Pacific Journal of Mathematics

ERRATUM: "UNIQUENESS AND EXISTENCE PROPERTIES OF BOUNDED OBSERVABLES"

Stanley P. Gudder

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should be replaced by $\overset{\wr}{z}$, and $\tilde{Z}$ should be replaced by $\overset{\wr}{Z}$. The symbols $\tilde{\mathfrak{A}}_m$ and $\tilde{\mathfrak{A}}^0_m$ should be replaced throughout by $\overset{\wr}{\mathfrak{A}}_m$ and $\overset{\wr}{\mathfrak{A}}^0_m$, respectively; however, $\tilde{\mathfrak{A}}_n$ and $\tilde{\mathfrak{A}}^0_n$ remain unchanged. The first equation of line 14 page 235 should be '$\overset{\wr}{\mathfrak{A}}_n = \tilde{\mathfrak{A}}'_n$."

Correction to

DUALITY AND TYPES OF COMPLETENESS IN LOCALLY CONVEX SPACES

WILLIAM B. JONES

Volume 18 (1966), 525–544

Proposition 2.14 is an obvious consequence of Lemma 2.8.

p. 538, line 5: The second equality is false in general for all α (see [4]).

Some misprints:

p. 526	§ 2 should start "$(\alpha, \beta) - \cdots$"
	line 3 of § 2, "α" instead of "a"
p. 528	last line, remove final "}"
p. 532	line 14, second "ε" should be "$\in$"
p. 535	line 2, should read $\cdots \leqq \dfrac{\varepsilon}{r}(r - \cdots$
p. 537	line 8, second "$=$" should be "$-$"
p. 541	line 9, "λ_0" instead of "1_0"

Correction to

UNIQUENESS AND EXISTENCE PROPERTIES OF BOUNDED OBSERVABLES

S. P. GUDDER

Volume 19 (1966), 81–93

The author recently discovered that the proof of the corollary to Theorem 4.5 is incorrect, thus invalidating Theorem 4.6. We show now that Theorem 4.6 is still true for a class of observables with infinite spectra and prove a generalization of Theorem 4.5.

An observable x is *semi-bounded above* (*below*) if there is a number

$-\infty < c < \infty$ such that $\sigma(x) \subset \{\lambda : \lambda \leqq c\}$ $(\sigma(x) \subset \{\lambda : \lambda \geqq c\})$. The following not only generalizes Theorem 4.5 but gives a much simpler proof.

THEOREM 1.1. *Let x and y be observables on a quite full logic which are semi-bounded above and suppose that $m(x)$ exists if and only if $m(y)$ exists and in that case $m(x) = m(y)$. Then* $\lambda_0 = \max\{\lambda : \lambda \in \sigma(x)\} = \max\{\lambda : \lambda \in \sigma(y)\}$ *and* $x(\lambda_0) = y(\lambda_0)$.

Proof. The first part of the conclusion follows just as in Theorem 4.5. Now suppose $m[x(\lambda_0)] = 1$, and $m[y(\lambda_0)] \neq 1$. Then there is a number $\mu < \lambda_0$ such that $m[y(-\infty, \mu)] > 0$. Now since $m(x)$ exists, so does $m(y)$ and we have

$$\begin{aligned}\lambda_0 = m(x) = m(y) &= \int_{(-\infty,\lambda_0]} \lambda m[y(d\lambda)] = \left(\int_{(-\infty,\mu)} + \int_{[\mu,\lambda_0]}\right)\lambda m[y(d\lambda)] \\ &\leqq \mu m[y(-\infty, \mu)] + \lambda_0 m[y[\mu, \lambda_0)] < \lambda_0 .\end{aligned}$$

which is a contradiction. Thus $m[y(\lambda_0)] = 1$ whenever $m[x(\lambda_0)] = 1$ and hence $x(\lambda_0) \leqq y(\lambda_0)$. By symmetry $x(\lambda_0) = y(\lambda_0)$.

Of course the same result holds for observables which are semi-bounded from below.

THEOREM 1.2. *Let x and y be bounded observables on a quite full logic and suppose the spectrum of x has at most one limit point. If $m(x) = m(y)$ for all $m \in M$ then $x = y$.*

Proof. The most general such x has a point $\lambda_0 \in \sigma(x)$ which is a limit point from both above and below of elements of $\sigma(x)$. The other cases will follow in a similar manner. We can assume without loss of generality that $\lambda_0 = 0$. Let the points of $\sigma(x)$ be ordered as follows: $\mu_1 < \mu_2 < \cdots < \lambda_0 < \cdots < \lambda_2 < \lambda_1$. Now by Theorem 1.1 $\max\{\lambda : \lambda \in \sigma(y)\} = \lambda_1$ and $y(\lambda_1) = x(\lambda_1)$. Now let $x_1 = x - \lambda_1\chi_{\lambda_1}(x)$ and let $y_1 = y - \lambda_1\chi_{\lambda_1}(y)$. Letting f be the identity function $f(\lambda) = \lambda$ we have for $E \in B(R)$

$$\begin{aligned}x_1(E) &= (f - \lambda_1\chi_{\lambda_1})(x)(E) = x[(f - \lambda_1\chi_{\lambda_1})^{-1}(E)] \\ &= \begin{cases} x(E) \wedge x(\lambda_1)' & \text{if } 0 \notin E \\ x(E) \vee x(\lambda_1) & \text{if } 0 \in E \end{cases} \qquad \cdots\cdots (1).\end{aligned}$$

It is now easy to see that

$$\sigma(x_1) = \sigma(x) \cap \{\lambda_1\}'; \; x_1(\lambda_i) = x\lambda_i), \; i = 2, 3, \cdots ;$$

and

$$x_1(\mu_i) = x(\mu_i), \; i = 1, 2, \cdots .$$

Now

$$m(x_1) = m(x) - \lambda_1 m[x(\lambda_1)] = m(y) - \lambda_1 m[y(\lambda_1)] = m(y_1) .$$

Applying Theorem 1.1, $\lambda_2 = \max\{\lambda : \lambda \in \sigma(y_1)\}$ and $y_1(\lambda_2) = x_1(\lambda_2) = x(\lambda_2)$. It now follows by applying (1) to y_1 and y that λ_2 is the second largest number in $\sigma(y)$ and $y(\lambda_2) = y_1(\lambda_2) = x(\lambda_2)$. Continuing this process with the λ_i's and also the μ_i's we have $\{\lambda_i, \mu_i : i = 1, 2, \cdots\} \subset \sigma(y)$ and $y(\lambda_i) = x(\lambda_i)$, $y(\mu_i) = x(\mu_i)$, $i = 1, 2, \cdots$. Since λ_0 is a limit point of the λ_i's it follows that $\lambda_0 \in \sigma(y)$, $\{\lambda_i, \mu_i : i = 1, 2, \cdots\} = \sigma(y)$ and

$$\begin{aligned} y(\lambda_0) &= y(\{\lambda_i, \mu_i : i = 1, 2, \cdots\}') = [\Sigma y(\lambda_i) + \Sigma y(\mu_i)]' \\ &= [\Sigma x(\lambda_i) + \Sigma x(\mu_i)]' = x(\lambda_0) . \end{aligned}$$

Hence $y = x$.

A similar technique may be used to prove:

COROLLARY 1.3. *Let x and y be observables on a quite full logic which are semi-bounded from above (below) and suppose the spectrum of x has no finite limit point (this includes the possibility of a limit point at $-\infty(+\infty)$). Suppose $m(y)$ exists if and only if $m(x)$ exists and in that case $m(y) = m(x)$. Then $x = y$.*

We close with a slightly strengthened form of Lemma 6.2 [1].

LEMMA 1.4. *If L is quite full and has Property E, then L is a lattice and $m(a) = m(b) = 1$ implies $m(a \wedge b) = 1$.*

Proof. That L is a lattice follows from Lemma 6.2 [1]. If $m(a) = m(b) = 1$, then $m(x_a + x_b) = m(a) + m(b) = 2$ and hence $1 = m[(x_a + x_b)\{2\}] = m(a \wedge b)$.

This last lemma is of interest since it rules out the counter-example of Section 5 [1] and is thus a possible sufficient condition for Property E.

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