IN Variant SPLITTING in JORDAN AND ALTERNATIVE ALGEBRAS

EARL J. TAFT
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Let $A$ be a finite-dimensional Jordan or alternative algebra over a field $F$ of characteristic 0. Let $N$ denote the radical of $A$. Then $A$ possesses maximal semisimple subalgebras isomorphic to $A/N$, [5], [6], any two of which are strictly conjugate, [2], [9]. If $G$ is a finite group of automorphisms and antiautomorphisms of $A$, then $A$ possesses $G$-invariant maximal semisimple subalgebras, [10]. We investigate here the uniqueness question for such $G$-invariant maximal semisimple subalgebras. The result is that the strict conjugacy can be chosen to commute pointwise with $G$ and to be in the enveloping associative algebra generated by the right and left multiplications in $A$.

Similar results have been obtained for associative algebras, [11], and Lie algebras, [12]. However, in the associative case, the conjugacy can be obtained in terms of adjoints of $G$-symmetric elements, i.e., elements left fixed by the automorphisms in $G$ and sent into their negatives by the antiautomorphisms in $G$. In the Lie algebra case, one needs only to consider automorphisms, and the conjugacy is obtained in terms of adjoints of fixed points of $G$. In each case, the conjugacy is in the enveloping associative algebra of $A$. In both the Jordan and alternative cases, the automorphisms which occur would commute pointwise with $G$ if the elements of $A$ which occur in their formulation in terms of right and left multiplications were to be fixed points of $G$. However, we have not obtained the conjugacies in this form, and it seems to be an open question whether or not it is always possible to do so.

If $G$ is assumed fully reducible, instead of finite, then $A$ will also possess $G$-invariant maximal semisimple subalgebras. This is noted in the Jordan case in [4] when $G$ contains only automorphisms, and the same proof can be extended to cover the alternative case, even if $G$ also contains antiautomorphisms. We have answered the uniqueness question for the similar situation in the associative and Lie cases, [13]. For the Jordan and alternative case, the problem seems more complicated. We note here that it is easily answered if $N^2 = 0$, with the strict conjugacy commuting pointwise with $G$. However, the general question remains open.

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2. Preliminaries. If $a \in A$, we let $R_a$ and $L_a$ stand for right and left multiplication by $a$, i.e., $xR_a = xa$, $xL_a = ax$. The following two lemmas are easily proved by straightforward calculation.

**Lemma 1.** Let $g$ be an automorphism of $A$. Then $g^{-1}R_ag = R_ag$ and $g^{-1}L_ag = L_ag$.

**Lemma 2.** Let $g$ be an antiautomorphism of $A$. Then $g^{-1}R_ag = L_ag$, $g^{-1}L_ag = R_ag$.

A derivation of $A$ will be called inner if it is in the enveloping Lie algebra generated by the right and left multiplications in $A$, [7]. We will have occasion to use the following types of inner derivations. If $A$ is Jordan, and $x, s \in A$, then $[R_x, R_s] = R_xR_s - R_sR_x$ is an inner derivation of $A$ which, for $x \in N$, will be a nilpotent element of the radical of the enveloping associative algebra generated by multiplications in $A$ by elements of $A$, [1], [2], [8]. If $A$ is alternative, and $s, x \in A$, then $D_{s,x} = [R_s, R_x] + [L_s, R_x] + [L_s, L_x]$ is an inner derivation of $A$ which, for $x \in N$, will be a nilpotent element of the radical of the enveloping associative algebra generated by the left and right multiplications of $A$, [7], [9].

**Lemma 3.** If $A$ is alternative, $a, b \in A$, then $[R_a, L_b] = [L_a, R_b]$, and $D_{a,b} = -D_{b,a}$.

**Proof.** $x[R_a, L_b] = b(xa) - (bx)a = -(b, x, a)$, where $(b, x, a) = (bx)a - b(xa)$ is the associator of $b, x,$ and $a$. Also $x[L_a, R_b] = (ax)b - a(xb) = (a, x, b)$. The first part of Lemma 3 follows from the skew-symmetry of the associator function. Hence

$$D_{b,a} = [R_b, R_a] + [L_b, R_a] + [L_b, L_a]$$

$$= -[R_a, R_b] - [R_a, L_b] - [L_a, L_b]$$

$$= -[R_a, R_b] - [L_a, R_b] - [L_a, L_b] = -D_{a,b}.$$

**Lemma 4.** Let $A$ be Jordan, and $g$ an automorphism of $A$. Then $g^{-1}[R_a, R_s]g = [R_ag, R_bg]$.

This is immediate from Lemma 1.

**Lemma 5.** Let $A$ be alternative, and $g$ an automorphism or antiautomorphism of $A$. Then $g^{-1}D_{a,b}g = D_{a,b}$.

**Proof.** This is clear from Lemma 1 if $g$ is an automorphism. Let $g$ be an antiautomorphism. Then, using Lemma 2, $g^{-1}D_{a,b}g = [L_ag, L_bg] + [R_ag, L_bg] + [R_ag, R_bg] = D_{a,b}$ by Lemma 3.
If $D$ is a nilpotent derivation of $A$, then $\exp D = I + D + (D^2/2!) + \cdots$ is an automorphism of $A$. We assume familiarity with the Campbell-Hausdorff formula, [3], \((\exp D_1)(\exp D_2) = \exp D_3\), where $D_3$ is in the Lie algebra generated by $D_1$ and $D_2$.

3. The Jordan case.

**Theorem 1.** Let $A$ be a finite-dimensional Jordan algebra over a field $F$ of characteristic 0. Let $G$ be a finite group of automorphisms of $A$. Let $S$ be a $G$-invariant maximal semisimple subalgebra of $A$. Let $T$ be a $G$-invariant semisimple subalgebra of $A$. Then there exists an automorphism $U = \exp D$ of $A$ such that

1. $U$ maps $T$ into $S$,
2. $D$ (and hence $U$) commutes pointwise with $G$,
3. $D$ is a nilpotent inner derivation of $A$ which is in the radical of the enveloping associative algebra of $A$.

**Proof.** Let $N$ denote the radical of $A$. Let $s$ and $n$ denote the projections of the vector space $A = S \oplus N$ onto $S$ and $N$ respectively. Then $s$ and $n$ are linear mappings such that

1. $s(t_1t_2) = s(t_1)s(t_2)$
2. $n(t_1t_2) = n(t_1)n(t_2) + n(t_1)s(t_2) + s(t_1)n(t_2)$
3. $s(tg) = s(t)g$, $n(tg) = n(t)g$

for $t_1, t_2, t \in T$, $g \in G$.

(i) and (ii) follow since $N$ is an ideal. (iii) follows from the invariance of $T$, $S$ and $N$ under $G$.

Now set $N_1 = N$, $N_i = N_{i-1}^2 + AN_{i-1}^2$. By [5], the $N_i$ form a nonincreasing sequence of ideals terminating in 0. Now $T_i = T \cap A = S + N_i$. Suppose that we have found automorphisms $U_0 = \exp 0$, $U_1 = \exp D_1$, $\cdots$, $U_{i-1} = \exp (D_{i-1})$ of $A$ satisfying (2) and (3) of Theorem 1 such that $T_i = T U_i U_1 \cdots U_{i-1} \subseteq S + N_i$. Then we will show that there exists an automorphism $U_i$ of $A$ satisfying (2) and (3) of Theorem 1 such that $T_i U_i \subseteq S + N_{i+1}$. Hence if $N_k = 0$, then $U = U_0 U_1 \cdots U_{k-1}$ will be the desired automorphism by the Campbell-Hausdorff formula.

Now $T_i$ is a $G$-invariant semisimple subalgebra of $A$, so that (i), (ii), (iii) hold for $t_1, t_2, t \in T_i$. Consider the space $N_i | N_{i+1}$. We consider this as a $T_i$-module by defining $t \cdot \bar{n} = \bar{n} \cdot t = \bar{n}s(t)$ for $n \in N_i$, $t \in T_i$. Then by (ii), we have

(iv) $\bar{n}(t_1t_2) = \bar{n}(t_1) \cdot t_2 + t_1 \cdot \bar{n}(t_2)$.

(iv) says that the map $t \to \bar{n}(t)$ is a derivation of $T_i$ into the module $N_i | N_{i+1}$. Hence, by [2], there exist elements $x_1, \cdots, x_p$ in $N_i, t_1, \cdots, t_p \in T_i$ such that

(v) $\bar{n}(t) = \sum_{j=1}^{p} ((\bar{x}_j \cdot t) \cdot t_j - \bar{x}_j \cdot (tt_j))$ for $t \in T_i$ i.e.,
\[ n(t) = \sum_{j=1}^{n} (x_j s(t_j)) s(t_j) - x_j s(tt_j) \].

Using (i), we have

\[(vi) \quad n(t) \equiv s(t) \sum_{j=1}^{n} [R_{x_j}, R_{s(t_j)}] \pmod{N_{i+1}} \text{ for } t \in T_i.\]

Let \( g \in G \). Then

\[ [R_{x_jg}, R_{s(t_j)g}] = g^{-1}[R_{x_j}, R_{s(t_j)}]g \]

by Lemma 4. Hence

\[ s(t) \sum_{j=1}^{n} [R_{x_jg}, R_{s(t_j)g}] = s(t)g^{-1}\left(\sum_{j=1}^{n} [R_{x_j}, R_{s(t_j)}]\right)g \]
\[ = s(tg^{-1})\left(\sum_{j=1}^{n} [R_{x_j}, R_{s(t_j)}]\right)g \equiv n(tg^{-1})g = n(t) \pmod{N_{i+1}}. \]

It follows that if we set \( D_i = -(1/m) \sum_{g \in G} (\sum_{j=1}^{n} [R_{x_jg}, R_{s(t_j)g}])g \)
where \( m \) is the order of \( G \), then

\[(vii) \quad n(t) = -s(t)D_i \pmod{N_{i+1}} \quad \text{for } t \in T_i.\]

Now \( D_i \) clearly satisfies (3) of the Theorem, since the \( x_jg \in N \). To see that \( D_i \) satisfies (2) of the Theorem, we fix a value of \( j \). Then

\[ \sum_{g \in G} [R_{x_jg}, R_{s(t_j)g}] = \sum_{g \in G} g^{-1}[R_{x_j}, R_{s(t_j)}]g \]

clearly commutes pointwise with \( G \). Hence so does \( D_i \), which is a linear combination of such mappings.

Finally, set \( U_i = \exp D_i \). If \( t \in T_i \), then \( t U_i = t + tD_i + \frac{t}{2}D_i^2 + \cdots = s(t) + n(t) + s(t)D_i + n(t)D_i + \frac{t}{2}D_i^2 + \cdots. \)

Now \( n(t) \in N_i \), so that \( n(t)D_i \in N_{i+1} \). Also, since the \( x_1, \cdots, x_p \in N_i \), we have that \( (t/2)D_i^2 + \cdots \in N_{i+1} \). Therefore

\[ t U_i \equiv s(t) + n(t) + s(t)D_i \pmod{N_{i+1}} \]
\[ \equiv s(t) \pmod{N_{i+1}} \text{ by (vii)}. \]

Hence \( T_i U_i \subseteq S + N_{i+1} \). This completes the proof of the Theorem.

**Corollary 1.** Let \( A \) be a finite-dimensional Jordan algebra over a field of characteristic 0. Let \( G \) be a finite group of automorphisms of \( A \). Let \( S \) and \( T \) be \( G \)-invariant maximal semisimple subalgebras of \( A \). Then \( S \) and \( T \) are strictly conjugate via an automorphism of \( A \) of the type described in Theorem 1.

**Corollary 2.** Let \( A \) and \( G \) be as in Corollary 1. Let \( T \) be any \( G \)-invariant semisimple subalgebra of \( A \). Then \( T \) is contained in a \( G \)-invariant maximal semisimple subalgebra of \( A \).

Corollary 1 is an immediate consequence of Theorem 1. Corollary 2 follows from the existence of a \( G \)-invariant maximal semisimple
subalgebra $S$ of $A$. For then if $U$ is an automorphism of $A$ which maps $T$ into $S$, and which commutes with $G$ pointwise, it follows that $SU^{-1}$ is a $G$-invariant maximal semisimple subalgebra of $A$ which contains $T$.

4. The alternative case.

**Theorem 2.** Let $A$ be a finite-dimensional alternative algebra over a field $F$ of characteristic $0$. Let $G$ be a finite group of automorphisms and antiautomorphism of $A$. Let $S$ be a $G$-invariant maximal semisimple subalgebra of $A$. Let $T$ be a semisimple subalgebra of $A$. Then there exists an automorphism $U = \exp D$ of $A$ such that

1. $U$ maps $T$ into $S$,
2. $D$ (and hence $U$) commutes pointwise with $G$,
3. $D$ is a nilpotent inner derivation of $A$ which is in the radical of the enveloping associative algebra of $A$.

**Proof.** The proof is similar to Theorem 1. We define $s$ and $n$ as in Theorem 1, but use $N_i = N^i$ instead. We consider $N^i | N^{i+1}$ as a two-sided $T_i$-module by $t \cdot \bar{n} = s(t)\bar{n}$ and $\bar{n} \cdot t = n\bar{s}(t)$. Then (i), (ii), (iii) and (iv) are valid. Hence, by [9], there exist elements $x_1, \ldots, x_p \in N^i$ and $t_1, \ldots, t_p \in T_i$ such that

$(v) \quad n(t) = t \sum_{j=1}^p D_{s(t), x_j} \bar{t}_j$ for $t \in T_i$

where $D_{s(t), x_j}$ is the inner derivation $[R_{s(t), x_j}, [L_{s(t), x_j}, L_{s(t), x_j}]]$ of $T_i$ into its two-sided module $N^i | N^{i+1}$. As in Theorem 1, we obtain

$(vi) \quad n(t) = s(t) \sum_{j=1}^p D_{s(t), x_j} \bar{t}_j (\mod N^{i+1})$ for $t \in T_i$

where $D_{s(t), x_j}$ is the inner derivation $[R_{s(t), x_j}, [L_{s(t), x_j}, L_{s(t), x_j}]]$ of $A$.

Now let $g \in G$. Then by Lemma 5, we have $g^{-i}(D_{s(t), x_j})g = D_{s(t), x_j}$. Hence, for any $g \in G$, $s(t) \sum_{j=1}^p D_{s(t), x_j} \bar{t}_j = s(t)g^{-t} \sum_{j=1}^p D_{s(t), x_j} \bar{t}_j = n(t)(\mod N^{i+1})$ by (iii) and (v).

Now set $D_i = -(1/m) \sum_{j=1}^p D_{s(t), x_j} \bar{t}_j$, where $m$ is the order of $G$. Then we have

$(vii) \quad n(t) = -s(t)D_i (\mod N^{i+1})$ for $t \in T_i$.

$D_i$ satisfies (3) of the Theorem since the $x_j g \in N$. To see that $D_i$ satisfies (2) of the Theorem, we fix a value of $j$. Then $\sum_{g \in G} D_{s(t), x_j} \bar{g} = \sum_{g \in G} g^{-1} D_{s(t), x_j} \bar{g}$ commutes pointwise with $G$. Hence so does $D_i$, which is a linear combination of such mappings.

Now we set $U_i = \exp D_i$, and get that $T_i U_i \subseteq S + N^{i+1}$ as in Theorem 1. Finally, we put $U = U_0 U_1 \cdots U_{k-1}$, where $N^k = 0$, and use the Campbell-Hausdorff formula to complete the proof of the
Theorem.

As in the Jordan case, we have the following two corollaries of Theorem 2.

**Corollary 1.** Let $A$ be a finite-dimensional alternative algebra over a field of characteristic 0. Let $G$ be a finite group of automorphisms and antiautomorphisms of $A$. Let $S$ and $T$ be $G$-invariant maximal semisimple subalgebras of $A$. Then $S$ and $T$ are strictly conjugate via an automorphism of $A$ of the type described in Theorem 2.

**Corollary 2.** Let $A$ and $G$ be an in Corollary 1. Let $T$ be any $G$-invariant semisimple subalgebra of $A$. Then $T$ is contained in a $G$-invariant maximal semisimple subalgebra of $A$.

5. The fully reducible case. Let $A$ be a finite-dimensional Jordan or alternative algebra over a field of characteristic zero. If $G$ is a fully reducible group of automorphisms and antiautomorphisms of $A$, then it follows from [4] that $G$ will leave invariant a maximal semisimple subalgebra of $A$. The analogue of Corollaries 1 has not been answered as yet for this case. However, if $N^2 = 0$, then any automorphism of the form described in the proofs of Theorems 1 and 2 which carries a $G$-invariant maximal semisimple subalgebra $T$ onto another one, $S$, is unique, and hence will commute pointwise with $G$.

For let $U_1 = \exp D_1, U_2 = \exp D_2$ be of this form and both map $T$ onto $S$. Then $D_1^2 = D_2^2 = 0$, so that $U_1 = I + D_1, U_2 = I + D_2$. If $t \in T$, then $tU_1 = t + tD_1 \in S$ and $tU_2 = t + tD_2 \in S$. Hence their difference $tD_1 - tD_2 \in S \cap N = 0$, since $D_1$ and $D_2$ have range in $N$. Hence $D_1 = D_2$ on $T$. Also $D_1$ and $D_2$ are both 0 on $N$ since $N^2 = 0$. Hence $D_1 = D_2$ since $A = T + N$.

Now let $g \in G$. Then $g^{-1}U_1g = I + g^{-1}D_1g$ will map $T$ onto $S$ and $g^{-1}D_1g$ is a derivation of square zero having range in $N$. Hence, by the above, $g^{-1}D_1g = D_1$, that is, $D_1$, and hence $U_1$, commutes pointwise with $G$.

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<td>Correction to: On continuity of multiplication in a complemented algebra</td>
<td>1474</td>
</tr>
<tr>
<td>Basil Gordon</td>
<td>Correction to: A generalization of the coset decomposition of a finite group</td>
<td>1474</td>
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