Let \( \mathcal{F} = (E, p, B) \) be a (Hurewicz) fibre space and let \( \lambda \) be a lifting function for \( \mathcal{F} \). For \( W \) a subset of \( B \), a map \( f: p^{-1}(W) \to E \) is called a fibre map if \( p(e) = p(f(e')) \) implies \( p(f(e)) = p(f(e')) \). Define \( \hat{f}: W \to B \) to be the map such that \( \hat{f}p = pf \). If \([W \cup \hat{f}(W)] \subseteq V \subseteq B\) where \( V \) is pathwise connected, define \( f^\#_\lambda: p^{-1}(b) \to p^{-1}(b') \), for \( b \in W \), by \( f^\#_\lambda(e) = \lambda(f(e), \omega)(1) \) where \( \omega: I \to V \) is a path such that \( \omega(0) = \hat{f}(b) \) and \( \omega(1) = b \). Let \( i \) be a fixed point index defined on the category of compact ANR’s and let \( Q \) denote the rationals. The main result of this paper is:

**Theorem 1.** Let \( \mathcal{F} = (E, p, B) \) be a fibre space such that \( E, B, \) and all the fibres are compact ANR’s. Let \( f: E \to E \) be a fibre map. If \( U \) is an open subset of \( B \) such that \( \hat{f}(b) \neq b \) for all \( b \in bd(U) \) and \( cl[U \cup \hat{f}(U)] \subseteq V \subseteq B \) where \( V \) is open and pathwise connected and \( \mathcal{F}|_V = (p^{-1}(V), p, V) \) is \( Q \)-orientable, then

\[
i(\hat{f}, p^{-1}(V)) = i(f, U). \quad L(f^\#_\lambda)
\]

where \( L(f^\#_\lambda) \) is the Lefschetz number of \( f^\#_\lambda \) for any \( b \in U \).

Independence of \( L(f^\#_\lambda) \). For \( \mathcal{F} = (E, p, B) \) a Hurewicz fibre space with lifting function \( \lambda \) [7] and \( \omega \) a loop in \( B \) based at \( b \), define \( \varphi: p^{-1}(b) \to p^{-1}(b') \) by \( \varphi(e) = \lambda(e, \omega)(1) \). The fibre space \( \mathcal{F} \) is called \( Q \)-orientable if

\[
\varphi^*: H^*(p^{-1}(b); Q) \to H^*(p^{-1}(b'); Q)
\]

is the identity isomorphism for all pairs \((b, \omega)\) where \( b \in B \) and \( \omega \) is a loop in \( B \) based at \( b \).

**Lemma.** Let \( \mathcal{F} = (E, p, B) \) be a \( Q \)-orientable fibre space and let \( \omega_i: I \to B, i = 1, 2, \) be paths such that \( \omega_i(0) = b \) and \( \omega_i(1) = b' \). Define \( \varphi_i: p^{-1}(b) \to p^{-1}(b') \) by \( \varphi_i(e) = \lambda(e, \omega_i)(1) \), then

\[
\varphi_i^* = \varphi_i^*: H^*(p^{-1}(b'); Q) \xrightarrow{\cong} H^*(p^{-1}(b); Q).
\]

**Proof.** By Proposition 2 of [4], each \( \varphi_i \) is a homotopy equivalence with homotopy inverse \( \varphi_i^*: p^{-1}(b') \to p^{-1}(b) \) given by \( \varphi_i^*(e') = \lambda(e', \bar{\omega}_i)(1) \) where \( \bar{\omega}_i(s) = \omega_i(1 - s) \). Therefore, \( \varphi_i^*: H^*(p^{-1}(b'); Q) \to H^*(p^{-1}(b); Q) \) is an isomorphism and \( \psi_i^* = (\varphi_i^*)^{-1} \). Consider \( \omega: I \to B \) defined by

\[
\begin{align*}
\omega(s) = \begin{cases} 
\omega_i(2s) & 0 \leq s \leq 1/2 \\
\bar{\omega}_i(1 - 2s) & 1/2 \leq s \leq 1
\end{cases}
\end{align*}
\]
then $\omega$ is a loop in $B$ based at $b$ and since $\mathcal{F}$ is $Q$-orientable, for $\varphi(e) = \lambda(e, \omega)(1)$, $\varphi^*$ is the identity isomorphism. It follows from [4] that $\varphi$ is homotopic to $\psi_2 \varphi_1$ so $\varphi^* = \varphi_1^* \psi_2^*$ and $\psi_2^* = (\varphi^*_1)^{-1}$. Hence $\psi_2^* = \psi_1^*$ and $\varphi^*_1 = \varphi^*_2$.

**Theorem 2.** Let $\mathcal{F} = (E, p, B)$ be a $Q$-orientable fibre space where $B$ is pathwise connected and $H^*(p^{-1}(b); Q)$ is finitely generated for $b \in B$. For $W \subseteq B$, let $f: p^{-1}(W) \to E$ be a fibre map, then $L(f_b) = L(f_{b'})$ for all $b, b' \in W$, where $f_b$ means $f_b^p$.

**Proof.** Since $f_b = \varphi_i(f | p^{-1}(b))$, the lemma implies that

$$f_b^*: H^*(p^{-1}(b); Q) \to H^*(p^{-1}(b); Q)$$

is independent of the choice of the path $\omega_i$ from $\bar{f}(b)$ to $b$. Let $\omega_0, \omega_1: I \to B$ such that $\omega_i(0) = \bar{f}(b), \omega_i(1) = \omega_i(0) = b$, and $\omega_i(1) = b'$. Define $\omega_2: I \to B$ by

$$\omega_2(s) = \begin{cases} f(\omega_i(2s)) & 0 \leq s \leq 1/2 \\ \omega_i(2s - 1) & 1/2 \leq s \leq 1. \end{cases}$$

We first show that diagram (1) is homotopy commutative, where $\varphi_i(e) = \lambda(e, \omega_i)(1), i = 0, 1, 2$.

$$\begin{array}{cccccc}
\varphi_1 & \downarrow & \varphi_2 \\
p^{-1}(b) & \xrightarrow{(f | p^{-1}(b))} & p^{-1}(\bar{f}(b)) & \xrightarrow{\varphi_0} & p^{-1}(b) \\
p^{-1}(b') & \xrightarrow{(f | p^{-1}(b'))} & p^{-1}(\bar{f}(b')) \\
\end{array}$$

(1)

Define the homotopy $H: p^{-1}(b) \times I \to p^{-1}(b)$ by

$$H(e, t) = \lambda[f(\lambda(e, \omega_i)(1 - t)), \omega^i](1)$$

where

$$\omega^i(s) = \begin{cases} \bar{f}(\omega_i(2s + t)) & 0 \leq s \leq \frac{1 - t}{2} \\ \omega_i\left(\frac{2s + t - 1}{t + 1}\right) & \frac{1 - t}{2} \leq s \leq 1. \end{cases}$$

Then $H(e, 0) = \varphi_2 f \varphi_1(e)$ and $H(e, 1) = \varphi_0 f(e)$ as required. By the lemma and [4], $(f_b^p)^* = (\varphi_2^* \varphi_1(f | p^{-1}(b')))^*$. Furthermore,

$$(\psi_1^* f_b^p \varphi_1)^* = (\psi_1^* \varphi_1(f | p^{-1}(b'))) \varphi_1^* = (\varphi_2(f | p^{-1}(b'))) \varphi_1^* = (\varphi_0(f | p^{-1}(b'))) = f_b^p.$$
vector spaces and \( \varphi^*, f^*, \psi_1^* \) are linear transformations. Pick bases for \( H^k(p^{-1}(b); Q) \) and \( H^k(p^{-1}(b'); Q) \) and let \( \Phi, F'' \), and \( \Psi \) be the matrices with respect to these bases representing \( \varphi^* \), \( f^* \), and \( \psi_1^* \) respectively. Since \( \psi_1^* = (\varphi_1^*)^{-1} \), \( \Psi \Phi = E_n \), the \( n \times n \) identity matrix, where \( n \) is the dimension of \( H^k(p^{-1}(b); Q) \). Therefore, \( \text{trace} (\Phi F'' \Psi) = \text{trace} (F'' \Psi) \) which implies that \( L(f_b) = L(\psi_1 f_b \varphi_1) \). The theorem now follows because \( (\psi_1 f_b \varphi_1)^* = f_b^* \) implies \( L(\psi_1 f_b \varphi_1) = L(f_b) \).

2. Extension of a theorem of Leray. Let \( B \) and \( F \) be topological spaces and let \( (B \times F, \pi^1, B) \) be the trivial fibre space. Suppose \( W \) is a subset of \( B \) and \( f: W \times F \rightarrow B \times F \) is a fibre map. Define \( f_b: F \rightarrow F \) by \( f_b = \pi^1 f j_b \), where \( j_b: F \rightarrow W \times F \) is given by \( j_b(x) = (b, x) \) and \( \pi^1: B \times F \rightarrow F \) is projection. Theorem 3 is a restatement of Theorem 27 of [9] in the somewhat specialized form in which we shall use it.

**Theorem 3 (Leray).** Let \( (B \times F, \pi^1, B) \) be the trivial fibre space where \( B \) and \( F \) are finite polyhedra. For \( U \) an open connected subset of \( B \), let \( f: \text{cl}(U) \times F \rightarrow B \times F \) be a fibre map.\(^1\) If \( \bar{f}(b) \neq b \) for all \( b \in \text{bd}(U) \), then

\[
\tilde{i}(f, U \times F) = \tilde{i}(\bar{f}, U) \cdot L(f_b)
\]

for all \( b \in U \), where \( \tilde{i} \) denotes the Leray fixed point index.

By Theorem 22 and Corollary 26-27 of [9], the Leray index [9, p. 208] satisfies the O'Neill axioms [10, p. 500]. (We will use the formulation of the axioms and the terminology of [1]). Therefore, an index \( i \) for the category of compact ANR’s, satisfying the O’Neill axioms, may be obtained from the index \( \tilde{i} \) in the following manner [2, p. 20]. Let \( X \) be a compact ANR and let \( \alpha \) be a finite open cover of \( X \), then there exists a finite polyhedron \( K \) and maps \( \varphi: X \rightarrow K \), \( \psi: K \rightarrow X \) such that \( \psi \varphi \) is \( \alpha \)-homotopic to the identity map on \( X \), i.e. there exists a map \( H: X \times I \rightarrow X \) such that \( H(x, 0) = x \), \( H(x, 1) = \psi \varphi(x) \), and for each \( x \in X \), the set \( \{H(x, t) | t \in I\} \) lies in a single element of \( \alpha \) [5, Theorem 6.1]. Write \( \psi \varphi \sim_\alpha 1_x \). For \( U \) an open subset of \( X \) and \( f: X \rightarrow X \) a map such that \( f(x) \neq x \) for all \( x \in \text{bd}(U) \), let

\[
i_\alpha(f, U) = \tilde{i}(\varphi f \psi, \psi^{-1}(U)).
\]

Browder [2, Theorem 2, p. 20] showed that there exists a finite open cover \( \kappa_f(U) \) of \( X \) such that if \( \alpha \) is a refinement of \( \kappa_f(U) \), then \( i_\alpha(f, U) \) is well-defined and independent of \( \alpha, \varphi, \) and \( \psi \). Write \( i_\alpha = i \) for all

---

\(^1\) The notation \( \text{cl}(U) \) denotes the closure of \( U \). We use \( \text{bd}(U) \) for the boundary of \( U \).
THEOREM 4. Let \((B \times F, \pi_1, B)\) be the trivial fibre space where \(B\) is a finite polyhedron and \(F\) is a compact ANR. For \(U\) a connected open subset of \(B\), let \(f: \text{cl}(U) \times F \to B \times F\) be a fibre map. If \(\tilde{f}(b) \neq b\) for all \(b \in \text{bd}(U)\), then
\[
i(f, U \times F) = \tilde{i}(\tilde{f}, U) \cdot L(f_b)
\]
for all \(b \in U\).

Proof. Let \(F\) be dominated by a finite polyhedron \(K\) by means of maps \(\varphi: F \to K\) and \(\psi: K \to F\). Define \(f^* : B \times K \to B \times K\) by \(f^*(b, k) = (\tilde{f}(b), \varphi_f \psi(k))\) then \(f^*\) is a fibre map with respect to \((B \times K, \pi_1, B)\) and \(\tilde{f}^* = \tilde{f}\). Since \(\psi \varphi\) is homotopic to the identity map on \(F\), \(L(f^*) = L(f_b)\) (see the proof that \(L(f^*) = L(\psi_f \varphi_b)\) in Theorem 2). Let \(\alpha\) be a finite open cover of \(B\) which refines \(\kappa(\pi(U))\) and, trivially,
\[
i(f, U \times F) = \tilde{i}(f, U \times K).
\]
Therefore, by Theorem 3,
\[
i(f, U \times F) = \tilde{i}(\tilde{f}, U) \cdot L(f_b).
\]

3. Proof of Theorem 1. We first assume that \(B\) is a finite polyhedron. By a theorem of Hopf [6, Theorem 5], given \(\varepsilon > 0\), there exists a map \(g: B \to B\) homotopic to \(\tilde{f}\) by a homotopy \(h: B \times I \to B\) such that \(h(b, 0) = \tilde{f}(b), h(b, 1) = g(b)\) and \(\rho(h(b, t), h(b, t')) < \varepsilon\) for \(b \in B, t, t' \in I\), where \(\rho\) is the metric of \(B\). The map \(g\) has a finite number of fixed points \(b_1, \ldots, b_s\) where, with respect to some barycentric subdivision of \(B\), each \(b_j\) lies in the interior of a different simplex \(\sigma_j\) of \(B\), where \(\sigma_j\) is not a face of any other simplex of \(B\). Since \(\tilde{f}\) has no fixed points on \(\text{bd}(U)\), \(\inf \{\rho(b, \tilde{f}(b)) \mid b \in \text{bd}(U)\} = \varepsilon_1 > 0\). Let \(\varepsilon_2 > 0\) be the distance from \(\text{cl} [U \cup \tilde{f}(U)]\) to \(B - V\) (if \(V = B\), take \(\varepsilon_2 = \infty\)). Let \(\varepsilon = \min(\varepsilon_1, \varepsilon_2)\) then \(h(b, t) \neq b\) for all \(b \in \text{bd}(U)\). Hence \(i(f, U) = i(\tilde{g}, U)\) by the homotopy axiom. Furthermore, \(\text{cl} [U \cup \tilde{g}(U)] \subseteq V\). The homotopy \(h\) induces \(h': B \to B'\). Let \(\lambda\) be regular lifting function for \(\mathcal{F}\) and define \(H': E \to E'\) by
\[
H'(e)(t) = \lambda(f(e), h'(p(e)))(t).
\]
Define \(g: E \to E\) by \(g(e) = H'(e)(1)\) then \(g\) is a fibre map homotopic to \(f\) by a homotopy without fixed points on \(\text{bd}(p^{-1}(U))\) so \(i(f, p^{-1}(U)) = i(g, p^{-1}(U))\). Furthermore, \(pg = \tilde{g}p\). Since \(f_b\) is precisely \(g_{b}\) if we use the path \(h'(b_j)\) to define \(f_{b_j}\) and the constant path to define \(g_{b_j}\),
then $L(f^\gamma_j) = L(g^\gamma_j)$. We have shown that when $B$ is a finite polyhedron, it is sufficient to verify the conclusion for the map $g$.

Let $U_j$ be a $\delta$-neighborhood of $b_j$ where $\delta$ is chosen small enough so that $[\text{cl} (U_j) \cup \bar{g}(\text{cl} (U_j))] \subseteq \sigma_j$. We may contract $\sigma_j$ to $b_j$ so that $b_j$ stays fixed throughout the contraction and such that the restriction to $\text{cl} (U_j)$ contracts $\text{cl} (U_j)$ through itself to $b_j$. The contraction induces fibre homotopy equivalences

$$
\alpha: p^{-1}(\sigma_j) \cong \sigma_j \times F; \beta
$$

$$
\alpha': p^{-1}(\text{cl} (U_j)) \cong \text{cl} (U_j) \times F; \beta'
$$

where the primes denote restriction and $F' = p^{-1}(b_j)$ [4, Proposition 4]. Consider the diagram

$$
\begin{array}{ccc}
\text{cl} (U_j) \times F & \xrightarrow{\alpha'} & p^{-1}(\text{cl} (U_j)) \\
\downarrow p & & \downarrow p \\
\text{cl} (U_j) & \xrightarrow{\bar{g}} & \sigma_j
\end{array}
$$

(2)

where $g' = \alpha g \beta'$. By Theorem 4,

$$
i(g', U_j \times F) = \overline{i}(\bar{g}, U) \cdot L(g^\gamma_j).
$$

If we use the constant path to define $g_{v_j}$, then $g_{v_j} = g^\gamma_j$, so $L(g^\gamma_j) = L(g^\gamma_j)$. Let $\mu = g \beta' : p^{-1}(\text{cl} (U_j)) \rightarrow \sigma_j \times F$, then by the commutativity axiom

$$
i(\alpha \mu, U_j \times F) = i(\mu \alpha', p^{-1}(U_j)) .
$$

Now $i(\alpha \mu, U_j \times F) = i(g', U_j \times F)$ by definition. On the other hand, $\mu \alpha' = g \beta' \alpha'$ is homotopic to $g$ by a homotopy which has no fixed points on $\text{bd}(p^{-1}(U_j))$ since $\bar{g}$ has no fixed points on $\text{bd}(U_j)$ and the homotopy between $\beta' \alpha'$ and the identity is fibre-preserving, so by the homotopy axiom $i(\mu \alpha', p^{-1}(U_j)) = i(g, p^{-1}(U_j))$. Therefore

$$
i(g, p^{-1}(U_j)) = \overline{i}(\bar{g}, U_j) \cdot L(g^\gamma_j).
$$

Renumber the fixed points of $\bar{g}$ so that $b_1, \ldots, b_q$ are the fixed points which lie in $U$. Since $g(e) = e$ implies $p(e) = b_j$ for some $j = 1, \ldots, s$, $g$ has no fixed points on $[p^{-1}(\text{cl} (U_j)) - \bigcup_{j=1}^s p^{-1}(U_j)]$. Hence by the additivity axiom,
Now suppose that $B$ is a compact ANR, let $K$ be a finite polyhedron and let $\varphi: B \to K$, $\psi: K \to B$ be maps such that $\psi \varphi \sim_\alpha 1_B$ where $\alpha$ refines $\kappa_f(U)$ and $\alpha(\bar{f}(U))$, the union of all $A \in \alpha$ such that $A \cap \bar{f}(U) \neq \emptyset$, is contained in $V$. Let $\psi^t(\mathcal{F}) = (\psi^t(E), p^t, K)$ where

$$\psi^t(E) = \{(k, e) \in K \times E \mid \psi(k) = p(e)\}$$

and $p^t(k, e) = k$, then $\psi^t(\mathcal{F})$ is a fibre space with lifting function $\lambda^t$ given by

$$\lambda^t((k, e), \omega)(t) = (\omega(t), \lambda(e, \psi \omega)(t))$$

where $\lambda$ is the lifting function of $\mathcal{F}$. Let $h: B \times I \to B$ be the $\alpha$-homotopy such that $h(b, 0) = b$, $h(b, 1) = \psi \varphi(b)$, then $h$ induces $h': B \to B'$. Define $\varphi': E \to \psi^t(E)$ by

$$\varphi'(e) = (\varphi p(e), \lambda(e, h'(p(e))))(1)$$

Consider

$$\psi^t(E) \xleftarrow{\varphi'} E \xrightarrow{f} \bar{E}$$

$$\psi^t(E) \xrightarrow{\psi} E \xrightarrow{p} p$$

$$K \xrightarrow{\bar{f}} B$$

$$K \xleftarrow{\varphi} B$$

where $\varphi'(k, e) = e$ and $f^t = \varphi' f \psi'$. Since $\bar{f}^t = \varphi \bar{f} \psi$ and $\psi \varphi \sim_\alpha 1_B$, then $i(\bar{f}, U) = \bar{i}(\bar{f}^t, \psi^{-1}(U))$. We let $\nu = \varphi' f: E \to \psi^t(E)$, then by the commutativity axiom,

$$i(\varphi' \nu, p^{-1}(U)) = i(\psi \nu, \psi^{-1} p^{-1}(U)) .$$

Define $H: E \times I \to E$ by $H(e, t) = \lambda(e, h'(p(e)))(t)$. If $H(f(e), t) = e$ for any $e \in bd(p^{-1}(U))$, $t \in I$, then $h(f(p(e)), t) = p(e)$ which is impossible since $\alpha$ refines $\kappa_f(U)$ [2, p. 20], so $\varphi' \nu = \varphi' \varphi f$ is homotopic to $f$ by a homotopy without fixed points on $bd(p^{-1}(U))$ and by the homotopy axiom

$$i(\varphi' \nu, p^{-1}(U)) = i(f, p^{-1}(U)) .$$
On the other hand, \( i(\psi', \psi^{-1}p^{-1}(U)) = i(f^k, p^k(\psi^{-1}(U))) \). If \( k \in \psi^{-1}(U) \), then \( \tilde{f}^k(k) \in \psi^{-1}(V) = W \) since \( \alpha(f(U)) \subseteq V \). Let \( \omega: I \to W \) be a path such that \( \omega(0) = \tilde{f}^k(k) \) and \( \omega(1) = k \). Define \( \omega': I \to V \) by

\[
\omega'(s) = \begin{cases} h'(\tilde{f}^k(k))(2s) & 0 \leq s \leq 1/2 \\ \omega(2s - 1) & 1/2 \leq s \leq 1 \end{cases}
\]

and let \( f_{\psi,k} \) be given by \( f_{\psi,k}(e) = \lambda(f(e), \omega')(1) \). Define \( f_{\psi,k}: p^{-1}(\psi(k)) \to p^{-1}(\psi(k)) \) by

\[
f_{\psi,k}(e) = \lambda(f(e), h'(\tilde{f}^k(k))(1), \psi\omega)(1)
\]

then by [4], \( f_{\psi,k} \) is homotopic to \( f_{\psi,k} \). But \( f_k^k(k, e) = \lambda^k((k, e), \omega)(1) = (k, f_{\psi,k}(e)) \). Therefore \( L(f_k^w) \) is equal to \( L(f_k^w) \) and is independent of \( k \) and \( \omega \). Applying the first part of the proof to the fibre space \( \psi^*(\mathcal{F}) \), the map \( f^k \), and the open set \( \psi^{-1}(U) \subseteq K \), we get

\[
i(f^k, p^k(\psi^{-1}(U))) = i(\tilde{f}^k, \psi^{-1}(U)) \cdot L(f_k^w).
\]

Therefore,

\[
i(f, p^{-1}(U)) = i(\tilde{f}, U) \cdot L(f_k^w)
\]

which completes the proof of Theorem 1.

4. The index of a fixed point class. Let \( X \) be a compact ANR and let \( f: X \to X \) be a map. Denote the fixed point classes of \( f \) by \( F_1, \ldots, F_r \). Let \((\bar{X}, \bar{p}, X)\) be the universal covering space of \( X \), then by [2, pp. 43-44] there is a map \( \bar{f}^j: \bar{X} \to \bar{X} \) such that \( \bar{p}\bar{f}^j = f\bar{p} \) which has the following properties: (1) if \( \bar{f}^j(e) = e \), then \( p(e) \in F_j \), (2) for each \( b \in F_j \) there exists \( e \in \bar{p}^{-1}(b) \) such that \( \bar{f}^j(e) = e \). We say that \( \bar{f}^j \) covers \( F_j \). There is an open set \( U_j \) in \( X \) containing \( F_j \) such that \( \text{cl}(U_j) \cap F_k = \emptyset \) for \( k \neq j \). The index of \( F_j \) is defined by \( i(F_j) = i(f, U_j) \) and is independent of the choice of \( U_j \).

**Theorem 5.** Let \( X \) be a compact ANR with finite fundamental group. Let \( f: X \to X \) be a map, let \( F \) be a fixed point class of \( f \), and let \( \bar{f}: \bar{X} \to \bar{X} \) cover \( F \). If there exists an open subset \( U \) of \( X \) such that for \( x \in U \), \( f(x) = x \) if, and only if, \( x \in F \), \( f(x) \neq x \) for \( x \in \text{bd}(U) \), and \( \text{cl}(U \cup f(U)) \subseteq V \), where \( V \) is an open connected simply-connected subset of \( X \), then

\[
i(F) = L(\bar{f})/L(\bar{f}^y)
\]

for \( x \in U \).

**Proof.** We first observe that \( L(\bar{f}^y) \neq 0 \). Take \( x \in F \), then since the fibre is discrete \( L(\bar{f}^y) \) is just the number of fixed points of \( \bar{f} \).
restricted to \( \tilde{p}^{-1}(x) \) which, since \( \tilde{f} \) covers \( F \), must be greater than zero. Since \( \pi_1(X) \) is finite, \( X \) is compact and we can apply Theorem 1 to obtain

\[
i(f, U) = i(\tilde{f}, \tilde{p}^{-1}(U))/L(\tilde{f}_X).
\]

Since \( \tilde{f} \) has no fixed points outside of \( \tilde{p}^{-1}(U) \),

\[
i(\tilde{f}, \tilde{p}^{-1}(U)) = L(\tilde{f}).
\]

The existence of the simply-connected set \( V \) in the hypotheses of Theorem 5 is not as severe a restriction as it may appear. For example, if \( X \) is a finite polyhedron, (or a compact topological manifold, with or without boundary) \( f \) is homotopic to a map \( g \) which has only isolated fixed points [6, Theorem 5] [3, Theorem 2] and the homotopy carries \( F \) to a fixed point class \( F' \) of \( g \) of the same index [2, Theorem 3, p. 36]. Hence we can apply Theorem 5 to \( g \) and \( F' \) to compute \( i(F) \) (compare Theorem 5.2 of [8]).

References


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