TENSOR PRODUCTS OVER $H^*$-ALGEBRAS

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Throughout, $A$, $B$, and $C$ denote (semi-simple) $H^*$-algebras whose respective decompositions into minimal closed ideals are $A = \sum A_a$, $B = \sum B_b$, and $C = \sum C_c$. It is assumed that $A$ is a right $C$-module and $B$ is a left $C$-module. We define a tensor product $A \otimes_0 B$ that is again an $H^*$-algebra, and show that it is isometric and isomorphic with an ideal in $A \otimes B \otimes C$. As a corollary, $A \otimes_0 B$ is strongly semi-simple if $A$, $B$, and $C$ are each strongly semi-simple. The converse to the corollary is shown to be false. When $A$, $B$, and $C$ are closed ideals in some $H^*$-algebra, with ordinary multiplication as the module action, then $A \otimes_0 B$ is shown to be isomorphic with the direct sum of all the one-dimensional ideals in $A \cap B \cap C$. When $A = L^2(G)$, $B = L^2(H)$, and $C = L^2(K)$, for suitable related compact groups $G$, $H$, and $K$, then the module actions are defined, and $A \otimes_0 B$ can be constructed. When $G = H = K$, it is shown that $A \otimes_0 B \cong L^2(G/N)$, where $N$ is the closure of the commutator subgroup of $G$. A conjecture is stated that would generalize this result to the case where $K$ is a closed subgroup of $G \cap H$.

Since $A \otimes_0 B$ will be represented in terms of ordinary tensor products $A \otimes B$ of $H^*$-algebras, the requisite facts concerning $A \otimes B$ are stated here (details may be found in [2]).

$A \otimes B$ is the Hilbert space completion of the space $A \otimes' B$ of all conjugate bilinear functionals $T$ on $A \times B$ of the form $T = \sum_i a_i \otimes b_i$, where $T(a, b) = \Sigma (a_i, a)(b_i, b)$ (see [3]). We define $(a \otimes b)(c \otimes d) = ac \otimes bd$, and extend by linearity and continuity to multiplication on $A \otimes B$. Then

I. $A \otimes B$ is an $H^*$-algebra and each $A_a \otimes B_b$ may be identified with a closed ideal in $A \otimes B$.

II. $A \otimes B = \Sigma \otimes (A_a \otimes B_b)$ is the decomposition of $A \otimes B$ into minimal closed ideals.

III. $A \otimes B$ is strongly semi-simple (see [5], p. 59) if and only if both $A$ and $B$ are strongly semi-simple.

1. Tensor products.

Definition. $F_0(A, B)$ will denote the collection of all finite formal
splits of the form
\[ \sum_{i=1}^{n} c_i(a_i, b_i), \]
with \( a_i \in A, b_i \in B, \) and \( c_i \in C; \) i.e. \( F_0(A, B) \) is the
free \( C \)-module generated by \( A \times B. \)

\( F_0(A, B) \) becomes an algebra and a pseudo-inner product space if
the operations are defined by the formulas:

\[
(c(a, b)) \cdot (c'(a', b')) = cc'(aa', bb'),
\]

\[
\lambda \sum c_i(a_i, b_i) = \sum (\lambda c_i)(a_i, b_i), \lambda \text{ complex, and}
\]

\[
(c(a, b), c'(a', b')) = (c, c')(a, a')(b, b')
\]

(the first and third must be extended by linearity). The positive
semi-definiteness of the pseudo-inner product follows from the fact that
\( (c(a, b), c'(a', b')) = (a \otimes b \otimes c, a' \otimes b' \otimes c') \); the other properties required
of an inner product obviously hold.

Let \( I'_1 \) be the ideal in \( F_0(A, B) \) spanned by the set of all elements
of the following forms:

1. \( c(a_1 + a_2, b) - c(a_1, b) - c(a_2, b), \)
2. \( c(a, b_1 + b_2) - c(a, b_1) - c(a, b_2), \)
3. \( (c_1 + c_2)(a, b) - c_1(a, b) - c_2(a, b), \)
4. \( \lambda c(a, b) - c(\lambda a, b), \) and
5. \( \lambda c(a, b) - c(a, \lambda b) \)

for arbitrary \( a, a_i \in A; b, b_i \in B; c, c_i \in C; \) and complex numbers \( \lambda. \)

Let \( I'_2 \) be the ideal in \( F_0(A, B) \) generated by the set of all elements
of the forms:

6. \( c_1 c_2(a, b) - c_1(ac, b), \) and
7. \( c_1 c_2(a, b) - c_2(a, c_1 b) \)

for arbitrary \( a \in A, b \in B, \) and \( c_i \in C. \) Then let \( I' = I'_1 \vee I'_2 = I'_1 + I'_2, \)
the ideal generated by the set of all elements of the forms (1)–(7).

**Proposition 1.** \( I'_1 = \{ X \in F_0(A, B): (X, X) = 0 \}. \)

**Proof.** Straightforward computations show that \( (X, Y) = 0 \) if \( X \)
is of one of the forms (1)–(5) and \( Y = c'(a', b'). \) Extending by linearity
we have immediately that \( (X, Y) = 0 \) for all \( X \in I'_1, Y \in F_0(A, B). \)
Suppose then that \( X = \sum_{i=1}^{n} c_i(a_i, b_i) \) and that \( (X, X) = 0. \) It must be
shown that \( X \in I'_1. \)

If \( \{ c_i \}_{i=1}^{n} \) is not linearly independent, then we may assume that
\( c_n = \sum_{i=1}^{n-1} \lambda_i c_i, \) and so
The expression in brackets is clearly an element of $I_i$, call it $\gamma_i$. Thus we have

\[
X = \sum_{i=1}^{n-1} c_i(a_i, b_i) + \left(\sum_{i=1}^{n-1} \lambda_i c_i\right)(a_n, b_n)
\]

\[
= \sum_{i=1}^{n-1} c_i(a_i, b_i) + \sum_{i=1}^{n-1} c_i(\lambda_i a_n, b_n)
\]

\[
+ \left[\left(\sum_{i=1}^{n-1} \lambda_i c_i\right)(a_n, b_n) - \sum_{i=1}^{n-1} c_i(\lambda_i a_n, b_n)\right].
\]

where $a_i = a_i$, $a_{i2} = \lambda_i a_n$, $b_i = b_i$, $b_{i2} = b_n$. Repeating the process as many times as is necessary we obtain

\[
X = \sum_{j=1}^{2^p} \left(\sum_{i=1}^{2^p-q(i)} c_i(a_{ij}, b_{ij})\right) + \gamma_1,
\]

where $\gamma_p \in I'_1$ and $\{c_{ij}\}_{i=1}^{n-p}$ is linearly independent. Then, for each fixed index $i$, by using an argument similar to the one above, we can write

\[
\sum_{j=1}^{2^p} c_i(a_{ij}, b_{ij}) = \sum_{k=1}^{2^{p-q(i)}} \left(\sum_{j=1}^{2^{p-q(i)}} c_i(a_{ijk}, b_{ijk})\right) + \gamma_{i, q(i)}
\]

where $\gamma_{i, q(i)} \in I'_1$ and $\{a_{ij}: j = 1, \ldots, 2^p - q(i)\}$ is linearly independent. As a result, we have

\[
X = \sum_{i=1}^{n-p} \sum_{j=1}^{2^{p-q(i)}} \sum_{k=1}^{2^{q(i)}} c_i(a_{ij}, b_{ijk}) + \gamma,
\]

where $\{c_i\}$ is linearly independent, $\{a_{ij}\}$ is linearly independent for each fixed $i$, and $\gamma \in I'_1$.

Fix any pair $<i, j>$ of indices. By the Hahn-Banach Theorem and the Riesz Theorem there exist $a' \in A$ and $c' \in C$ such that

\[
\|c'\| = \|a'\| = 1, \quad \langle c_i, c' \rangle = d_i > 0, \quad \langle a_{ij}, a' \rangle = d_{ij} > 0,
\]

($c_i'$, $c'$) = 0 if $i' \neq i$, and ($a_{ij}'$, $a'$) = 0 if $j' \neq j$. Since $F_p(A, B)$ is a pseudo-inner product space, the Schwarz inequality holds. Thus if we let $b' = \sum\{b_{ijk}: k = 1, \ldots, 2^{\nu(i)}\}$, we have

\[
|\langle X, c'(a', b') \rangle| \leq \langle X, X \rangle \langle c'(a', b'), c'(a', b') \rangle = 0.
\]

On the other hand,

\[
\langle X, c'(a', b') \rangle = \sum_{m, n, k} c_m c'(a_{mn}, a')(b_{mnk}, b') = d_i d_{ij} \|b'\|^2 = 0,
\]

so that $b' = 0$. If we now write
\[
\sum_k c_i(a_{ij}, b_{ijk}) = c_i(a_{ij}, \sum_i b_{ijk}) + [\sum_k c_i(a_{ij}, b_{ijk}) - c_i(a_{ij}, \sum_i b_{ijk})]
= c_i(a_{ij}, 0) + \gamma'_{ij},
\]
where \(\gamma'_{ij}\) is the expression in brackets, which is clearly an element of \(I'_i\), then we have

\[
X = \sum_i c_i(a_{ij}, 0) + \gamma',
\]
where \(\gamma' = \sum_i \gamma'_{ij}\), and so \(X \in I'_i\).

\(F'_0(A, B)\) is a pseudo-normed space, with \(||X||^* = (X, X)\). Let us denote by \(F'_0(A, B)\) its pseudo-normed completion, i.e. the collection of all Cauchy sequences from \(F'_0(A, B)\). Define a mapping

\[
\phi: F'_0(A, B) \to A \otimes B \otimes C
\]
as follows:

\[
\phi(\Sigma c_i(a_i, b_i)) = \Sigma a_i \otimes b_i \otimes c_i.
\]
It is immediate that \(\phi\) is a linear, homogeneous, multiplicative isometry, and that its range is dense. Thus \(\phi\) can be extended to an isometric homomorphism on \(F'_0(A, B)\) onto \(A \otimes B \otimes C\). Note that \(||XY|| \leq ||X|| \cdot ||Y||\) for all \(X, Y \in F'_0(A, B)\), since \(A \otimes B \otimes C\) is a Banach algebra. Thus the operations defined on \(F'_0(A, B)\) can be extended to \(F'_0(A, B)\), as usual.

Let \(I_1, I_2, \text{ and } I\) denote the closures, in \(F'_0(A, B)\), of \(I'_1, I'_2, \text{ and } I'\), respectively. It is obvious from Proposition 1 that \(I_1 = \{X \in F'_0(A, B): ||X|| = 0\}\), i.e. \(I_1\) is the closure of \((0)\). Thus \(I_1\) is a subset of every closed subspace of \(F'_0(A, B)\), which means, in particular, that \(I = I_2\). In other words, \(I\) can be described quite simply as the closed ideal of \(F'_0(A, B)\) generated by the collection of all elements of the forms (6) and (7).

**DEFINITION.** \(A \otimes_0 B\), the tensor product of \(A\) and \(B\), over \(C\), is the quotient algebra \(F'_0(A, B)/I\).

\(A \otimes_0 B\) is a normed space (as is always the case when a pseudo-normed space is factored by a closed subspace). We proceed to identify it with an ideal in \(A \otimes B \otimes C\). Let \(D = \phi(I)\) and define a map \(\gamma: A \otimes_0 B \to (A \otimes B \otimes C)/D\) by the formula \(\gamma(X + I) = \phi(X) + D\). It is clear that \(\gamma\) is linear, and since \(\gamma(I) = \phi(0) + D = D\), \(\gamma\) is well defined; it is multiplicative since \(\phi\) is multiplicative. Finally, \(\gamma\) is an isometry. For if \(T = X + I \in A \otimes_0 B\), then

\[
||\gamma T|| = ||\phi X + D|| = \inf \{||\phi X + Z||: Z \in D\}
= \inf \{||\phi X + \phi Y||: Y \in I\}
= \inf \{||X + Y||: Y \in I\} = ||T||,
\]
since $\varphi$ is an isometric homomorphism.

Since $D$ is a closed ideal in the $H^*$-algebra $A \otimes B \otimes C$, $(A \otimes B \otimes C)/D$ is isomorphic and isometric with the closed ideal $D^\perp$, which we shall denote by $E$. We summarize the foregoing information in the next theorem.

**Theorem.** There is an isometric isomorphism from $A \otimes_0 B$ into $A \otimes B \otimes C$; its range is the closed ideal $E$ which is the orthogonal complement of the closed ideal $D$ generated by all elements of the forms

(i) $a \otimes b \otimes c_i - ac_i \otimes b \otimes c_i$,

(ii) $a \otimes b \otimes c_i - a \otimes c_i b \otimes c_i$.

Consequently, $A \otimes_0 B$ is an $H^*$-algebra; its minimal closed ideals can be identified with those minimal closed ideals $A_\alpha \otimes B_\beta \otimes C_\gamma$ of $A \otimes B \otimes C$ that are orthogonal to $D$.

**Corollary.** If $A, B,$ and $C$ are strongly semi-simple, then $A \otimes_0 B$ is strongly semi-simple.

The following proposition provides means by which it is easy to construct examples for which the converse to the above corollary is false.

**Proposition 2.** If $A_\alpha \otimes B_\beta \otimes C_\gamma$ is a minimal closed ideal in $E$, then $C_\gamma$ is of dimension one.

**Proof.** Choose a canonical basis $\{a_{ij} \otimes b_{kl} \otimes c_{mn}\}$ for $A_\alpha \otimes B_\beta \otimes C_\gamma$ (see [2]). Since $a_{ij} \otimes b_{kl} \otimes c_{mn} \in E$, it must be orthogonal to

$$a_{ij} \otimes b_{kl} \otimes c_{mp} c_{pn} - a_{ij} c_{pn} \otimes b_{kl} \otimes c_{mp}.$$

If the dimension of $C_\gamma$ were greater than one, then it would be possible to choose $n \neq p$, and we would have

$$0 = (a_{ij} \otimes b_{kl} \otimes c_{mn}, a_{ij} \otimes b_{kl} \otimes c_{mn} - a_{ij} c_{pn} \otimes b_{kl} \otimes c_{mp})$$

$$= \|a_{ij}\|^2 \|b_{kl}\|^2 \|c_{mn}\|^2.$$

since $(c_{mn}, c_{mp}) = 0$. This, of course, is a contradiction.

**Corollary.** If $C$ has no one-dimensional minimal ideals, then $A \otimes_0 B = (0)$.

2. **Examples.** Perhaps the easiest method of obtaining examples of $H^*$-algebras $A, B,$ and $C$ related as above is to let $A, B,$ and $C$ be
closed ideals in some $H^*$-algebra $\mathcal{A}$. The structure of $A \otimes_o B$, under such circumstances, is described in the next proposition.

**Proposition 3.** Suppose that $A$, $B$, and $C$ are closed ideals in an $H^*$-algebra $\mathcal{A}$. If $A$ and $B$ are viewed as $C$-modules with ordinary multiplication in $\mathcal{A}$ as the module action, then $A \otimes_o B$ is isomorphic with the direct sum of all the one-dimensional minimal ideals in $A \cap B \cap C$. The isomorphism is an isometry if and only if the identity of each one-dimensional minimal ideal in $A \cap B \cap C$ has norm one.

**Proof.** Choose a canonical basis $\{u^a_{pq}\}$ for $\mathcal{A}$. Then $\{a_{ij}\} = A = \cap \{u^a_{pq}\}$, $\{b_{jk}^a\} = B = \cap \{u^a_{pq}\}$, and $\{c_{mn}^a\} = C = \cap \{u^a_{pq}\}$ are canonical bases for $A$, $B$, and $C$, respectively and $\{a_{ij}^a \otimes b_{jk}^a \otimes c_{mn}^a\}$ is a canonical basis for $A \otimes B \otimes C$. If $a_{ij}^a \otimes b_{jk}^a \otimes c_{mn}^a \in E$, then, by Proposition 2, $c_{mn}^a = e^a$ is the identity of a one-dimensional minimal ideal. If $\alpha \neq \gamma$, then

$$a_{ij}^a \otimes b_{jk}^a \otimes e^a e^a = a_{ij}^a \otimes b_{jk}^a \otimes e^a = a_{ij}^a \otimes b_{jk}^a \otimes e^a \in D.$$ 

Similarly, if $\beta \neq \gamma$, then $a_{ij}^a \otimes b_{jk}^a \otimes e^a \in D$. Thus if an element of a canonical basis is to be in $E$ it must be of the form $e^a \otimes e^a \otimes e^a$. Relatively straightforward computations show that each such basis element is orthogonal to $D$, and the proof is completed.

Suppose now that $G$, $H$, and $K$ are compact groups, and that $\theta: K \rightarrow G$ and $\varphi: K \rightarrow H$ are continuous homomorphisms. Then $\theta(K)$ and $\varphi(K)$ are closed subgroups of $G$ and $H$, respectively, $L^2(G)$ and $L^2(H)$ become modules over $L^2(K)$, with the module action defined by:

$$g \ast k(x) = \int_K g(x(\theta z)^{-1}) k(z) dz,$$

$$k \ast h(y) = \int_K k(z) h((\varphi z)^{-1} y) dz,$$

for all $g \in L^2(G)$, $h \in L^2(H)$, $k \in L^2(K)$, $x \in G$, and $y \in H$ (all integrations are with respect to normalized Haar measures). If we let $A = L^2(G)$, $B = L^2(H)$, $C = L^2(K)$, then $A \otimes_o B$ is a well-defined $H^*$-algebra. As was remarked in [2], $A \otimes_o B \otimes C$ can be identified with $L^2(G \times H \times K)$, and so, by the Theorem of §1, $A \otimes_o B$ can be identified with a closed ideal $J$ in $L^2(G \times H \times K)$. At one extreme, suppose $\theta$ and $\varphi$ map $K$ onto the identities of $G$ and $H$, respectively. It is not difficult to see that in this case $A \otimes_o B$ can be identified with $L^2(G \times H)$.

At what might be considered another extreme, suppose that $G$ and $H$ are closed subgroups of some compact group, that $K$ is a closed subgroup of $G \cap H$, and that $\theta$ and $\varphi$ are the inclusion maps. Define an equivalence relation on $G \times H \times K$ as follows: $(x, y, z) \sim (u, v, w)$
if and only if $F(x, y, z) = F(u, v, w)$ for all $F \in J$. Then $M = \{(x, y, z): (x, y, z) \sim (e, e, e)\}$ is a closed normal subgroup of $G \times H \times K$, and its cosets are the equivalence classes of $\sim$. All functions $F \in J$ are thus constant on the cosets of $M$, providing a mapping $\psi$ from $J$ to $L^2((G \times H \times K)/M)$. The map $\psi$ is an isometric isomorphism and its image is an ideal. On the basis of the Tannaka Duality Theorem (see [4], p. 439) it seems reasonable to conjecture that $\psi$ is surjective, so that $A \otimes_\sigma B$ can be identified with $L^2((G \times H \times K)/M)$. The conjecture has not been settled in general, but let us consider the very special case where $G = H = K$. Then, by Proposition 3, $A \otimes_\sigma B$ can be identified with the direct sum of all one-dimensional minimal ideals in $L^2(G)$, which in turn is isomorphic and isometric with $L^2(G/N)$, where $N$ is the closure of the commutator subgroup of $G$. Since $G/N$ and $(G \times G \times G)/M$ are isomorphic via the mapping $xN \rightarrow (x, e, e)M$, the conjecture is verified in this special case.

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Journal of Mathematics, 103 Highland Boulevard, Berkeley 8, California.

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), No. 6,
2-chome, Fujimi-cho, Chiyoda-ku, Tokyo, Japan.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

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