

# Pacific Journal of Mathematics

**CORRECTION TO: NON-LINEAR DIFFERENTIAL EQUATIONS  
ON CONES IN BANACH SPACES**

CHARLES VERNON COFFMAN

shall leave the matter so. A similar remark applies to Theorem 4.2.

### REFERENCE

1. John Lamperti and Patrick Suppes, 'Chains of infinite order and their application to learning theory,' Pacific J. Math. **9** (1959), 739-754.

Correction to

## NON-LINEAR DIFFERENTIAL EQUATIONS ON CONES IN BANACH SPACES

CHARLES V. COFFMAN

Volume 14 (1964), 9-15

In [1] the proof of a main lemma, Lemma 3.1, contains an error. The lemma itself is false without stronger hypotheses. The purpose of this note is to state and prove a lemma which can be used in place of Lemma 3.1 in the proofs of Theorem 4.1 and 5.1 in [1].

Let  $Y$  be a Banach space, let  $I$  be a closed linear manifolds in  $Y^*$  which is total for  $Y$ .<sup>1</sup> Assume that  $I$  is some real interval. The differential equation with which [1] is concerned is

$$(1) \quad dy/dt = f(t, y),$$

where  $f$  is a function from  $I \times C \rightarrow Y$  which is continuous with respect to the weak  $I$ -topology on  $Y$ ;  $C$  is a subset of  $Y$ . The notation and terminology used here will be the same as that employed in [1]; the definition of a weak  $I$ -derivative, a weak  $I$ -solution of (1), etc., are to be found in [1].

Let  $\mathcal{C}$  be the space of weakly  $I$ -continuous functions on  $I$  with values in  $C$ , furnished with the topology of uniform convergence (in the weak  $I$ -topology) on compact subintervals of  $I$ . If  $C$  is compact in the weak  $I$ -topology, then Ascoli's theorem implies that a set of equicontinuous functions in  $\mathcal{C}$  is relatively compact in  $\mathcal{C}$ . However unless the topology on  $\mathcal{C}$  satisfies the first axiom of countability one cannot conclude from Ascoli's theorem, as is done in [1], that an equicontinuous sequence of functions in  $\mathcal{C}$  has a convergent subsequence. ( $\mathcal{C}$  will satisfy the first axiom of countability, for example,

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Received March 3, 1965.

<sup>1</sup> In [1] a total manifold is defined but is incorrectly called a determining manifold. The author wishes to thank the referee of this note for pointing out this mistake as well as for correcting an omission in the original proof of the lemma stated here.

if  $C$  is bounded and  $\Gamma$  is separable in its norm topology.)

Let  $Y, \Gamma, C, I$  and  $\mathcal{E}$  be as above, the following Lemma can be used in place of Lemma 3.1 of [1] in the proofs of Theorem 4.1 and 5.1.

**LEMMA.** *Let  $\{y_n(t)\}$  be a sequence of weakly  $\Gamma$ -continuous functions defined on  $I$  with values in  $C$ . Let  $C$  be compact in the weak  $\Gamma$ -topology. For each neighborhood  $V$  of 0 in  $Y$ , in the weak  $\Gamma$ -topology, and for each compact subinterval  $I'$  of  $I$ , let there exist an  $N = N(V, I')$  such that for all  $n \geq N$ ,  $y_n(t)$  is a  $V$ -approximate weak  $\Gamma$ -solution of (1) on  $I'$ . Then, in  $\mathcal{E}$ , the sequence  $\{y_n(t)\}$  has a cluster point  $y_0(t)$  and  $y_0(t)$  is a weak  $\Gamma$ -solution of (1) on  $I$ .*

*Proof.* As is shown in the proof of Lemma 3.1 in [1], the sequence  $\{y_n(t)\}$  is equicontinuous in the weak  $\Gamma$ -topology on  $Y$ , thus it follows from Ascoli's theorem that the sequence  $\{y_n(t)\}$  has a cluster point  $y_0(t)$  in  $\mathcal{E}$ . To complete the proof it will be shown that given  $\gamma \in \Gamma$ , there exists a subsequence  $\{y_{n_k}(t)\}$  of the original sequence such that

$$(2) \quad \gamma(y_{n_k}(t)) \rightarrow \gamma(y_0(t)) \quad \text{as } k \rightarrow \infty ,$$

and

$$(3) \quad \gamma(f(t, y_{n_k}(t))) \rightarrow \gamma(f(t, y_0(t))) \quad \text{as } k \rightarrow \infty ,$$

uniformly on compact subintervals of  $I$ . To this end let  $\{I_k\}$  be an expanding sequence of compact intervals whose union is  $I$ . Since  $f(t, y)$  is uniformly continuous on  $I_k \times C$  for each  $k$ , there is a neighborhood  $V_k$  of 0 such that  $|\gamma(f(t, y'(t)) - f(t, y_0(t)))| < (1/k)$  on  $I_k$  for any function  $y'(t)$  with  $y'(t) - y_0(t) \in V_k$  on  $I_k$ . Let

$$V\left[\gamma, \frac{1}{k}\right] = \left\{y \in Y : |\gamma(y)| < \frac{1}{k}\right\}, \quad \text{if } V'_k = V_k \cap V\left[\gamma, \frac{1}{k}\right]$$

then for each  $k$  it is possible to choose an element  $\{y_{n_k}(t)\}$  of the original sequence such that  $y_{n_k}(t) - y_0(t) \in V'_k$  on  $I_k$ . It easily follows that a subsequence selected in this manner satisfies (2) and (3), and the limits are uniform on compact subintervals of  $I$ . Finally since the hypothesis implies that

$$\gamma(D_\Gamma y_{n_k}(t) - f(t, y_{n_k}(t))) \rightarrow 0, \quad \text{as } k \rightarrow \infty ,$$

uniformly on compact subintervals of  $I$ , it follows from (2) and (3) that

$$(4) \quad \gamma(y_0(t_1) - y_0(t_0)) = \int_{t_0}^{t_1} \gamma(f(t, y_0(t))) dt, \quad t_1, t_0 \in I.$$

As  $\gamma$  was arbitrary, (4) holds for each  $\gamma \in \Gamma$ , consequently  $D_\Gamma y_0(t)$

exists on  $I$  and  $y_0(t)$  is a weak  $I$ -solution of (1) on  $I$ .

#### REFERENCES

1. C. V. Coffman, *Non-linear differential equations on cones in Banach spaces*, Pacific J. Math. **14** (1964), 9-15.

CARNEGIE INSTITUTE OF TECHNOLOGY

Correction to

### A SUFFICIENT CONDITION THAT AN ARC IN $S^n$ BE CELLULAR

P. H. DOYLE

Volume 14 (1964), 501-503

In Corollary 1 add to the hypothesis: each subarc of  $A_2$  is  $p$ -shrinkable.

Correction to

### ON CONTINUITY OF MULTIPLICATION IN A COMPLEMENTED ALGEBRA

PARFENY P. SAWOROTNOW

Volume 14 (1964), 1399-1403

Page 1400, line 6 from the bottom: Should read  $\|R_x\|$  instead of  $\|R\|$ .

Page 1401, line 15: Should read  $|\lambda - \lambda_0| \|y_{\lambda_0} x\| < 1$  instead of  $|\lambda - \lambda_0| \|y_{\lambda_0} x\| < 1$ .

Correction to

### A GENERALIZATION OF THE COSET DECOMPOSITION OF A FINITE GROUP

BASIL GORDON

Volume 15 (1965), 503-509

Page 508, line 15: Change §2 to read §3.

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The *Pacific Journal of Mathematics* is published quarterly, in March, June, September, and December. Effective with Volume 13 the price per volume (4 numbers) is \$18.00; single issues, \$5.00. Special price for current issues to individual faculty members of supporting institutions and to individual members of the American Mathematical Society: \$8.00 per volume; single issues \$2.50. Back numbers are available.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley 8, California.

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), No. 6, 2-chome, Fujimi-cho, Chiyoda-ku, Tokyo, Japan.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

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\* Basil Gordon, Acting Managing Editor until February 1, 1966.

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