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**INVARIANT SPLITTING IN JORDAN AND ALTERNATIVE  
ALGEBRAS**

EARL J. TAFT

## INVARIANT SPLITTING IN JORDAN AND ALTERNATIVE ALGEBRAS

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**Let  $A$  be a finite-dimensional Jordan or alternative algebra over a field  $F$  of characteristic 0. Let  $N$  denote the radical of  $A$ . Then  $A$  possesses maximal semisimple subalgebras isomorphic to  $A/N$ , [5], [6], any two of which are strictly conjugate, [2], [9]. If  $G$  is a finite group of automorphisms and antiautomorphisms of  $A$ , then  $A$  possesses  $G$ -invariant maximal semisimple subalgebras, [10]. We investigate here the uniqueness question for such  $G$ -invariant maximal semisimple subalgebras. The result is that the strict conjugacy can be chosen to commute pointwise with  $G$  and to be in the enveloping associative algebra generated by the right and left multiplications in  $A$ .**

Similar results have been obtained for associative algebras, [11], and Lie algebras, [12]. However, in the associative case, the conjugacy can be obtained in terms of adjoints of  $G$ -symmetric elements, i.e., elements left fixed by the automorphisms in  $G$  and sent into their negatives by the antiautomorphisms in  $G$ . In the Lie algebra case, one needs only to consider automorphisms, and the conjugacy is obtained in terms of adjoints of fixed points of  $G$ . In each case, the conjugacy is in the enveloping associative algebra of  $A$ . In both the Jordan and alternative cases, the automorphisms which occur would commute pointwise with  $G$  if the elements of  $A$  which occur in their formulation in terms of right and left multiplications were to be fixed points of  $G$ . However, we have not obtained the conjugacies in this form, and it seems to be an open question whether or not it is always possible to do so.

If  $G$  is assumed fully reducible, instead of finite, then  $A$  will also possess  $G$ -invariant maximal semisimple subalgebras. This is noted in the Jordan case in [4] when  $G$  contains only automorphisms, and the same proof can be extended to cover the alternative case, even if  $G$  also contains antiautomorphisms. We have answered the uniqueness question for the similar situation in the associative and Lie cases, [13]. For the Jordan and alternative case, the problem seems more complicated. We note here that it is easily answered if  $N^2 = 0$ , with the strict conjugacy commuting pointwise with  $G$ . However, the general question remains open.

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2. Preliminaries. If  $a \in A$ , we let  $R_a$  and  $L_a$  stand for right and left multiplication by  $a$ , i.e.,  $xR_a = xa$ ,  $xL_a = ax$ . The following two lemmas are easily proved by straightforward calculation.

LEMMA 1. *Let  $g$  be an automorphism of  $A$ . Then  $g^{-1}R_ag = R_{ag}$  and  $g^{-1}L_ag = L_{ag}$ .*

LEMMA 2. *Let  $g$  be an antiautomorphism of  $A$ . Then  $g^{-1}R_ag = L_{ag}$ ,  $g^{-1}L_ag = R_{ag}$ .*

A derivation of  $A$  will be called inner if it is in the enveloping Lie algebra generated by the right and left multiplications in  $A$ , [7]. We will have occasion to use the following types of inner derivations. If  $A$  is Jordan, and  $x, s \in A$ , then  $[R_x, R_s] = R_xR_s - R_sR_x$  is an inner derivation of  $A$  which, for  $x \in N$ , will be a nilpotent element of the radical of the enveloping associative algebra generated by multiplications in  $A$  by elements of  $A$ , [1], [2], [8]. If  $A$  is alternative, and  $s, x \in A$ , then  $D_{s,x} = [R_s, R_x] + [L_s, R_x] + [L_s, L_x]$  is an inner derivation of  $A$  which, for  $x \in N$ , will be a nilpotent element of the radical of the enveloping associative algebra generated by the left and right multiplications of  $A$ , [7], [9].

LEMMA 3. *If  $A$  is alternative,  $a, b \in A$ , then  $[R_a, L_b] = [L_a, R_b]$ , and  $D_{a,b} = -D_{b,a}$ .*

*Proof.*  $x[R_a, L_b] = b(xa) - (bx)a = -(b, x, a)$ , where  $(b, x, a) = (bx)a - b(xa)$  is the associator of  $b, x$ , and  $a$ . Also  $x[L_a, R_b] = (ax)b - a(xb) = (a, x, b)$ . The first part of Lemma 3 follows from the skew-symmetry of the associator function. Hence

$$\begin{aligned} D_{b,a} &= [R_b, R_a] + [L_b, R_a] + [L_b, L_a] \\ &= -[R_a, R_b] - [R_a, L_b] - [L_a, L_b] \\ &= -[R_a, R_b] - [L_a, R_b] - [L_a, L_b] = -D_{a,b}. \end{aligned}$$

LEMMA 4. *Let  $A$  be Jordan, and  $g$  an automorphism of  $A$ . Then  $g^{-1}[R_a, R_b]g = [R_{ag}, R_{bg}]$ .*

This is immediate from Lemma 1.

LEMMA 5. *Let  $A$  be alternative, and  $g$  an automorphism or antiautomorphism of  $A$ . Then  $g^{-1}D_{a,b}g = D_{ag,bg}$ .*

*Proof.* This is clear from Lemma 1 if  $g$  is an automorphism. Let  $g$  be an antiautomorphism. Then, using Lemma 2,  $g^{-1}D_{a,b}g = [L_{ag}, L_{bg}] + [R_{ag}, L_{bg}] + [R_{ag}, R_{bg}] = D_{ag,bg}$  by Lemma 3.

If  $D$  is a nilpotent derivation of  $A$ , then  $\exp D = I + D + (D^2/2!) + \dots$  is an automorphism of  $A$ . We assume familiarity with the Campbell-Hausdorff formula, [3],  $(\exp D_1)(\exp D_2) = \exp D_3$ , where  $D_3$  is in the Lie algebra generated by  $D_1$  and  $D_2$ .

3. The Jordan case.

**THEOREM 1.** *Let  $A$  be a finite-dimensional Jordan algebra over a field  $F$  of characteristic 0. Let  $G$  be a finite group of automorphisms of  $A$ . Let  $S$  be a  $G$ -invariant maximal semisimple subalgebra of  $A$ . Let  $T$  be a  $G$ -invariant semisimple subalgebra of  $A$ . Then there exists an automorphism  $U = \exp D$  of  $A$  such that*

- (1)  $U$  maps  $T$  into  $S$ ,
- (2)  $D$  (and hence  $U$ ) commutes pointwise with  $G$ ,
- (3)  $D$  is a nilpotent inner derivation of  $A$  which is in the radical of the enveloping associative algebra of  $A$ .

*Proof.* Let  $N$  denote the radical of  $A$ . Let  $s$  and  $n$  denote the projections of the vector space  $A = S \oplus N$  onto  $S$  and  $N$  respectively. Then  $s$  and  $n$  are linear mappings such that

- (i)  $s(t_1 t_2) = s(t_1) s(t_2)$
- (ii)  $n(t_1 t_2) = s(t_1) n(t_2) + n(t_1) s(t_2) + n(t_1) n(t_2)$
- (iii)  $s(tg) = s(t)g, \quad n(tg) = n(t)g$

for  $t_1, t_2, t \in T, g \in G$ .

(i) and (ii) follow since  $N$  is an ideal. (iii) follows from the invariance of  $T, S$  and  $N$  under  $G$ .

Now set  $N_1 = N, N_i = N_{i-1}^2 + AN_{i-1}$ . By [5], the  $N_i$  form a nonincreasing sequence of ideals terminating in 0. Now  $T_1 = T \subseteq A = S + N_1$ . Suppose that we have found automorphisms  $U_0 = \exp 0, U_1 = \exp D_1, \dots, U_{i-1} = \exp(D_{i-1})$  of  $A$  satisfying (2) and (3) of Theorem 1 such that  $T_i = TU_0U_1 \dots U_{i-1} \subseteq S + N_i$ . Then we will show that there exists an automorphism  $U_i$  of  $A$  satisfying (2) and (3) of Theorem 1 such that  $T_i U_i \subseteq S + N_{i+1}$ . Hence if  $N_k = 0$ , then  $U = U_0 U_1 \dots U_{k-1}$  will be the desired automorphism by the Campbell-Hausdorff formula.

Now  $T_i$  is a  $G$ -invariant semisimple subalgebra of  $A$ , so that (i), (ii), (iii) hold for  $t_1, t_2, t \in T_i$ . Consider the space  $N_i | N_{i+1}$ . We consider this as a  $T_i$ -module by defining  $t \cdot \bar{n} = \bar{n} \cdot t = \overline{ns(t)}$  for  $n \in N_i, t \in T_i$ . Then by (ii), we have

$$(iv) \quad \overline{n(t_1 t_2)} = \overline{n(t_1)} \cdot t_2 + t_1 \cdot \overline{n(t_2)}.$$

(iv) says that the map  $t \rightarrow \overline{n(t)}$  is a derivation of  $T_i$  into the module  $N_i | N_{i+1}$ . Hence, by [2], there exist elements  $x_1, \dots, x_p$  in  $N_i, t_1, \dots, t_p \in T_i$  such that

$$(v) \quad \overline{n(t)} = \sum_{j=1}^p ((\bar{x}_j \cdot t) \cdot t_j - \bar{x}_j \cdot (tt_j)) \text{ for } t \in T_i \text{ i.e.,}$$

$$\overline{n(t)} = \sum_{j=1}^p \overline{(x_j s(t))s(t_j)} - \overline{x_j s(tt_j)} .$$

Using (i), we have

$$(vi) \quad n(t) \equiv s(t) \sum_{j=1}^p [R_{x_j}, R_{s(t_j)}] \pmod{N_{i+1}} \text{ for } t \in T_i .$$

Let  $g \in G$ . Then

$$[R_{x_j g}, R_{s(t_j)g}] = g^{-1}[R_{x_j}, R_{s(t_j)}]g$$

by Lemma 4. Hence

$$\begin{aligned} s(t) \sum_{j=1}^p [R_{x_j g}, R_{s(t_j)g}] &= s(t)g^{-1} \left( \sum_{j=1}^p [R_{x_j}, R_{s(t_j)}] \right) g \\ &= s(tg^{-1}) \left( \sum_{j=1}^p [R_{x_j}, R_{s(t_j)}] \right) g \equiv n(tg^{-1})g = n(t) \pmod{N_{i+1}} . \end{aligned}$$

It follows that if we set  $D_i = -(1/m) \sum_{g \in G} (\sum_{j=1}^p [R_{x_j g}, R_{s(t_j)g}])$ , where  $m$  is the order of  $G$ , then

$$(vii) \quad n(t) \equiv -s(t)D_i \pmod{N_{i+1}} \quad \text{for } t \in T_i .$$

Now  $D_i$  clearly satisfies (3) of the Theorem, since the  $x_j g \in N$ . To see that  $D_i$  satisfies (2) of the Theorem, we fix a value of  $j$ . Then  $\sum_{g \in G} [R_{x_j g}, R_{s(t_j)g}] = \sum_{g \in G} g^{-1}[R_{x_j}, R_{s(t_j)}]g$  clearly commutes pointwise with  $G$ . Hence so does  $D_i$ , which is a linear combination of such mappings.

Finally, set  $U_i = \exp D_i$ . If  $t \in T_i$ , then  $tU_i = t + tD_i + (t/2)D_i^2 + \dots = s(t) + n(t) + s(t)D_i + n(t)D_i + (t/2)D_i^2 + \dots$ .

Now  $n(t) \in N_i$ , so that  $n(t)D_i \in N_{i+1}$ . Also, since the  $x_1, \dots, x_p \in N_i$ , we have that  $(t/2)D_i^2 + \dots \in N_{i+1}$ . Therefore

$$\begin{aligned} tU_i &\equiv s(t) + n(t) + s(t)D_i \pmod{N_{i+1}} \\ &\equiv s(t) \pmod{N_{i+1}} \text{ by (vii).} \end{aligned}$$

Hence  $T_i U_i \subseteq S + N_{i+1}$ . This completes the proof of the Theorem.

**COROLLARY 1.** *Let  $A$  be a finite-dimensional Jordan algebra over a field of characteristic 0. Let  $G$  be a finite group of automorphisms of  $A$ . Let  $S$  and  $T$  be  $G$ -invariant maximal semisimple subalgebras of  $A$ . Then  $S$  and  $T$  are strictly conjugate via an automorphism of  $A$  of the type described in Theorem 1.*

**COROLLARY 2.** *Let  $A$  and  $G$  be as in Corollary 1. Let  $T$  be any  $G$ -invariant semisimple subalgebra of  $A$ . Then  $T$  is contained in a  $G$ -invariant maximal semisimple subalgebra of  $A$ .*

Corollary 1 is an immediate consequence of Theorem 1. Corollary 2 follows from the existence of a  $G$ -invariant maximal semisimple

subalgebra  $S$  of  $A$ . For then if  $U$  is an automorphism of  $A$  which maps  $T$  into  $S$ , and which commutes with  $G$  pointwise, it follows that  $SU^{-1}$  is a  $G$ -invariant maximal semisimple subalgebra of  $A$  which contains  $T$ .

4. The alternative case.

**THEOREM 2.** *Let  $A$  be a finite-dimensional alternative algebra over a field  $F$  of characteristic 0. Let  $G$  be a finite group of automorphisms and antiautomorphism of  $A$ . Let  $S$  be a  $G$ -invariant maximal semisimple subalgebra of  $A$ . Let  $T$  be a semisimple subalgebra of  $A$ . Then there exists an automorphism  $U = \exp D$  of  $A$  such that*

- (1)  $U$  maps  $T$  into  $S$ ,
- (2)  $D$  (and hence  $U$ ) commutes pointwise with  $G$ ,
- (3)  $D$  is a nilpotent inner derivation of  $A$  which is in the radical of the enveloping associative algebra of  $A$ .

*Proof.* The proof is similar to Theorem 1. We define  $s$  and  $n$  as in Theorem 1, but use  $N_i = N^i$  instead. We consider  $N^i | N^{i+1}$  as a two-sided  $T_i$ -module by  $t \cdot \bar{n} = \overline{s(t)n}$  and  $\bar{n} \cdot t = \overline{ns(t)}$ . Then (i), (ii), (iii) and (iv) are valid. Hence, by [9], there exist elements  $x_1, \dots, x_p \in N^i$  and  $t_1, \dots, t_p \in T_i$  such that

$$(v) \quad \overline{n(t)} = t \sum_{j=1}^p D_{t_j, \bar{x}_j} \quad \text{for } t \in T_i$$

where  $D_{t_j, \bar{x}_j}$  is the inner derivation  $[R_{t_j}, R_{\bar{x}_j}] + [L_{t_j}, R_{\bar{x}_j}] + [L_{t_j}, L_{\bar{x}_j}]$  of  $T_i$  into its two-sided module  $N^i | N^{i+1}$ . As in Theorem 1, we obtain

$$(vi) \quad n(t) \equiv s(t) \sum_{j=1}^p D_{s(t_j), x_j} \pmod{N^{i+1}} \text{ for } t \in T_i,$$

where  $D_{s(t_j), x_j}$  is the inner derivation  $[R_{s(t_j)}, R_{x_j}] + [L_{s(t_j)}, R_{x_j}] + [L_{s(t_j)}, L_{x_j}]$  of  $A$ .

Now let  $g \in G$ . Then by Lemma 5, we have  $g^{-1}(D_{s(t_j), x_j})g = D_{s(t_j)g, x_jg}$ . Hence, for any  $g \in G$ ,  $s(t) \sum_{j=1}^p D_{s(t_j)g, x_jg} = s(t)g^{-1}(\sum_{j=1}^p D_{s(t_j), x_j})g = s(tg^{-1})(\sum_{j=1}^p D_{s(t_j), x_j})g \equiv n(t) \pmod{N^{i+1}}$  by (iii) and (v).

Now set  $D_i = -(1/m) \sum_{g \in G} (\sum_{j=1}^p D_{s(t_j)g, x_jg})$ , where  $m$  is the order of  $G$ . Then we have

$$(vii) \quad n(t) \equiv -s(t)D_i \pmod{N^{i+1}} \text{ for } t \in T_i.$$

$D_i$  satisfies (3) of the Theorem since the  $x_jg \in N$ . To see that  $D_i$  satisfies (2) of the Theorem, we fix a value of  $j$ . Then  $\sum_{g \in G} D_{s(t_j)g, x_jg} = \sum_{g \in G} g^{-1}D_{s(t_j), x_j}g$  commutes pointwise with  $G$ . Hence so does  $D_i$ , which is a linear combination of such mappings.

Now we set  $U_i = \exp D_i$ , and get that  $T_i U_i \subseteq S + N^{i+1}$  as in Theorem 1. Finally, we put  $U = U_0 U_1 \dots U_{k-1}$ , where  $N^k = 0$ , and use the Campbell-Hausdorff formula to complete the proof of the

Theorem.

As in the Jordan case, we have the following two corollaries of Theorem 2.

**COROLLARY 1.** *Let  $A$  be a finite-dimensional alternative algebra over a field of characteristic 0. Let  $G$  be a finite group of automorphisms and antiautomorphisms of  $A$ . Let  $S$  and  $T$  be  $G$ -invariant maximal semisimple subalgebras of  $A$ . Then  $S$  and  $T$  are strictly conjugate via an automorphism of  $A$  of the type described in Theorem 2.*

**COROLLARY 2.** *Let  $A$  and  $G$  be as in Corollary 1. Let  $T$  be any  $G$ -invariant semisimple subalgebra of  $A$ . Then  $T$  is contained in a  $G$ -invariant maximal semisimple subalgebra of  $A$ .*

5. The fully reducible case. Let  $A$  be a finite-dimensional Jordan or alternative algebra over a field of characteristic zero. If  $G$  is a fully reducible group of automorphisms and antiautomorphisms of  $A$ , then it follows from [4] that  $G$  will leave invariant a maximal semisimple subalgebra of  $A$ . The analogue of Corollaries 1 has not been answered as yet for this case. However, if  $N^2 = 0$ , then any automorphism of the form described in the proofs of Theorems 1 and 2 which carries a  $G$ -invariant maximal semisimple subalgebra  $T$  onto another one,  $S$ , is unique, and hence will commute pointwise with  $G$ .

For let  $U_1 = \exp D_1, U_2 = \exp D_2$  be of this form and both map  $T$  onto  $S$ . Then  $D_1^2 = D_2^2 = 0$ , so that  $U_1 = I + D_1, U_2 = I + D_2$ . If  $t \in T$ , then  $tU_1 = t + tD_1 \in S$  and  $tU_2 = t + tD_2 \in S$ . Hence their difference  $tD_1 - tD_2 \in S \cap N = 0$ , since  $D_1$  and  $D_2$  have range in  $N$ . Hence  $D_1 = D_2$  on  $T$ . Also  $D_1$  and  $D_2$  are both 0 on  $N$  since  $N^2 = 0$ . Hence  $D_1 = D_2$  since  $A = T + N$ .

Now let  $g \in G$ . Then  $g^{-1}U_1g = I + g^{-1}D_1g$  will map  $T$  onto  $S$  and  $g^{-1}D_1g$  is a derivation of square zero having range in  $N$ . Hence, by the above,  $g^{-1}D_1g = D_1$ , that is,  $D_1$ , and hence  $U_1$ , commutes pointwise with  $G$ .

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