of the lattice of complemented ideals within the unit group</i>	779
Paul Civin, <i>Correction to "Some ergodic theorems involving two operators"</i>	795