

Pacific Journal of Mathematics

ON A THEOREM BY HOFFMAN AND RAMSAY

STELIOS A. NEGREPONTIS

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Hoffman and Ramsay recently constructed, using the continuum hypothesis, a proper subalgebra of the algebra $C(\beta N)$ of all complex-valued continuous functions on βN which is uniformly closed and separates points of βN . (This example settles in the negative a conjecture by Badé and Curtis). The purpose of this note is to indicate that this result does not need the continuum hypothesis.

Their construction, in [3], naturally falls into two parts, the first of which is devoted to the proof that the space X of maximal ideals of the algebra L^∞ of all essentially bounded measurable complex-valued functions on the unit circle is homeomorphic to a subset of $\beta N - N$. It is in this part that they are forced to use the continuum hypothesis.

1. We have a simple proof of this fact without appeal to the continuum hypothesis.

THEOREM. *The space X is homeomorphic to a subset of $\beta N - N$.*

Proof. We first show that there is a compact separable space S in which X is embedded. This can be seen in at least two distinct ways. By a theorem in [2, p.174] we can regard X as a subset of $\mathcal{M}(H^\infty)$, the space of maximal ideals of H^∞ . (For a definition of H^∞ , the reader should consult [2]). According to the "Corona" conjecture proved by Carleson, the open unit disc is dense in $\mathcal{M}(H^\infty)$, and thus, $\mathcal{M}(H^\infty)$ is a separable space. Thus, we may take $S = \mathcal{M}(H^\infty)$. An alternative, and more elementary, way is to notice that $L^\infty = C(X)$, a fact proved in [2, p.170], and hence that $C(X)$ has cardinality $\mathcal{C} = 2^{\aleph_0}$; but this clearly implies that we can embed X in the compact "cube" $[0, 1]^{\mathcal{C}}$, which according to [4, 3N] is separable. Thus, we may take $S = [0, 1]^{\mathcal{C}}$.

In any case, let $\varphi: N \rightarrow S$ be any mapping onto a dense subset of S . The Stone-Cech extension of φ , denoted by $\bar{\varphi}$, provides a mapping of βN onto S . As it is well-known, using Zorn's lemma, we can find a closed subset A of $\bar{\varphi}^{-1}(X)$ such that $\bar{\varphi}: A \rightarrow X$ is irreducible, i.e. $\bar{\varphi}$ is onto, and $\bar{\varphi}(B) \neq X$ for every proper closed subset B of A . Gleason's lemma [1, 2.3], which applies since X is extremally disconnected, states that $\bar{\varphi}$ is a homeomorphism. We can embed βN into $\beta N - N$, and hence, the theorem is proved.

2. REMARKS. (i) The initial attempt to prove the above theorem, which failed, was the following: Let S^1 be the unit circle in the plane, and let $\sigma : X \rightarrow S^1$ be the natural projection (as described in [2, p.171]). A quick proof of the theorem would follow, if it could be shown that σ were irreducible; but this is not the case.

(ii) The space X is the only example known to the author of an extremally disconnected subspace of βN which is not separable. Surely X is not a retract of βN . It follows that although the projective objects (in the category of compact spaces and continuous maps) are "the retracts of the free objects" (in the terminology of Rainwater [5]), it is not true that a projective object is a retract of every free object in which it is embedded.

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