

Pacific Journal of Mathematics

ISOMORPHISME D'HYPERGROUPOÏ DES ISOTOPES

A. SADE

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Un *Hypergroupeïde* [2], est un ensemble, E , muni d'une loi de composition faisant correspondre à tout couple ordonné d'éléments $x, y \in E$ un sous-ensemble non vide de E , $x * y = (a, b, c, \dots) \subseteq E$. Un *scalaire*, [1, p. 707], est un élément s , tel que, $\forall x \in E$, $x * s$ et $s * x$ se réduisent à un seul élément. Un Hypergroupeïde est *associatif* si $\forall x, y, z \in E$, $(x * y) * z = x * (y * z)$. Une *unité scalaire* est un élément, u , tel que $\forall x \in E$, $x * u = u * x = x$. Deux hypergroupeïdes $H(*)$ et $H'(\times)$, définis sur le même ensemble E , sont *isotopes* si, ξ, η, ζ , étant trois applications biunivoques de E sur lui-même,

$$(1) \quad \forall x, y, z \in E, \quad x * y = z \overset{\zeta}{\rightleftarrows} x\xi \times y\eta = z\zeta,$$

ou $x * y = (x\xi \times y\eta)\zeta^{-1}$. Ils sont *isomorphes* si $\xi = \eta = \zeta$.

THÉOREME. *Si un hypergroupeïde $H(\times)$, avec l'unité scalaire u , est isotope (ξ, η, ζ) d'un hypergroupeïde associatif $G(*)$, défini sur le même ensemble, alors G et H coïncident par l'isomorphisme $x \rightarrow x\theta = x\xi\xi^{-1}\eta$.*

Preuve. Puisque G est associatif,

$$\forall x, y, z \in E, \quad x * (y * z) = (x * y) * z.$$

Donc, sur l'isotope H , (1)

$$(2) \quad (x\xi \times (y\xi \times z\eta)\zeta^{-1}\eta)\zeta^{-1} = ((x\xi \times y\eta)\zeta^{-1}\xi \times z\eta)\zeta^{-1}.$$

ξ, η étant des permutations de E ,

$$(\exists) x, z \in E, \quad x\xi = u, \quad z\eta = u.$$

Puisque (2) est vérifiée $\forall x, y$, en faisant $x\xi = u$, on a

$$(y\xi \times z\eta)\zeta^{-1}\eta = y\eta\zeta^{-1}\xi \times z\eta,$$

$$(3) \quad (x * y)\eta = x\eta\zeta^{-1}\xi \times y\eta.$$

En faisant au contraire $z\eta = u$, on a

$$x\xi \times y\xi\zeta^{-1}\eta = (x\xi \times y\eta)\zeta^{-1}\xi,$$

$$(4) \quad \forall t \in E, \quad t \times y\theta = (t \times y\eta)\zeta^{-1}\xi.$$

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Enfin, en faisant à la fois $x\xi = u$ et $z\eta = u$, on a

$$(5) \quad \forall y, y\eta\xi^{-1} = y\xi\xi^{-1}\eta = y\theta .$$

Donc, d'après (3),

$$(x * y)\eta\xi^{-1} = (x\eta\xi^{-1} \times y\eta)\xi^{-1} .$$

D'après (5)

$$(x * y)\theta = (x\theta \times y\eta)\xi^{-1} ;$$

d'après (4)

$$(x * y)\theta = x\theta \times y\theta .$$

La démonstration ne serait plus valable si u n'était pas scalaire bilatère et si l'associativité se réduisait à l'inclusion

$$(x * y) * z \supseteq x * (y * z) .$$

RÉFÉRENCES

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2. Yuzo Utumi, *On Hypergroups of group right cosets*, Osaka Math. J. **1** (1949), 73-80.

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