

# Pacific Journal of Mathematics

**LINEAR TRANSFORMATIONS ON GRASSMAN SPACES**

ROY WESTWICK

# LINEAR TRANSFORMATIONS ON GRASSMANN SPACES

R. WESTWICK

1. Let  $U$  denote an  $n$ -dimensional vector space over an algebraically closed field  $F$ , and let  $G_{nr}$  denote the set of nonzero pure  $r$ -vectors of the Grassmann product space  $\Lambda^r U$ . Let  $T$  be a linear transformation of  $\Lambda^r U$  which sends  $G_{nr}$  into  $G_{nr}$ . In this note we prove that  $T$  is nonsingular, and then, by using the results of Wei-Liang Chow in [1], we determine the structure of  $T$ .

For each  $z = x_1 \wedge \cdots \wedge x_r \in G_{nr}$ , we let  $[z]$  denote the  $r$ -dimensional subspace of  $U$  spanned by the vectors  $x_1, \cdots, x_r$ . By Lemma 5 of [1], two independent elements  $z_1$  and  $z_2$  of  $G_{nr}$  span a subspace all of whose nonzero elements are in  $G_{nr}$  if and only if  $\dim([z_1] \cap [z_2]) = r - 1$ ; that is, if and only if  $[z_1]$  and  $[z_2]$  are adjacent. If  $V \subseteq \Lambda^r U$  is a subspace such that each nonzero vector in  $V$  is in  $G_{nr}$  and if  $V$  is maximal (that is, not contained in a larger such subspace) then  $\{[z] \mid z \in V, z \neq 0\}$  is a maximal set of pairwise adjacent  $r$ -dimensional subspaces of  $U$ . These sets of subspaces are of two types; namely, the set of all  $r$ -dimensional subspaces of  $U$  containing a common  $(r - 1)$ -dimensional subspace, and the set of all  $r$ -dimensional subspaces of an  $(r + 1)$ -dimensional subspace of  $U$ . We adopt the usual convention of calling these sets of subspaces maximal sets of the first and second kind respectively. We will let  $A_r$  denote the set of those maximal  $V$  which determine a set of pairwise adjacent subspaces of the first kind, and we will let  $B_r$  denote the set of those maximal  $V$  which determine a set of pairwise adjacent subspaces of the second kind.

2. In this section we prove that if  $T$  sends each member of  $B_r$  into a member of  $B_r$ , then  $T$  is nonsingular.

Let  $U_1, \cdots, U_t$  be  $k$ -dimensional pairwise adjacent subspaces of  $U$  and let  $z_i \in G_{nk}$  be such that  $[z_i] = U_i$  for  $i = 1, \cdots, t$ . Then  $\{U_1, \cdots, U_t\}$  is said to be independent if and only if  $\{z_1, \cdots, z_t\}$  is an independent subset of  $\Lambda^k U$ . We note the following facts concerning an independent set  $\{U_1, \cdots, U_t\}$ . If it is of the first kind (in the sense of the previous section) then there is an independent set of vectors  $\{x_1, \cdots, x_{k-1}, y_1, \cdots, y_t\}$  of  $U$  such that for  $i = 1, \cdots, t$ ,  $U_i = \langle x_1, \cdots, x_{k-1}, y_i \rangle$  denotes the linear subspace spanned by the vectors enclosed. If it is of the second kind, then there is an independent set of vectors  $\{x_1, \cdots, x_{k+1}\}$  such that  $U_i = \langle x_1, \cdots, x_{i-1}, x_{i+1}, \cdots, x_{k+1} \rangle$ , for  $i = 1, \cdots, t$ . It is easily

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deduced from this that  $\dim (\bigwedge^r U_1 + \cdots + \bigwedge^r U_i)$  is equal to  $t \binom{k-1}{r-1} + \binom{k-1}{r}$  or  $\sum_{i=0}^{t-1} \binom{k-i}{r-1}$  according as the set of subspaces  $\{U_i\}$  is of the first or second kind. We adopt the usual convention that  $\binom{m}{n} = 0$  if  $m < n$ . Finally, if the set  $\{U_1, \dots, U_i\}$  is not independent, then for some  $i$ ,  $\bigwedge^r U_i \subseteq \bigwedge^r U_1 + \cdots + \bigwedge^r U_{i-1}$ . In fact, the choice of  $i$  such that  $\{z_1, \dots, z_{i-1}\}$  is independent and  $z_i \in \langle z_1, \dots, z_{i-1} \rangle$  will do.

We require the

LEMMA 1. *Let  $\{U_1, \dots, U_{s+1}\}$  be a set of pairwise adjacent  $k$ -dimensional subspaces of  $U$ . Suppose further that the set is independent and is of the second kind. Let  $V \subseteq \bigwedge^r U_1 + \cdots + \bigwedge^r U_{s+1}$  be a subspace with dimension  $\binom{k-s}{r-s}$ , where  $s \leq r \leq k$ . Then there is a set  $\{V_1, \dots, V_s\}$  of pairwise adjacent  $k$ -dimensional subspaces of  $U$  such that  $V \cap (\bigwedge^r V_1 + \cdots + \bigwedge^r V_s) \neq \{0\}$ .*

*Proof.* Let  $m = \binom{k-s}{r-s}$  and let  $\{z_1, \dots, z_m\}$  be a basis of  $V$ . Choose an independent set of vectors  $\{x_1, \dots, x_{k+1}\}$  of  $U$  such that for  $i = 1, \dots, s+1$ ,  $U_i = \langle x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_{k+1} \rangle$ . We can write

$$z_i = z_1^i + x_1 \wedge \cdots \wedge x_{s-1} \wedge x_s \wedge z_2^i + x_1 \wedge \cdots \wedge x_{s-1} \wedge x_{s+1} \wedge z_3^i$$

where

$$z_1^i \in \bigwedge^r U_1 + \cdots + \bigwedge^r U_{s-1} \quad \text{and} \quad z_2^i, z_3^i \in \bigwedge^{r-s} \langle x_{s+2}, \dots, x_{k+1} \rangle$$

for  $i = 1, \dots, m$ . In the case that  $s = 1$ , we take  $z_1^i \in \bigwedge^r \langle x_3, \dots, x_{k+1} \rangle$ . In the case that  $s = r$ , we take  $z_2^i, z_3^i \in F$ . If  $\{z_2^1, \dots, z_2^m\}$  or  $\{z_3^1, \dots, z_3^m\}$  is dependent, then we can form a linear combination of  $z_1, \dots, z_m$  which will be in  $\bigwedge^r U_1 + \cdots + \bigwedge^r U_{s-1} + \bigwedge^r U_{s+1}$  or  $\bigwedge^r U_1 + \cdots + \bigwedge^r U_{s-1} + \bigwedge^r U_s$  respectively. If, on the other hand, both sets are independent then each is a basis of  $\bigwedge^{r-s} \langle x_{s+2}, \dots, x_{k+1} \rangle$  since  $\dim (\bigwedge^{r-s} \langle x_{s+2}, \dots, x_{k+1} \rangle) = \binom{k-s}{r-s} = m$ . Let  $z_2^i = \sum_{j=1}^m a_{ij} z_2^j$ ,  $i = 1, \dots, m$ . Choose  $\lambda \neq 0$  and  $b_i \in F$ , not all equal to zero, such that

$$\lambda b_j = \sum_{i=1}^m b_i a_{ij}, \quad j = 1, \dots, m.$$

Then

$$\begin{aligned} 0 \neq \sum_{j=1}^m b_j z_j &= \sum_{j=1}^m z_1^j + \sum_{j=1}^m x_1 \wedge \cdots \wedge x_{s-1} \wedge (x_s + \lambda^{-1} x_{s+1}) \wedge b_j z_2^j \\ &\in \bigwedge^r U_1 + \cdots + \bigwedge^r U_{s-1} + \bigwedge^r V_1 \end{aligned}$$

where  $V_1 = \langle x_1, \dots, x_{s-1}, x_s + \lambda^{-1} x_{s+1}, x_{s+2}, \dots, x_{k+1} \rangle$ . The subspaces

$U_1, \dots, U_{s-1}, V_1$  are pairwise adjacent and so the Lemma is proved.

The nonsingularity of  $T$  is now proved as follows. Let  $W$  be a subspace of  $U$ . We prove, by induction on the dimension of  $W$ , that  $T$  is one-to-one on  $\Lambda^r W$  and that the image of  $\Lambda^r W$  under  $T$  is  $\Lambda^r W'$  for some subspace  $W'$  of  $U$  with  $\dim(W) = \dim(W')$ . When  $\dim(W) = r + 1$  this is clear since we are assuming that  $B_r$  is sent into  $B_r$  by  $T$ . Suppose that the statement has been proved for  $k$ -dimensional subspaces, and consider a  $(k + 1)$ -dimensional subspace  $W$  of  $U$ . Let  $s$  be the largest integer such that for any set  $\{W_1, \dots, W_s\}$  of pairwise adjacent  $k$ -dimensional subspaces of  $W$ ,  $T$  is one-to-one on  $\Lambda^r W_1 + \dots + \Lambda^r W_s$ . If  $s \geq r + 1$  then  $T$  is one-to-one on  $\Lambda^r W$ , since in this case, for an independent set  $\{W_1, \dots, W_s\}$  we must have  $\Lambda^r W = \Lambda^r W_1 + \dots + \Lambda^r W_s$ . Suppose then that  $1 \leq s \leq r$  and let  $\{U_1, \dots, U_{s+1}\}$  be any set of  $s + 1$  pairwise adjacent  $k$ -dimensional subspaces of  $W$ . If the set is dependent then  $T$  is one-to-one  $\Lambda^r U_1 + \dots + \Lambda^r U_{s+1}$  since we may drop one of the terms. Therefore we assume that the set is independent. Choose  $k$ -dimensional subspaces  $U'_1, \dots, U'_{s+1}$  such that  $T(\Lambda^r U_i) = \Lambda^r U'_i$  for  $i = 1, \dots, s + 1$ . For each  $j \leq s$ ,  $T$  maps  $\Lambda^r U_1 + \dots + \Lambda^r U_j$  onto  $\Lambda^r U'_1 + \dots + \Lambda^r U'_j$ . Therefore, since  $T$  is one-to-one on  $\Lambda^r U_1 + \dots + \Lambda^r U_s$ , the set  $\{U'_1, \dots, U'_s\}$  is independent. Furthermore, the set  $\{U'_1, \dots, U'_{s+1}\}$  is also independent. If not, then the image under  $T$  of both  $\Lambda^r U_1 + \dots + \Lambda^r U_s$  and  $\Lambda^r U_1 + \dots + \Lambda^r U_{s+1}$  is  $\Lambda^r U'_1 + \dots + \Lambda^r U'_s$ . But then the dimension of the null space of  $T$  in  $\Lambda^r U_1 + \dots + \Lambda^r U_{s+1}$  is at least as large as the difference in the dimensions of  $\Lambda^r U_1 + \dots + \Lambda^r U_{s+1}$  and  $\Lambda^r U_1 + \dots + \Lambda^r U_s$ , that is,  $\binom{k-s}{r-s}$ . We apply Lemma 1 to contradict the choice of  $s$ . It follows that  $T$  is one-to-one on all of  $\Lambda^r W$ . Finally, let  $\{W_1, \dots, W_{k+1}\}$  be an independent set of  $k$ -dimensional pairwise adjacent subspaces of  $W$  (necessarily of the second kind). Let  $W'_i$  be chosen so that  $T(\Lambda^r W_i) = \Lambda^r W'_i$ . It follows easily that  $\{W'_1, \dots, W'_{k+1}\}$  is of the second kind also, so that the image of  $\Lambda^r W$  is  $\Lambda^r W'$  where  $W'$  is the  $(k + 1)$ -dimensional subspace of  $U$  containing  $W'_1, \dots, W'_{k+1}$ . By taking  $W = U$  we see that  $T$  is one-to-one on  $\Lambda^r U$ .

3. It is necessary to investigate whether a general  $T$  does necessarily send each element of  $B_r$  into  $B_r$ . For the cases  $n > 2r$ ,  $n < 2r$ , this is proved directly, using Lemma 2. The case  $n = 2r$  requires a more delicate argument, given at the end of this section; there it is shown that if some element of  $B_r$  is sent into  $B_r$  by  $T$ , then  $T$  sends  $B_r$  into  $B_r$ .

LEMMA 2. *Let  $r < n$  and let  $V_1$  and  $V_2$  be in  $A_r$  such that  $V_1 \cap V_2 \neq \{0\}$ . Then, if  $V \subseteq V_1 + V_2$  and  $\dim(V) = n - r$ , we have  $V \cap G_{nr} \neq \phi$ .*

*Proof.* Let  $U_i$  be the  $(r - 1)$ -dimensional subspace of  $U$  determined by  $V_i$  for  $i = 1, 2$ . Since  $V_1 \cap V_2 \neq \{0\}$ , either  $U_1 = U_2$  or  $\dim(U_1 \cap U_2) = r - 2$ .

If  $U_1 = U_2$  then  $V_1 = V_2$ , so that in this case it is clear that  $V \cap G_{nr} \neq \phi$ .

Suppose that  $\dim(U_1 \cap U_2) = r - 2$  and let  $\{x_1, \dots, x_{r-2}\}$  be a basis of this intersection. Choose  $y_i$  such that  $U_i = \langle x_1, \dots, x_{r-2}, y_i \rangle$  for  $i = 1, 2$ . Choose  $u_i$  and  $v_i$  in  $U$ ,  $i = 1, \dots, n - r$ , such that

$$\{z_i = x_1 \wedge \dots \wedge x_{r-2} \wedge (y_1 \wedge u_i + y_2 \wedge v_i) \mid i = 1, \dots, n - r\}$$

forms a basis of  $V$ . If

$$\{x_1, \dots, x_{r-2}, y_1, y_2, v_1, \dots, v_{n-r}\} \quad \text{or} \quad \{x_1, \dots, x_{r-2}, y_1, y_2, u_1, \dots, u_{n-r}\}$$

is dependent, then there is a linear combination of the  $z_i$  which is in  $V_1$  or  $V_2$  respectively. If, on the other hand, both sets are independent, then they are both bases for  $U$  and we may write

$$u_i = w_i + c_i y_2 + \sum_{j=1}^{n-r} a_{ij} v_j, \quad i = 1, \dots, n - r,$$

where  $w_i \in \langle x_1, \dots, x_{r-2}, y_1 \rangle$  and  $c_i, a_{ij} \in F$ . We note that  $\det(a_{ij}) \neq 0$  so we can choose  $\lambda \neq 0$  and  $b_j$  for  $j = 1, \dots, n - r$ , not all zero, such that  $\lambda b_j = \sum_{i=1}^{n-r} b_i a_{ij}$ . Then

$$0 \neq \sum_{j=1}^{n-r} b_j z_j = x_1 \wedge \dots \wedge x_{r-2} \wedge (y_1 + \lambda^{-1} y_2) \wedge \left[ \left( \sum_{j=1}^{n-r} b_j c_j \right) y_2 + \lambda \sum_{j=1}^{n-r} b_j v_j \right]$$

is an element of  $V \cap G_{nr}$ . This proves the Lemma.

For  $n \neq 2r$  the image under  $T$  of an element of  $B_r$  is an element of  $B_r$ . For  $n < 2r$  this is clearly so since the subspaces of  $\bigwedge^r U$  in  $B_r$  have dimension  $r + 1$ , which is greater than the dimension  $(n - r + 1)$  of the subspaces in  $A_r$ .

For  $n > 2r$  we proceed as follows. The image of an  $A_r$  is an  $A_r$ . Suppose that the image of a  $W \in B_r$  is a subspace of a  $V \in A_r$ . Choose two elements  $V_1$  and  $V_2$  of  $A_r$  such that  $V_1 \cap V_2 \neq \{0\}$  and  $\dim(V_1 \cap W) = \dim(V_2 \cap W) = 2$ . One does this by choosing  $V_1$  and  $V_2$  so that the  $(r - 1)$ -dimensional subspaces of  $U$  determined by them are adjacent subspaces of the  $(r + 1)$ -dimensional subspace determined by  $W$ . Now,  $T(V_1) = T(V_2) = V$  since each is in  $A_r$  and each intersects  $V$  in at least two dimensions. Therefore  $T(V_1 + V_2) = V$  and so the null space of  $T$  in  $V_1 + V_2$  has dimension equal to  $(2n - 2r + 1) - (n - r + 1) = n - r$ . By Lemma 2, it follows that the null space of  $T$  intersects  $G_{nr}$  which contradicts the hypothesis that  $T$  sends  $G_{nr}$  into  $G_{nr}$ .

In the case that  $n = 2r$  the image of a  $B_r$  may be an  $A_r$  since the dimensions are equal. However, we prove that if some  $B_r$  is sent into a  $B_r$  by  $T$ , then the image of each  $B_r$  is a  $B_r$ . Suppose not. Then we can choose  $(r + 1)$ -dimensional subspaces  $W_1$  and  $W_2$  of  $U$  such that  $T(\bigwedge^r W_1) \in A_r$  and  $T(\bigwedge^r W_2) \in B_r$ . Furthermore, we can choose  $W_1$  and  $W_2$  adjacent, so that  $\dim(W_1 \cap W_2) = r$ . Choose three distinct elements  $V_1, V_2$ , and  $V_3$  of  $A_r$  such that the  $(r - 1)$ -dimensional subspaces of  $U$  determined by these elements are contained in  $W_1 \cap W_2$ . Then  $\dim(V_i \cap \bigwedge^r W_j) = 2$  for  $i = 1, 2, 3$  and  $j = 1, 2$ , so that  $T(V_i)$  intersects  $T(\bigwedge^r W_j)$  in at least two dimensions for each  $i, j$ . This implies that each  $T(V_i)$  is equal to one of  $T(\bigwedge^r W_j)$  and so two of them are equal. The argument of the previous paragraph now leads to a contradiction.

4. By essentially the same argument as used by Chow in [1] to prove his Theorem 1, we can prove that; if  $S$  is a nonsingular linear transformation of  $\bigwedge^r U$  sending  $G_{nr}$  into  $G_{nr}$ , and if the image of each  $B_r$  is a  $B_r$ , then  $S$  is a compound. (By a compound we mean a linear transformation of  $\bigwedge^r U$  which is induced by a linear transformation of  $U$ .)

In the case that  $n \neq 2r$  it follows that  $T$  is necessarily a compound. For  $n = 2r$ ,  $T$  is a compound if some  $B_r$  is sent into a  $B_r$ . If we let  $T_0$  denote a linear transformation of  $\bigwedge^r U$  induced by a correlation of the  $r$ -dimensional subspaces of  $U$ , then  $T_0$  is nonsingular and sends  $G_{nr}$  onto  $G_{nr}$ . The image of each  $A_r$  under  $T_0$  is a  $B_r$ . Therefore, if a  $B_r$  is sent by  $T$  into an  $A_r$ , the  $T_0T$  is a compound. We have proved the

**THEOREM.** *Let  $U$  be an  $n$ -dimensional vector space over an algebraically closed field and let  $T$  be a linear transformation of  $\bigwedge^r U$  which sends  $G_{nr}$  into  $G_{nr}$ . Then  $T$  is a compound except, possibly, when  $n = 2r$ , in which case  $T$  may be the composite of a compound and a linear transformation induced by a correlation of the  $r$ -dimensional subspaces of  $U$ .*

**REFERENCE**

1. Wei-Liang Chow, *On the Geometry of Algebraic Homogeneous Spaces*, Annals of Math., **50** (1949), 32-67.



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# Pacific Journal of Mathematics

Vol. 14, No. 3

July, 1964

Erik Balslev and Theodore William Gamelin, <i>The essential spectrum of a class of ordinary differential operators</i> .....	755
James Henry Bramble and Lawrence Edward Payne, <i>Bounds for derivatives in elliptic boundary value problems</i> .....	777
Hugh D. Brunk, <i>Integral inequalities for functions with nondecreasing increments</i> .....	783
William Edward Christilles, <i>A result concerning integral binary quadratic forms</i> .....	795
Peter Crawley and Bjarni Jónsson, <i>Refinements for infinite direct decompositions of algebraic systems</i> .....	797
Don Deckard and Carl Mark Percy, <i>On continuous matrix-valued functions on a Stonian space</i> .....	857
Raymond Frank Dickman, Leonard Rubin and P. M. Swingle, <i>Another characterization of the <math>n</math>-sphere and related results</i> .....	871
Edgar Earle Enochs, <i>A note on reflexive modules</i> .....	879
Vladimir Filippenko, <i>On the reflection of harmonic functions and of solutions of the wave equation</i> .....	883
Derek Joseph Haggard Fuller, <i>Mappings of bounded characteristic into arbitrary Riemann surfaces</i> .....	895
Curtis M. Fulton, <i>Clifford vectors</i> .....	917
Irving Leonard Glicksberg, <i>Maximal algebras and a theorem of Radó</i> .....	919
Kyong Taik Hahn, <i>Minimum problems of Plateau type in the Bergman metric space</i> .....	943
A. Hayes, <i>A representation theory for a class of partially ordered rings</i> .....	957
J. M. C. Joshi, <i>On a generalized Stieltjes transform</i> .....	969
J. M. C. Joshi, <i>Inversion and representation theorems for a generalized Laplace transform</i> .....	977
Eugene Kay McLachlan, <i>Extremal elements of the convex cone <math>B_n</math> of functions</i> .....	987
Robert Alan Melter, <i>Contributions to Boolean geometry of <math>p</math>-rings</i> .....	995
James Ronald Retherford, <i>Basic sequences and the Paley-Wiener criterion</i> .....	1019
Dallas W. Sasser, <i>Quasi-positive operators</i> .....	1029
Oved Shisha, <i>On the structure of infrapolynomials with prescribed coefficients</i> .....	1039
Oved Shisha and Gerald Thomas Cargo, <i>On comparable means</i> .....	1053
Maurice Sion, <i>A characterization of weak* convergence</i> .....	1059
Morton Lincoln Slater and Robert James Thompson, <i>A permanent inequality for positive functions on the unit square</i> .....	1069
David A. Smith, <i>On fixed points of automorphisms of classical Lie algebras</i> .....	1079
Sherman K. Stein, <i>Homogeneous quasigroups</i> .....	1091
J. L. Walsh and Oved Shisha, <i>On the location of the zeros of some infrapolynomials with prescribed coefficients</i> .....	1103
Ronson Joseph Warne, <i>Homomorphisms of <math>d</math>-simple inverse semigroups with identity</i> .....	1111
Roy Westwick, <i>Linear transformations on Grassman spaces</i> .....	1123