Pacific Journal of Mathematics

A GENERALISATION OF W^*-ALGEBRAS

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Vol. 15, No. 3 November 1965

PACIFIC JOURNAL OF MATHEMATICS
Vol. 15, No. 3, 1965

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Using the theory of double centralisers due to B. E. Johnson, we define a QW^*-algebra as being a B^*-algebra, A, such that the algebra of double centralisers of each closed *-subalgebra B is contained in a suitable related closed *-subalgebra B_{00}.

After obtaining explicit descriptions of the algebras of double centralisers of commutative and noncommutative B^*-algebras, we prove that in the general noncommutative case a W^*-algebra is a QW^*-algebra, and a QW^*-algebra is an AW^*-algebra, while in the commutative case the QW^* and AW^* conditions are equivalent.

We prove that if A is QW^* then so are its centre, any maximal commutative *-subalgebra, and any subalgebra of the form eAe for e a projection in A.

We shall be concerned with centraliser theory, for the basic details of which reference may be made to Johnson [2], [3].

I should like to take this opportunity of expressing my sincere gratitude to Dr. J. H. Williamson, my research supervisor, for his advice and encouragement.

DEFINITION 1. A *left centraliser* $\mathscr{T}$ of the algebra A is a linear map $\mathscr{T}$ of A into itself such that $\mathscr{T}(xy) = (\mathscr{T}x)y$ for all $x, y \in A$.

A *right centraliser* $\mathscr{S}$ is a linear operator on A such that $\mathscr{S}(xy) = x(\mathscr{S}y)$ for all $x, y \in A$.

A *double centraliser* (the concept is due to Johnson [2]) is a pair of linear operators $(\mathscr{T}, \mathscr{S})$ such that $x\cdot(\mathscr{T}y) = (\mathscr{S}x)\cdot y$ for all $x, y \in A$.

The set of all double centralisers on A is denoted by $Q(A)$.

We will assume throughout that $xA = 0$ or $Ax = 0$ only holds for $x = 0$. We note that this holds for B^*-algebras since $xA = 0 \Rightarrow xx^* = 0 \Rightarrow x = 0$, and $Ax = 0 \Rightarrow x^*x = 0 \Rightarrow x = 0$.

It is not difficult to see that defining $(\mathscr{T}_x, \mathscr{S}_x) \in Q(A)$ for $x \in A$ by $\mathscr{T}_x(y) = xy$, $\mathscr{S}_x(y) = yx$, and algebraic operations in $Q(A)$ by

$$\lambda_1(\mathscr{T}_1, \mathscr{S}_1) + \lambda_2(\mathscr{T}_2, \mathscr{S}_2) = (\lambda_1\mathscr{T}_1 + \lambda_2\mathscr{T}_2, \lambda_1\mathscr{S}_1 + \lambda_2\mathscr{S}_2)$$
$$(\mathscr{T}_1, \mathscr{S}_1)\cdot(\mathscr{T}_2, \mathscr{S}_2) = (\mathscr{T}_1\mathscr{T}_2, \mathscr{S}_2\mathscr{S}_1)$$

Received August 27, 1964.

we have A embedded as a subalgebra of $Q(A)$, which is an algebra with identity. $A = Q(A)$ if and only if A has an identity. Also, for $(\mathscr{T}, \mathscr{S}) \in Q(A)$, $\mathscr{T}$ is a left centraliser and $\mathscr{S}$ is a right centraliser, and either of $\mathscr{T}, \mathscr{S}$ determines the other uniquely.

If A is commutative, the notions of right, left and double centraliser coincide, and for $(\mathscr{T}, \mathscr{S}) \in Q(A)$ we have $\mathscr{T} = \mathscr{S}$.

PROPOSITION 1. If A is a Banach algebra then all double centralisers are continuous.

Proof. Suppose $(\mathscr{T}, \mathscr{S}) \in Q(A)$ and say $x_n \to x$, $\mathscr{T}x_n \to y$. Then

$$\begin{aligned} z\cdot(\mathscr{T}x_n) &= (\mathscr{S}z)\cdot x_n \\ \to z\cdot y \qquad & \to (\mathscr{S}z)\cdot x = z\cdot(\mathscr{T}x)\ . \end{aligned}$$

So $z(y - \mathscr{T}x) = 0$ for all $z \in A$ i.e. $A(y - \mathscr{T}x) = 0$ and so $y = \mathscr{T}x$. Therefore $\mathscr{T}$ is a closed operator on the Banach space A, hence by the Closed Graph Theorem, $\mathscr{T}$ is continuous. Likewise so is $\mathscr{S}$.

We are particularly interested in C^*-algebras and in both the commutative and noncommutative cases explicit descriptions of their centraliser algebras may be given.

By the Gelfand Representation Theorem a commutative B^*-algebra is isometrically isomorphic to the space $C_0(Z)$ of all continuous functions vanishing at infinity on its carrier space, Z, a locally compact Hausdorff space.

PROPOSITION 2. For a locally compact Hausdorff space Z we have $QC_0(Z) = C(Z)$, the space of all bounded continuous functions on Z.

Proof. Certainly any $h \in C(Z)$ defines an element $\mathscr{T}_h$ of $QC_0(Z)$ by $\mathscr{T}_h f = h\cdot f$ for $f \in C_0(Z)$, for

$$f \in C_0(Z),\ h \in C(Z) \Rightarrow hf \in C_0(Z)$$

and

$$h(fg) = (hf)g\ .$$

We clearly have $\|\mathscr{T}_h\| \leq \|h\|_\infty$. Suppose conversely we are given a centraliser $\mathscr{T}$ on $C_0(Z)$. Then for $f, g \in C_0(Z)$ we have

$$(\mathscr{T}f)g = \mathscr{T}(fg) = \mathscr{T}(gf) = (\mathscr{T}g)f$$

so for $z \in Z$ taking any $f \in C_0(Z)$ such that $f(z) \neq 0$ and defining $h(z) = \mathscr{T}f(z)/f(z)$ we have $h(z)$ well defined independently of f.

Being a quotient of continuous functions, h is continuous at z, for each $z \in Z$. And for any $g \in C_0(Z)$,

$$\mathscr{T}g(z) = \frac{\mathscr{T}f(z)}{f(z)} g(z) = h(z)g(z)$$

so

$$\mathscr{T}g = hg = \mathscr{T}_h g .$$

Now by Proposition 1, $\mathscr{T}$ is a bounded operator, so taking $f \in C_0(Z)$ such that $0 \leqq f \leqq 1$ and $f(z) = 1$ we have $h(z) = \mathscr{T}f(z)$ and $|\mathscr{T}f(z)| \leqq \|\mathscr{T}f\|_\infty \leqq \|\mathscr{T}\| \|f\|_\infty = \|\mathscr{T}\|$ so $\|h\|_\infty \leqq \|\mathscr{T}\|$ and we see $h \in C(Z)$.

Hence all $\mathscr{T}$ are of the form $\mathscr{T}_h$ and $\|\mathscr{T}\| = \|h\|_\infty$. So $QC_0(Z) = C(Z)$.

PROPOSITION 3. *If A is a C^*-algebra over H, principal identity E, then $Q(A)$ is isometrically isomorphic to*

$$\{T \in \mathscr{B}(H) \colon T = ETE,\ TA \cup AT \subset A\} .$$

Proof. Recall that the principal identity of a C^*-algebra A is defined to be the orthogonal projection of H onto $M = H \ominus N$ where $N = \{\xi \in H \colon A\xi = 0\}$. Equivalently M is the closure of

$$M_1 = \{T\xi : T \in A, \xi \in H\} .$$

Suppose given $(\mathscr{T}, \mathscr{S}) \in Q(A)$, then $\mathscr{T}$ is a bounded left centraliser.

Since A is a C^*-algebra it has an approximate identity (Segal [6]), $(Z_\lambda)_{\lambda \in \Lambda}$ say, so $\|Z_\lambda\| = 1$, and $SZ_\lambda \to S$, $Z_\lambda S \to S$ for each $S \in A$. So $\mathscr{T}(Z_\lambda S) \to \mathscr{T}(S)$. But $\mathscr{T}(Z_\lambda S) = \mathscr{T}(Z_\lambda)S = T_\lambda S$ where $T_\lambda = \mathscr{T}(Z_\lambda)$, so $\mathscr{T}(S) = \lim_\lambda T_\lambda S$ and $\|T_\lambda\| \leqq \|\mathscr{T}\| \|Z_\lambda\| = \|\mathscr{T}\|$. For $\xi \in M_1$, $\xi = S\eta$ some $S \in A, \eta \in H$ so $\mathscr{T}(S)\eta = \lim_\lambda T_\lambda S\eta = \lim_\lambda T_\lambda \xi$. Define $T\xi = \lim_\lambda T_\lambda \xi = \mathscr{T}(S)\eta$, then T maps M_1 into M and $\|T\xi\| \leqq \|\mathscr{T}\| \|\xi\|$ so $\|T\| \leqq \|\mathscr{T}\|$.

So extend T to a map of M into M and define $T = 0$ on $H \ominus M$, so we have $T = ETE$ and $\mathscr{T}(S)\eta = \lim_\lambda T_\lambda S\eta = TS\eta$. Therefore $\mathscr{T}(S) = TS$ and $\|\mathscr{T}\| \leqq \|T\|$. So $\|\mathscr{T}\| = \|T\|$.

We have

$$\begin{aligned} (\mathscr{S}S)Z_\lambda &= S(\mathscr{T}Z_\lambda) = STZ_\lambda \\ \to \mathscr{S}S & \qquad\qquad\quad \to ST . \end{aligned}$$

So $\mathscr{S}(S) = ST$ for all $S \in A$, and as for $\mathscr{T}$, $\|\mathscr{S}\| = \|T\|$. Since $TS, ST \in A$ for all $S \in A$ we have $TA \cup AT \subset A$. Conversely given any

T such that $T = ETE$ and $TA \cup AT \subset A$, the maps $S \rightarrow TS$, $S \rightarrow ST$ both map A into itself and define a double centraliser of A. Hence result.

Denote the set $\{T \in \mathscr{B}(H): T = ETE, TA \cup AT \subset A\}$ by $I(A)$, the idealiser of A in $E \cdot \mathscr{B}(H) \cdot E$.

Now let us suppose that B is a closed *-subalgebra of the B^*-algebra A. We define $B_0 = \{x \in A: Bx = xB = 0\}$ and $B_{00} = (B_0)_0$. Then B_{00} is a closed *-subalgebra of A containing B. Should it be necessary to make explicit mention of the algebra A we will write $B_0(A)$, etc.

Suppose two elements x_1, x_2 of B_{00} give the same double centraliser on B, so $x_1 y = x_2 y$ and $y x_1 = y x_2$ for all $y \in B$. Then $(x_1 - x_2)B = B(x_1 - x_2) = 0$ so $x_1 - x_2 \in B_0$. But $(x_1 - x_2)^* \in B_{00}$ so we have

$$(x_1 - x_2)^*(x_1 - x_2) = 0$$

and hence $x_1 - x_2 = 0$. So $x_1 = x_2$.

DEFINITION 2. A B^*-algebra A is said to be a *QW^*-algebra* if for each closed *-subalgebra B of A all double centralisers of B are given by elements of B_{00}. We see that for each double centraliser the corresponding element of B_{00} is unique, and so we may briefly say that A is QW^* if and only if $Q(B) \subset B_{00}$ for all closed *-subalgebras B.

We recall the definition of an AW^*-algebra (Kaplansky [4]).

DEFINITION 3. A B^*-algebra A is said to be an *AW^*-algebra* if

(i) every set of orthogonal projections in A has a least upper bound in A.

(ii) every maximal commutative *-subalgebra B of A is generated by its projections.

We also recall that a W^*-algebra is a C^*-algebra, over H say, which is closed in the weak operator topology defined by seminorms $\| T \|_{\xi,\eta} = |\langle T\xi, \eta \rangle|$ for $\xi, \eta \in H$. Denote weak closure by ${}^{-w}$.

PROPOSITION 4. For A a C^*-algebra, $I(A) \subset A^{-w}$.

Proof. By von Neumann's Double Commutant Theorem, $A^{-w} = \{T \in \mathscr{B}(H): T = ETE, T \in A''\}$ where as usual A'' denotes the double commutant of A.

Suppose $T \in I(A), S \in A', R \in A$, then certainly $T = ETE$ and $(ST - TS)R = S(TR) - T(SR) = TRS - TRS = 0$. So $(ST - TS)E = 0$

and therefore $ST = TSE$. Since $T^* \in I(A)$, $S^* \in A'$ we have $S^*T^* = T^*S^*E$ so $TS = EST$. Thus $TS = EST = ETSE = TSE = ST$ and so $T \in A''$. Hence $I(A) \subset A''$.

THEOREM 1. *For a B^*-algebra A, $W^* \Rightarrow QW^* \Rightarrow AW^*$.*

If A is commutative, carrier space Z, then A is $QW^* \Leftrightarrow A$ is $AW^* \Leftrightarrow Z$ is extremally disconnected.

Proof. If A is a W^*-algebra and B is a closed *-subalgebra of A with principal identity E, then since A is W^* we note $E \in A$, and by Proposition 4, $I(B) \subset B^{-w} \subset A^{-w} = A$. Also we easily see that $B_0 = (I - E)A(I - E)$ so $B_{00} = EAE$. Thus $Q(B) \subset B_{00}$ by Proposition 3 and hence A is QW^*.

Suppose now that A is a commutative B^*-algebra, carrier space Z, so by the Gelfand Representation Theorem A is isometrically isomorphic to $C_0(Z)$.

It is well known that A is AW^* if and only if Z is an extremally disconnected compact Hausdorff space.

Suppose A is QW^*, then taking $B = A$ we see that A has an identity, so Z is compact Hausdorff.

Let U be any open dense subset of Z.

Then taking $B = \{f \in C(Z): f = 0 \text{ on } Z \backslash U\} = C_0(U)$, B is a closed *-ideal in A so $Q(B) = C(U) \subset A$.

So any continuous function f on U is extendible to Z. Therefore Z is extremally disconnected (see Gillman and Jerison [1], p. 96).

Now suppose that Z is an extremally disconnected compact Hausdorff space, and suppose B is a closed *-subalgebra of $A = C(Z)$.

Let $(Z_\lambda)_{\lambda \in \Lambda}$ be the sets of constancy of B (see Rickart [5], Ch. 3, § 2), then these form an upper semicontinuous decomposition of Z, so the space of these sets, Z' say, is a compact Hausdorff space and B may be considered as a space of continuous functions on Z'.

B is self-adjoint and separates points of Z', so by the Stone-Weierstrass Theorem, *either* B consists of all continuous functions on Z', in which case B has an identity so $Q(B) = B$, *or* B consists of all continuous functions on Z' vanishing at some point Z_0 of Z'. So $Q(B) =$ all continuous functions on $Z' \backslash \{Z_0\}$.

Given any function on $Z' \backslash \{Z_0\}$ it corresponds to a function f on $Z \backslash Z_0 = Y$ say.

Y is open, so $\bar{Y}$ is a compact open subset of Z, and therefore $\bar{Y}$ is extremally disconnected (Gillman and Jerison [1], p. 23). So there exists an extension of f to $\bar{Y}$, and defining $f = 0$ on $Z \backslash \bar{Y}$ we extend f to a continuons function on Z.

Now since

$$B_0 = \{g \in C(Z) \colon g = 0 \text{ on } Y\}$$
$$= \{g \in C(Z) \colon g = 0 \text{ on } \bar{Y}\}$$

and

$$B_{00} = \{g \in C(Z) \colon g = 0 \text{ on } Z \backslash \bar{Y}\}$$

we therefore have $Q(B) \subset B_{00}$.

So A is QW^* and we have proved our theorem for A commutative.

Now let us return to the general case and suppose A to be QW^*.

(i) Suppose (e_α) is a set of orthogonal projections in A (so $\alpha \neq \beta \Rightarrow e_\alpha e_\beta = 0$).

Let B = closed *-subalgebra of A generated by the e_α's.
= closed linear hull of the e_α's.

Now there exists a unique $e \in B_{00}$ such that $ex = xe = x$ for all $x \in B$ and $e^*, e^2 \in B_{00}$ with

$$e^*x = xe^* = x$$
$$e^2x = xe^2 = x \quad \text{for all } x \in B\,.$$

So $e^2 = e^* = e$ and thus e is a projection.

Also $ee_\alpha = e_\alpha e = e_\alpha$ all α, so $e \geqq e_\alpha$ all α.

Now suppose f is a projection in A such that $f \geqq$ all e_α. Then $fe_\alpha = e_\alpha f = e_\alpha$ all α, so since all $x \in B$ are limits of linear combinations of the e_α's, we have $fx = xf = x$ for all $x \in B$.

Now

$$y \in B_0 \Rightarrow yfx = yx = 0$$
$$xyf = 0 \qquad \text{all } x \in B \Rightarrow yf \in B_0$$

so for all $y \in B_0$,

$$fey = f0 = 0$$
$$yfe = 0 \qquad \text{thus } fe \in B_{00}\,.$$

But

$$fex = fx = x$$
$$xfe = xe = x$$

all $x \in B$, so since e is unique, $e = fe$.

So $ef = fe = e$ and $e \leqq f$.

Hence e is a least upper bound in A for the e_α's.

(ii) Suppose B is a maximal commutative *-subalgebra of A. Then by Proposition 5 below, B is QW^*, thus since B is commutative it follows from the above result that B is AW^*, and is a maximal commutative *-subalgebra of itself and therefore generated by its projections.

Thus we have both conditions for A to be AW^*.

The obvious question of interest arising from this theorem is whether or not the QW^* and the AW^* conditions are equivalent in the noncommutative case, but so far we have not been able to settle this problem.

We now prove some results for QW^*-algebras similar to those holding for W^*- and AW^*-algebras. We are indebted to the referee for pointing out case (iv) of Proposition 5 as generalising cases (i) and (ii).

PROPOSITION 5. If A is a QW^*-algebra then so also are the following closed $*$-subalgebras of A:

(i) the centre Z of A,

(ii) any maximal commutative $*$-subalgebra of A,

(iii) the subalgebra eAe for any projection e in A,

(iv) S'' for any subset S of A such that $S^* = S$, where S'' is the double commutant of S in A.

Proof. We first prove (iv) from which (i) and (ii) follow immediately.

(iv) Suppose B is a closed $*$-subalgebra of S''.

Since A is QW^* any double centraliser on B is given by some $x \in B_{00}(A)$.

To prove $x \in B_{00}(S'')$, since $B_0(S'') \subset B_0(A)$, we need only show $x \in S''$.

Let $y \in S'$, $z \in B \subset S''$, then

$$(xy - yx)z = x(yz) - y(xz) = xzy - xzy = 0$$
$$z(xy - yx) = (zx)y - (zy)x = yzx - yzx = 0$$

so $xy - yx \in B_0(A)$.

Now

$$u \in B_0(A) \Rightarrow yuz = 0$$
$$zyu = yzu = 0 \quad \text{all } z \in B \Rightarrow yu \in B_0(A)\ ,$$

and likewise $u \in B_0(A) \Rightarrow uy \in B_0(A)$.

Therefore since $x \in B_{00}(A)$, $xyu = 0$ and $uxy = 0$ for all $u \in B_0(A)$, so $xy \in B_{00}(A)$, and likewise $yx \in B_{00}(A)$. So $(xy - yx)^* \in B_{00}(A)$ and hence $xy - yx = 0$ for all $y \in S'$. Thus $x \in S''$ and the result follows.

(i) We have $Z = A'$, $Z' = A$ so $Z = Z''$, and clearly $Z = Z^*$, so the result follows from (iv).

(ii) Suppose C is a maximal commutative $*$-subalgebra of A, then by maximality C is closed and $C' = C$, so $C = C''$ and the result follows from (iv).

(iii) Let B be a closed $*$-subalgebra of eAe, then since A is QW^*

any double centraliser on B is given by some $x \in B_{00}(A)$. Since $B \subset eAe$ we have $y \in B_0(A) \Rightarrow ey, ye \in B_0(A)$ and $x \in B_{00}(A) \Rightarrow exe \in B_{00}(A)$.

But for $z \in A$ we have

$$zexe = (zx)e = zx$$
$$exez = e(xz) = xz$$

so by the uniqueness of x in $B_{00}(A)$ we have $x = exe$.

Thus $x \in eAe$ and so $x \in B_{00}(eAe)$. Hence eAe is QW^*.

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The *Pacific Journal of Mathematics* is published quarterly, in March, June, September, and December. Effective with Volume 13 the price per volume (4 numbers) is $18.00; single issues, $5.00. Special price for current issues to individual faculty members of supporting institutions and to individual members of the American Mathematical Society: $8.00 per volume; single issues $2.50. Back numbers are available.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley 8, California.

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), No. 6, 2-chome, Fujimi-cho, Chiyoda-ku, Tokyo, Japan.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies.

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