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**INVARIANT SPLITTING IN JORDAN AND ALTERNATIVE
ALGEBRAS**

EARL J. TAFT

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Let A be a finite-dimensional Jordan or alternative algebra over a field F of characteristic 0. Let N denote the radical of A . Then A possesses maximal semisimple subalgebras isomorphic to A/N , [5], [6], any two of which are strictly conjugate, [2], [9]. If G is a finite group of automorphisms and antiautomorphisms of A , then A possesses G -invariant maximal semisimple subalgebras, [10]. We investigate here the uniqueness question for such G -invariant maximal semisimple subalgebras. The result is that the strict conjugacy can be chosen to commute pointwise with G and to be in the enveloping associative algebra generated by the right and left multiplications in A .

Similar results have been obtained for associative algebras, [11], and Lie algebras, [12]. However, in the associative case, the conjugacy can be obtained in terms of adjoints of G -symmetric elements, i.e., elements left fixed by the automorphisms in G and sent into their negatives by the antiautomorphisms in G . In the Lie algebra case, one needs only to consider automorphisms, and the conjugacy is obtained in terms of adjoints of fixed points of G . In each case, the conjugacy is in the enveloping associative algebra of A . In both the Jordan and alternative cases, the automorphisms which occur would commute pointwise with G if the elements of A which occur in their formulation in terms of right and left multiplications were to be fixed points of G . However, we have not obtained the conjugacies in this form, and it seems to be an open question whether or not it is always possible to do so.

If G is assumed fully reducible, instead of finite, then A will also possess G -invariant maximal semisimple subalgebras. This is noted in the Jordan case in [4] when G contains only automorphisms, and the same proof can be extended to cover the alternative case, even if G also contains antiautomorphisms. We have answered the uniqueness question for the similar situation in the associative and Lie cases, [13]. For the Jordan and alternative case, the problem seems more complicated. We note here that it is easily answered if $N^2 = 0$, with the strict conjugacy commuting pointwise with G . However, the general question remains open.

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2. **Preliminaries.** If $a \in A$, we let R_a and L_a stand for right and left multiplication by a , i.e., $xR_a = xa$, $xL_a = ax$. The following two lemmas are easily proved by straightforward calculation.

LEMMA 1. *Let g be an automorphism of A . Then $g^{-1}R_ag = R_{ag}$ and $g^{-1}L_ag = L_{ag}$.*

LEMMA 2. *Let g be an antiautomorphism of A . Then $g^{-1}R_ag = L_{ag}$, $g^{-1}L_ag = R_{ag}$.*

A derivation of A will be called inner if it is in the enveloping Lie algebra generated by the right and left multiplications in A , [7]. We will have occasion to use the following types of inner derivations. If A is Jordan, and $x, s \in A$, then $[R_x, R_s] = R_xR_s - R_sR_x$ is an inner derivation of A which, for $x \in N$, will be a nilpotent element of the radical of the enveloping associative algebra generated by multiplications in A by elements of A , [1], [2], [8]. If A is alternative, and $s, x \in A$, then $D_{s,x} = [R_s, R_x] + [L_s, R_x] + [L_s, L_x]$ is an inner derivation of A which, for $x \in N$, will be a nilpotent element of the radical of the enveloping associative algebra generated by the left and right multiplications of A , [7], [9].

LEMMA 3. *If A is alternative, $a, b \in A$, then $[R_a, L_b] = [L_a, R_b]$, and $D_{a,b} = -D_{b,a}$.*

Proof. $x[R_a, L_b] = b(xa) - (bx)a = -(b, x, a)$, where $(b, x, a) = (bx)a - b(xa)$ is the associator of b, x , and a . Also $x[L_a, R_b] = (ax)b - a(xb) = (a, x, b)$. The first part of Lemma 3 follows from the skew-symmetry of the associator function. Hence

$$\begin{aligned} D_{b,a} &= [R_b, R_a] + [L_b, R_a] + [L_b, L_a] \\ &= -[R_a, R_b] - [R_a, L_b] - [L_a, L_b] \\ &= -[R_a, R_b] - [L_a, R_b] - [L_a, L_b] = -D_{a,b}. \end{aligned}$$

LEMMA 4. *Let A be Jordan, and g an automorphism of A . Then $g^{-1}[R_a, R_b]g = [R_{ag}, R_{bg}]$.*

This is immediate from Lemma 1.

LEMMA 5. *Let A be alternative, and g an automorphism or antiautomorphism of A . Then $g^{-1}D_{a,b}g = D_{ag,bg}$.*

Proof. This is clear from Lemma 1 if g is an automorphism. Let g be an antiautomorphism. Then, using Lemma 2, $g^{-1}D_{a,b}g = [L_{ag}, L_{bg}] + [R_{ag}, L_{bg}] + [R_{ag}, R_{bg}] = D_{ag,bg}$ by Lemma 3.

If D is a nilpotent derivation of A , then $\exp D = I + D + (D^2/2!) + \dots$ is an automorphism of A . We assume familiarity with the Campbell-Hausdorff formula, [3], $(\exp D_1)(\exp D_2) = \exp D_3$, where D_3 is in the Lie algebra generated by D_1 and D_2 .

3. The Jordan case.

THEOREM 1. *Let A be a finite-dimensional Jordan algebra over a field F of characteristic 0. Let G be a finite group of automorphisms of A . Let S be a G -invariant maximal semisimple subalgebra of A . Let T be a G -invariant semisimple subalgebra of A . Then there exists an automorphism $U = \exp D$ of A such that*

- (1) U maps T into S ,
- (2) D (and hence U) commutes pointwise with G ,
- (3) D is a nilpotent inner derivation of A which is in the radical of the enveloping associative algebra of A .

Proof. Let N denote the radical of A . Let s and n denote the projections of the vector space $A = S \oplus N$ onto S and N respectively. Then s and n are linear mappings such that

- (i) $s(t_1 t_2) = s(t_1) s(t_2)$
- (ii) $n(t_1 t_2) = s(t_1) n(t_2) + n(t_1) s(t_2) + n(t_1) n(t_2)$
- (iii) $s(tg) = s(t)g, \quad n(tg) = n(t)g$

for $t_1, t_2, t \in T, g \in G$.

(i) and (ii) follow since N is an ideal. (iii) follows from the invariance of T, S and N under G .

Now set $N_1 = N, N_i = N_{i-1}^2 + AN_{i-1}$. By [5], the N_i form a nonincreasing sequence of ideals terminating in 0. Now $T_1 = T \subseteq A = S + N_1$. Suppose that we have found automorphisms $U_0 = \exp 0, U_1 = \exp D_1, \dots, U_{i-1} = \exp(D_{i-1})$ of A satisfying (2) and (3) of Theorem 1 such that $T_i = TU_0U_1 \dots U_{i-1} \subseteq S + N_i$. Then we will show that there exists an automorphism U_i of A satisfying (2) and (3) of Theorem 1 such that $T_i U_i \subseteq S + N_{i+1}$. Hence if $N_k = 0$, then $U = U_0 U_1 \dots U_{k-1}$ will be the desired automorphism by the Campbell-Hausdorff formula.

Now T_i is a G -invariant semisimple subalgebra of A , so that (i), (ii), (iii) hold for $t_1, t_2, t \in T_i$. Consider the space $N_i | N_{i+1}$. We consider this as a T_i -module by defining $t \cdot \bar{n} = \overline{n \cdot t} = \overline{ns(t)}$ for $n \in N_i, t \in T_i$. Then by (ii), we have

$$(iv) \quad \overline{n(t_1 t_2)} = \overline{n(t_1)} \cdot t_2 + t_1 \cdot \overline{n(t_2)}.$$

(iv) says that the map $t \rightarrow \overline{n(t)}$ is a derivation of T_i into the module $N_i | N_{i+1}$. Hence, by [2], there exist elements x_1, \dots, x_p in $N_i, t_1, \dots, t_p \in T_i$ such that

$$(v) \quad \overline{n(t)} = \sum_{j=1}^p ((\bar{x}_j \cdot t) \cdot t_j - \bar{x}_j \cdot (tt_j)) \text{ for } t \in T_i \text{ i.e.,}$$

$$\overline{n(t)} = \sum_{j=1}^p \overline{(x_j s(t))s(t_j)} - \overline{x_j s(t t_j)} .$$

Using (i), we have

$$(vi) \quad n(t) \equiv s(t) \sum_{j=1}^p [R_{x_j}, R_{s(t_j)}] \pmod{N_{i+1}} \text{ for } t \in T_i .$$

Let $g \in G$. Then

$$[R_{x_j g}, R_{s(t_j)g}] = g^{-1}[R_{x_j}, R_{s(t_j)}]g$$

by Lemma 4. Hence

$$\begin{aligned} s(t) \sum_{j=1}^p [R_{x_j g}, R_{s(t_j)g}] &= s(t)g^{-1} \left(\sum_{j=1}^p [R_{x_j}, R_{s(t_j)}] \right) g \\ &= s(tg^{-1}) \left(\sum_{j=1}^p [R_{x_j}, R_{s(t_j)}] \right) g \equiv n(tg^{-1})g = n(t) \pmod{N_{i+1}} . \end{aligned}$$

It follows that if we set $D_i = -(1/m) \sum_{g \in G} (\sum_{j=1}^p [R_{x_j g}, R_{s(t_j)g}])$, where m is the order of G , then

$$(vii) \quad n(t) \equiv -s(t)D_i \pmod{N_{i+1}} \quad \text{for } t \in T_i .$$

Now D_i clearly satisfies (3) of the Theorem, since the $x_j g \in N_i$. To see that D_i satisfies (2) of the Theorem, we fix a value of j . Then $\sum_{g \in G} [R_{x_j g}, R_{s(t_j)g}] = \sum_{g \in G} g^{-1}[R_{x_j}, R_{s(t_j)}]g$ clearly commutes pointwise with G . Hence so does D_i , which is a linear combination of such mappings.

Finally, set $U_i = \exp D_i$. If $t \in T_i$, then $tU_i = t + tD_i + (t/2)D_i^2 + \dots = s(t) + n(t) + s(t)D_i + n(t)D_i + (t/2)D_i^2 + \dots$.

Now $n(t) \in N_i$, so that $n(t)D_i \in N_{i+1}$. Also, since the $x_1, \dots, x_p \in N_i$, we have that $(t/2)D_i^2 + \dots \in N_{i+1}$. Therefore

$$\begin{aligned} tU_i &\equiv s(t) + n(t) + s(t)D_i \pmod{N_{i+1}} \\ &\equiv s(t) \pmod{N_{i+1}} \text{ by (vii).} \end{aligned}$$

Hence $T_i U_i \subseteq S + N_{i+1}$. This completes the proof of the Theorem.

COROLLARY 1. *Let A be a finite-dimensional Jordan algebra over a field of characteristic 0. Let G be a finite group of automorphisms of A . Let S and T be G -invariant maximal semisimple subalgebras of A . Then S and T are strictly conjugate via an automorphism of A of the type described in Theorem 1.*

COROLLARY 2. *Let A and G be as in Corollary 1. Let T be any G -invariant semisimple subalgebra of A . Then T is contained in a G -invariant maximal semisimple subalgebra of A .*

Corollary 1 is an immediate consequence of Theorem 1. Corollary 2 follows from the existence of a G -invariant maximal semisimple

subalgebra S of A . For then if U is an automorphism of A which maps T into S , and which commutes with G pointwise, it follows that SU^{-1} is a G -invariant maximal semisimple subalgebra of A which contains T .

4. The alternative case.

THEOREM 2. *Let A be a finite-dimensional alternative algebra over a field F of characteristic 0. Let G be a finite group of automorphisms and antiautomorphism of A . Let S be a G -invariant maximal semisimple subalgebra of A . Let T be a semisimple subalgebra of A . Then there exists an automorphism $U = \exp D$ of A such that*

- (1) U maps T into S ,
- (2) D (and hence U) commutes pointwise with G ,
- (3) D is a nilpotent inner derivation of A which is in the radical of the enveloping associative algebra of A .

Proof. The proof is similar to Theorem 1. We define s and n as in Theorem 1, but use $N_i = N^i$ instead. We consider $N^i | N^{i+1}$ as a two-sided T_i -module by $t \cdot \bar{n} = \overline{s(t)n}$ and $\bar{n} \cdot t = \overline{ns(t)}$. Then (i), (ii), (iii) and (iv) are valid. Hence, by [9], there exist elements $x_1, \dots, x_p \in N^i$ and $t_1, \dots, t_p \in T_i$ such that

$$(v) \quad \overline{n(t)} = t \sum_{j=1}^p D_{t_j, \bar{x}_j} \quad \text{for } t \in T_i$$

where D_{t_j, \bar{x}_j} is the inner derivation $[R_{t_j}, R_{\bar{x}_j}] + [L_{t_j}, R_{\bar{x}_j}] + [L_{t_j}, L_{\bar{x}_j}]$ of T_i into its two-sided module $N^i | N^{i+1}$. As in Theorem 1, we obtain

$$(vi) \quad n(t) \equiv s(t) \sum_{j=1}^p D_{s(t_j), x_j} \pmod{N^{i+1}} \text{ for } t \in T_i,$$

where $D_{s(t_j), x_j}$ is the inner derivation $[R_{s(t_j)}, R_{x_j}] + [L_{s(t_j)}, R_{x_j}] + [L_{s(t_j)}, L_{x_j}]$ of A .

Now let $g \in G$. Then by Lemma 5, we have $g^{-1}(D_{s(t_j), x_j})g = D_{s(t_j)g, x_jg}$. Hence, for any $g \in G$, $s(t) \sum_{j=1}^p D_{s(t_j)g, x_jg} = s(t)g^{-1}(\sum_{j=1}^p D_{s(t_j), x_j})g = s(tg^{-1})(\sum_{j=1}^p D_{s(t_j), x_j})g \equiv n(t) \pmod{N^{i+1}}$ by (iii) and (v).

Now set $D_i = -(1/m) \sum_{g \in G} (\sum_{j=1}^p D_{s(t_j)g, x_jg})$, where m is the order of G . Then we have

$$(vii) \quad n(t) \equiv -s(t)D_i \pmod{N^{i+1}} \text{ for } t \in T_i.$$

D_i satisfies (3) of the Theorem since the $x_jg \in N$. To see that D_i satisfies (2) of the Theorem, we fix a value of j . Then $\sum_{g \in G} D_{s(t_j)g, x_jg} = \sum_{g \in G} g^{-1}D_{s(t_j), x_j}g$ commutes pointwise with G . Hence so does D_i , which is a linear combination of such mappings.

Now we set $U_i = \exp D_i$, and get that $T_i U_i \subseteq S + N^{i+1}$ as in Theorem 1. Finally, we put $U = U_0 U_1 \dots U_{k-1}$, where $N^k = 0$, and use the Campbell-Hausdorff formula to complete the proof of the

Theorem.

As in the Jordan case, we have the following two corollaries of Theorem 2.

COROLLARY 1. *Let A be a finite-dimensional alternative algebra over a field of characteristic 0. Let G be a finite group of automorphisms and antiautomorphisms of A . Let S and T be G -invariant maximal semisimple subalgebras of A . Then S and T are strictly conjugate via an automorphism of A of the type described in Theorem 2.*

COROLLARY 2. *Let A and G be as in Corollary 1. Let T be any G -invariant semisimple subalgebra of A . Then T is contained in a G -invariant maximal semisimple subalgebra of A .*

5. The fully reducible case. Let A be a finite-dimensional Jordan or alternative algebra over a field of characteristic zero. If G is a fully reducible group of automorphisms and antiautomorphisms of A , then it follows from [4] that G will leave invariant a maximal semisimple subalgebra of A . The analogue of Corollaries 1 has not been answered as yet for this case. However, if $N^2 = 0$, then any automorphism of the form described in the proofs of Theorems 1 and 2 which carries a G -invariant maximal semisimple subalgebra T onto another one, S , is unique, and hence will commute pointwise with G .

For let $U_1 = \exp D_1$, $U_2 = \exp D_2$ be of this form and both map T onto S . Then $D_1^2 = D_2^2 = 0$, so that $U_1 = I + D_1$, $U_2 = I + D_2$. If $t \in T$, then $tU_1 = t + tD_1 \in S$ and $tU_2 = t + tD_2 \in S$. Hence their difference $tD_1 - tD_2 \in S \cap N = 0$, since D_1 and D_2 have range in N . Hence $D_1 = D_2$ on T . Also D_1 and D_2 are both 0 on N since $N^2 = 0$. Hence $D_1 = D_2$ since $A = T + N$.

Now let $g \in G$. Then $g^{-1}U_1g = I + g^{-1}D_1g$ will map T onto S and $g^{-1}D_1g$ is a derivation of square zero having range in N . Hence, by the above, $g^{-1}D_1g = D_1$, that is, D_1 , and hence U_1 , commutes pointwise with G .

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