

Pacific Journal of Mathematics

MAXIMAL CONVEX FILTERS IN A LOCALLY CONVEX SPACE

FRANK J. WAGNER

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Let $E[\mathcal{S}]$ be a locally convex space, \mathfrak{B} a saturated covering of E by bounded sets, and E' the topological dual of $E[\mathcal{S}]$. Let $\mathcal{T}_{\mathfrak{B}}$ be the topology on E' of uniform convergence on sets of \mathfrak{B} and E'' the topological dual of $E'[\mathcal{T}_{\mathfrak{B}}]$. We assume E'' has the natural topology \mathcal{T}_n —that of uniform convergence on the equicontinuous sets of E' .

This article includes the following: (1) an intrinsic characterization for a bounded convex set B of E of the closure \bar{B} of B in E'' ; (2) an intrinsic characterization of the closure \bar{E} of E in E'' ; and (3) necessary and sufficient conditions that \bar{E} be E'' .

The spaces β . Let \mathfrak{M} be the class of all closed convex neighborhoods¹ of 0 in $E[\mathcal{S}]$, and $B \in \mathfrak{B}$. A filter \mathfrak{F} on B is called a *convex filter* if, for every $F \in \mathfrak{F}$, there exist $M, N \in \mathfrak{M}$ and $\chi \in E$ such that $\dot{M} \supset N$, $F \supset (M + \chi) \cap B$, and $(N + \chi) \cap B \in \mathfrak{F}$. Clearly if \mathfrak{F} and \mathfrak{G} are two convex filters on B , such that every set of \mathfrak{F} meets every set of \mathfrak{G} , then the least upper bound filter of \mathfrak{F} and \mathfrak{G} on B is also convex. Furthermore:

LEMMA 1. For $M, N \in \mathfrak{M}$, if $\dot{M} \supset N$, then there exists $K \in \mathfrak{M}$ such that $\dot{M} \supset K \supset \dot{K} \supset N$.

Proof. If p and q are the distance functions of M and N , then $1/2(p + q)$ is the distance function of such a K .

THEOREM 1. A convex filter \mathfrak{F} on B is a maximal convex filter on B if and only if, for every two closed convex bodies K and L of E such that $\dot{K} \supset L$, either $K \cap B \in \mathfrak{F}$ or $B \setminus L \in \mathfrak{F}$.

Proof. Assume \mathfrak{F} is maximal and let K and L be as above, and let $B \setminus L \notin \mathfrak{F}$. Let $x \in \dot{L}$ and define a sequence $\{M_n\}$ in \mathfrak{M} so that

$$\dot{K} - x \supset M_1 \supset \dot{M}_1 \supset L - x \quad \text{and} \quad \dot{M}_n \supset M_{n+1} \supset \dot{M}_{n+1} \supset L - x \quad (n \geq 1).$$

Then the filter \mathfrak{G} on B with base $\{(M_n + x) \cap B \mid n = 1, 2, 3, \dots\}$ is

Received July 8, 1964 and in revised form January 11, 1965. Supported by National Science Foundation grant NSF G-24865.

¹ The notation and definitions are principally those of Gottfried Köthe, *Topologische lineare Räume I*, Springer-Verlag, Berlin, 1960.

convex and $K \cap B \in \mathfrak{G} \subset \mathfrak{F}$.

Conversely let \mathfrak{F} and \mathfrak{G} be two convex filters on B such that \mathfrak{F} is strictly weaker than \mathfrak{G} . Let $G \in \mathfrak{G}$, $M, N \in \mathfrak{M}$, and $x \in E$ such that $G \notin \mathfrak{F}$, $\overset{\circ}{M} \supset N$, $G \supset (M + x) \cap B$, and $(N + x) \cap B \in \mathfrak{G}$. Then neither $(M + x) \cap B$ nor $B \setminus (L + x) \in \mathfrak{F}$.

REMARKS 1. For every $x \in B$, $\mathfrak{B}_B(x) = \{V \cap B \subset B \mid V \text{ a neighborhood of } x \text{ in } E\}$ is a maximal convex filter on B .

2. For a maximal convex filter \mathfrak{F} on B , there is $x \in B$ such that $\mathfrak{F} = \mathfrak{B}_B(x)$ if and only if \mathfrak{F} has nonempty intersection.

LEMMA 2. Every maximal convex filter on B is a weak Cauchy filter.

Proof. Let \mathfrak{F} be a maximal convex filter on B ,

$$u \in E', M = \{x \in E \mid |ux| \leq 1/2\} \text{ and } N = \{x \in E \mid |ux| \leq 1/4\}.$$

Then $M, N \in \mathfrak{M}$ and $\overset{\circ}{M} \supset N$. Since B is weakly precompact, there exist $x_1, x_2, \dots, x_n \in E$ such that $\bigcup_{i=1}^n (N + x_i) \supset B$, and so $(M + x_i) \cap B \in \mathfrak{F}$ for some $1 \leq i \leq n$. For $x, y \in (M + x_i) \cap B$, we have $|ux - uy| \leq 1$.

For a maximal convex filter \mathfrak{F} on B and $u \in E'$, let $\mathfrak{F}(u)$ denote the limit of the restriction of u to B according to the filter \mathfrak{F} .

LEMMA 3. For every maximal convex filter \mathfrak{F} on B , the mapping $u \rightarrow \mathfrak{F}(u)$ on E' is linear and $\mathcal{T}_{\mathfrak{B}}$ continuous.

Proof. Linearity is easily proved. Also let V be the polar set of the absolutely convex hull of $2B$, $u \in V$, and $F \in \mathfrak{F}$ such that $|ux - \mathfrak{F}(u)| \leq 1/2$ for every $x \in F$. Then, for such an x , we have $|\mathfrak{F}(u)| \leq |\mathfrak{F}(u) - ux| + |ux| \leq 1$.

We shall denote by $\beta = \beta_B$ the set of all maximal convex filters on B . By Lemma 3 there is a mapping π_B from β_B into E'' such that $\pi_B(\mathfrak{F})(u) = \mathfrak{F}(u)$ for every $\mathfrak{F} \in \beta_B$ and $u \in E'$.

THEOREM 2. If either \mathcal{T} is the weak topology or B is convex, then π_B is a one-to-one mapping of β_B onto the \mathcal{T}_n -closure \bar{B} of B in E'' .

Proof. For $\mathfrak{F} \in \beta_B$, $\pi_B(\mathfrak{F})$ is in the weak closure of B in E'' . For

given $u_1, \dots, u_n \in E'$ and $\varepsilon > 0$, let $F_1, \dots, F_n \in \mathfrak{F}$ such that $|u_i x - \mathfrak{F}(u_i)| \leq \varepsilon$ ($1 \leq i \leq n$) and $x \in \bigcap_{i=1}^n F_i$. Then $|\mathfrak{F}(u_i) - u_i x| \leq \varepsilon$, ($1 \leq i \leq n$).

Also, if B is convex, $\pi_B(\mathfrak{F})$ is in the \mathcal{S}_n -closure \bar{B} of B in E'' . Suppose the contrary. Then there is a continuous real linear functional w on E'' and a real number r such that $w(\pi_B(\mathfrak{F})) < r$ and $wz > r$ for every $z \in \bar{B}$.

Assume first that E is a real vector space. Let u be the restriction of w to E' , so $u \in E$. Let $F \in \mathfrak{F}$ such that $|ux - \mathfrak{F}(u)| < r - w(\pi_B(\mathfrak{F}))$ for every $x \in F$. Then, for such an x , we have $wx = ux - \mathfrak{F}(u) + \mathfrak{F}(u) < r$. But $x \in B$.

Now let E be a complex vector space. Then there is a complex linear functional v on E'' such that $w = \Re v$. Let u be the restriction of v to E and $F \in \mathfrak{F}$ such that $|ux - \mathfrak{F}(u)| \leq r - w(\pi_B(\mathfrak{F}))$ for every $x \in F$. Then for such an x we have $wx = \Re(vx) = \Re(ux - \mathfrak{F}(u) + \mathfrak{F}(u)) < r$. Again, we have a contradiction.

Thus $\pi_B(\beta_B) \subset \bar{B}$ if \mathcal{S} is the weak topology or B is convex.

On the other hand, if $z \in \bar{B}$, then :

$$\mathfrak{B}_B(z) = \{V \cap B \subset B \mid V \text{ a neighborhood of } z \text{ in } E''[\mathcal{S}_n]\} \in \beta_B$$

and $\pi_B(\mathfrak{B}_B(z)) = z$. Let V be a neighborhood of z in $E''[\mathcal{S}_n]$, and let U and W be closed convex neighborhoods of 0 in $E''[\mathcal{S}_n]$ such that $\overset{\circ}{U} \supset W$ and $U + U \subset V - z$. Let $\chi \in (U + z) \cap (-W + z) \cap B$, $M = U \cap E$, and $N = V \cap E$. Then $M, N \in \mathfrak{M}$ and $\overset{\circ}{M} \supset N$, $V \supset (M + \chi) \cap B$, and $(N + \chi) \cap B = (W + \chi) \cap B \in \mathfrak{B}_B(z)$. Thus $\mathfrak{B}_B(z)$ is convex.

Let K and L be closed convex bodies of E such that $\overset{\circ}{K} \supset L$. Let $x \in \overset{\circ}{L}$, $M = K - x$, and $N = L - x$. Either $z \in \text{interior } M^{\circ\circ} + x$ —in which case $K \cap B = (M + x) \cap B = (M^{\circ\circ} + x) \cap B \in \mathfrak{B}_B(z)$ —or $z \notin N^{\circ\circ} + x$ —in which case $E'' \setminus (N^{\circ\circ} + x)$ is a neighborhood of z in E'' and so $B \setminus L = [E'' \setminus (N^{\circ\circ} + x)] \cap B \in \mathfrak{B}_B(z)$. Thus $\mathfrak{B}_B(z) \in \beta_B$.

Finally, let $u \in E'$, $\varepsilon > 0$, and $F \in \mathfrak{B}_B(z)$ such that $|ux - \mathfrak{B}_B(z)(u)| \leq \varepsilon/2$ for every $x \in F$. Let $V = \{w \in E'' \mid |wu - zu| \leq \varepsilon/2\}$. Then, for $x \in F \cap V$, we have $|\mathfrak{B}_B(z)(u) - zu| \leq |\mathfrak{B}_B(z)(u) - ux| + |ux - zu| \leq \varepsilon$. Therefore, $\pi_B(\mathfrak{B}_B(z))(u) = zu$ for $u \in E''$, and so $\pi_B(\mathfrak{B}_B(z)) = z$.

REMARK. Thus $\pi_B(\mathfrak{B}_B(z)) = z$ for $z \in \bar{B}$ and $\mathfrak{F} = \mathfrak{B}_B(\pi_B(\mathfrak{F}))$ for $\mathfrak{F} \in \beta_B$.

COROLLARY 1. If either \mathcal{S} is the weak topology or B is convex, then every maximal convex filter on B is a \mathcal{S} -Cauchy filter.

COROLLARY 2. If either \mathcal{S} is the weak topology or B is convex,

then for every $\mathfrak{F} \in \beta_B$ and $M \in \mathfrak{M}$, there exist $x \in B$ such that $(M + x) \cap B \in \mathfrak{F}$.

Proof. Let $F \in \mathfrak{F}$ such that $F' - F \subset M$ and $x \in F$.

For $M \in \mathfrak{M}$ and $x \in B$ we define :

$$\begin{aligned} \nu_B(M, x) &= \{\mathfrak{F} \in \beta_B \mid (\overset{\circ}{M} + x) \cap B \in \mathfrak{F}\} \\ \mu_B(M, x) &= \{\mathfrak{F} \in \beta_B \mid \pi_B(\mathfrak{F}) \in \text{interior } M^{\circ\circ} + x\} . \end{aligned}$$

For $M, N \in \mathfrak{M}$ and $x, y \in B$, if $z \in (\overset{\circ}{M} + x) \cap (\overset{\circ}{N} + y) \cap B$ and $K = (M + x - z) \cap (N + y - z)$, then $\nu_B(M, x) \cap \nu_B(N, y) = \nu_B(K, z)$ and $\mu_B(M, x) \cap \mu_B(N, y) = \mu_B(K, z)$. Hence the class of all sets of the form $\nu_B(M, x)$ and the class of all sets of the form $\mu_B(M, x)$ (for $M \in \mathfrak{M}$ and $x \in B$) form bases of topologies, called the ν - and μ -topologies respectively, on β_B .

THEOREM 3. *If $\pi_B(\beta_B) \subset \bar{B}$ (in particular if either \mathcal{S} is the weak topology or B is convex), then ν - and μ -topologies coincide and π_B is a homeomorphism of β_B onto \bar{B} .*

Proof. If $\pi_B(\beta_B) \subset \bar{B}$, then, for $M \in \mathfrak{M}$ and $x \in B$, we have $\mu_B(M, x) \subset \nu_B(M, x)$, and so the identity mapping of β_B with the μ -topology onto β_B with the ν -topology is continuous.

Also π_B from β_B with the ν -topology onto \bar{B} is continuous. Let $\mathfrak{F} \in \beta_B$ and V a neighborhood of $\pi_B(\mathfrak{F})$ in $E''[\mathcal{S}_n]$. Let U be a closed convex neighborhood of 0 in E'' such that $U + U \subset V - \pi_B(\mathfrak{F})$, $M = U \cap E$, and $x \in (\overset{\circ}{U} + \pi_B(\mathfrak{F})) \cap B$. Then $(\overset{\circ}{M} + x) \cap B \in \mathfrak{B}_B(\pi_B(\mathfrak{F})) = \mathfrak{F}$, and so $\mathfrak{F} \in \nu_B(M, x)$. Also if $\mathfrak{G} \in \nu_B(M, x)$, there is a neighborhood W of $\pi_B(\mathfrak{G})$ such that $W \cap B = (M + x) \cap B = (U + x) \cap B$ so

$$\pi_B(\mathfrak{G}) \in \overline{W \cap B} \subset U + x \subset U + U + \pi_B(\mathfrak{F}) \subset V .$$

Finally π_B^{-1} from \bar{B} onto β_B with the μ -topology is continuous by the definition of the sets μ .

COROLLARY 1. *If either \mathcal{S} is the weak topology or B is convex, then B is closed in $E''[\mathcal{S}_n]$ if and only if every maximal convex filter on B has nonempty intersection.*

COROLLARY 2. *B is weakly compact if and only if every maximal weakly convex filter on B has nonempty intersection.*

2. The space η . Let \mathfrak{A} denote the class of all convex sets of \mathfrak{B} and $\alpha = \bigcup_{B \in \mathfrak{A}} \beta_B$ the topological union of the spaces β_B . Let π be

the continuous function from α into $E''[\mathcal{F}_n]$ defined by $\pi(\mathfrak{F}) = \pi_B(\mathfrak{F})$ if $\mathfrak{F} \in \beta_B$. For $A, B \in \mathfrak{A}$ such that $A \subset B$, define a mapping g_{BA} from β_A into β_B by $g_{BA}(\mathfrak{F}) = \mathfrak{B}_B(\pi_A(\mathfrak{F}))$ (for $\mathfrak{F} \in \beta_A$). Then $g_{BA} = \pi_B^{-1}\pi_A$ and consequently is a homeomorphism of β_A into β_B . Also, if $A \subset B \subset C$, then $g_{CA} = g_{CB}g_{BA}$.

THEOREM 4. *Let $A, B \in \mathfrak{A}$ such that $A \subset B$, and let $\mathfrak{F} \in \beta_A$ and $\mathfrak{G} \in \beta_B$. The following three conditions are equivalent;*

- (a) $\mathfrak{G} = g_{BA}(\mathfrak{F})$;
- (b) $\pi(\mathfrak{F}) = \pi(\mathfrak{G})$;
- (c) Every set of \mathfrak{G} contains a set of \mathfrak{F} .

Proof. $\mathfrak{F} = \mathfrak{B}_A(\pi_B(\mathfrak{F}))$, $\mathfrak{G} = \mathfrak{B}_B(\pi_B(\mathfrak{G}))$, and $g_{BA}(\mathfrak{F}) = \mathfrak{B}_B(\pi_B(\mathfrak{F}))$. Hence (a) and (b) are equivalent. Also (b) implies (c): Given $G \in \mathfrak{G}$ there is a neighborhood V of $\pi(\mathfrak{G}) = \pi(\mathfrak{F})$ such that $G = V \cap B \supset V \cap A \in \mathfrak{F}$. Also (c) implies (b): If $\pi(\mathfrak{F}) \neq \pi(\mathfrak{G})$, then $\pi(\mathfrak{F})$ and $\pi(\mathfrak{G})$ have disjoint neighborhoods V and W in E'' , and so $W \cap A$ is a set of \mathfrak{G} containing no set of \mathfrak{F} .

COROLLARY. *Let A and $B \in \mathfrak{A}$, $\mathfrak{F} \in \beta_A$, and $\mathfrak{G} \in \beta_B$. The following three conditions are equivalent:*

- (a) $\pi(\mathfrak{F}) = \pi(\mathfrak{G})$.
- (b) There exists $C \in \mathfrak{A}$ such that $C \supset A \cup B$ and $g_{CA}(\mathfrak{F}) = g_{CB}(\mathfrak{G})$.
- (c) There exists $C \in \mathfrak{A}$ and $\mathfrak{H} \in \beta_C$ such that $C \supset A \cup B$ and every set of \mathfrak{H} contains a set of \mathfrak{F} and a set of \mathfrak{G} .

Now let R be the equivalence relation $\pi(\mathfrak{F}) = \pi(\mathfrak{G})$ on α , η the quotient space α/R , ρ the canonical mapping of α onto η , and σ the mapping from η into E'' such that $\pi = \sigma\rho$.

THEOREM 5. *σ is a homeomorphism of η onto the \mathcal{F}_n -closure \bar{E} of E in E'' .*

Proof. We need only prove $\sigma(\eta) = \pi(\alpha) \supset \bar{E}$. Consider the dual system $\langle E', \bar{E} \rangle$. Since every $u \in E'$ is uniformly continuous on E , the topology induced on \bar{E} by \mathcal{F}_n is admissible for this dual system. For $z \in \bar{E}$, there is a closed absolutely convex set $B \in \mathfrak{B}$ such that $|zu| \leq 1$ for every $u \in B^\circ$. Hence, $z \in B^{\circ\circ} =$ the closure of B in any admissible topology = the \mathcal{F}_n -closure \bar{B} of B .

For $B \in \mathfrak{A}$, the weakest topology on β_B for which every function of the form $\mathfrak{F} \rightarrow \mathfrak{F}(u)$ (for $u \in E'$) is continuous will be called the *weak topology* of β_B . Clearly β_B in the weak topology is homeomorphic

with \bar{B} in the topology induced on \bar{B} by the weak-star topology of E'' .

THEOREM 6. *The following three conditions are equivalent:*

- (a) $\bar{E} = E''$;
- (b) \bar{B} is weak-star compact for every $B \in \mathfrak{A}$;
- (c) β_B is weakly compact for every $B \in \mathfrak{A}$.

Proof. Clearly (b) and (c) are equivalent. Also (a) implies (b); by the Alaoglu—Bourbaki theorem, for $B \in \mathfrak{A}$, the weak-star closure of B in $E'' = \bar{E}$ is weak-star compact; but since \mathcal{T}_n is an admissible topology for the dual system $\langle E', \bar{E} \rangle$, this weak-star closure is \bar{B} . Finally (b) implies (a): regarding \mathfrak{B} as a total class of bounded subsets of \bar{E} , by the Mackey-Arens theorem $\mathcal{T}_{\mathfrak{B}}$ is an admissible topology for the dual system $\langle E', \bar{E} \rangle$, and so $E'' = \bar{E}$.

THEOREM 7. *For $B \in \mathfrak{A}$, β_B is weakly compact if and only if for every maximal weakly-convex filter \mathfrak{F} on B , there is a maximal \mathcal{T} -convex filter on B which is stronger than \mathfrak{F} .*

Proof. Let β_B^w be the space of all maximal weakly convex filters on B and π_B^w the homeomorphism of β_B^w into E'' with the weak-star topology. In general $B \subset \pi_B(\beta_B) = \bar{B} \subset$ weak-star closure of $B = \pi_B^w(\beta_B^w)$.

If β_B is weakly compact, then $\pi_B^w(\beta_B^w) = \pi_B(\beta_B) = \bar{B}$. So, for $\mathfrak{F} \in \beta_B^w$, $\pi_B^w(\mathfrak{F}) \in \bar{B}$ and hence $\mathfrak{B}_B(\pi_B^w(\mathfrak{F})) \in \beta_B$ is stronger than \mathfrak{F} .

Conversely, let $\mathfrak{F} \in \beta_B^w$ and $\mathfrak{G} \in \beta_B$ stronger than \mathfrak{F} . Then $\pi_B^w(\mathfrak{F}) = \pi_B(\mathfrak{G})$, and so $\pi_B^w(\beta_B^w) \subset \pi_B(\beta_B)$.

COROLLARY. *$\bar{E} = E''$ if and only if, for every $B \in \mathfrak{A}$ and every maximal weakly-convex filter \mathfrak{F} on B , there is a \mathcal{T} -convex filter on B stronger than \mathfrak{F} .*

3. Acknowledgement. The author wished to express his gratitude to Professor Ky Fan for the help and encouragement given him in the work presented here.

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The *Pacific Journal of Mathematics* is published quarterly, in March, June, September, and December. Effective with Volume 13 the price per volume (4 numbers) is \$18.00; single issues, \$5.00. Special price for current issues to individual faculty members of supporting institutions and to individual members of the American Mathematical Society: \$8.00 per volume; single issues \$2.50. Back numbers are available.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley 8, California.

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), No. 6, 2-chome, Fujimi-cho, Chiyoda-ku, Tokyo, Japan.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

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* Basil Gordon, Acting Managing Editor until February 1, 1966.

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