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In this paper we study the class $\mathfrak{R}$ of Hausdorff compact spaces X which are obtainable as images of ordered compacta K under (continuous) maps $f: K \to X$ onto X. The topology of K is the order topology induced by a total (linear) ordering $<$ on K. We find that X is locally peripherally metric (Theorem 5), i.e., it has a basis of open sets with metrizable frontiers.

In fact, our main result is this stronger statement.

THEOREM 1. *Let X be a continuous image of an ordered compactum K and let G be an open F_σ-set in X. If* Cl G *is connected, then the frontier Fr G is metrizable.*

Theorems 1 and 5 answer in the affirmative two questions raised by the author in [3].

As an immediate consequence, we obtain

COROLLARY 1. *Let X be a continuous image of an ordered compactum K and let G be an open F_σ-set in X. If every point $x \in$* Fr *G has a connected open neighborhood in* Cl G, *then* Fr G *is metrizable.*

Another easy consequence of Theorem 1 is the following theorem of L. B. Treybig [10]:

COROLLARY 2 (*Treybig*). *Let X be a continuous image of an ordered compactum K. If X is connected and separable, then it is metrizable.*

The proof of Theorem 1 given in §5 depends on an apparently new metrization theorem for Hausdorff compact spaces (Theorem 2 of §1), on earlier work of the author on separation properties of images of ordered compacta [3], on the earlier joint work with P. Papić ([5], [6],) and on the following product theorem due to A. J. Ward [13] and L. B. Treybig [9] (see also [3] and [4]).

PRODUCT THEOREM (*Ward, Treybig*). *Let X and Y be infinite compacta such that $X \times Y$ is the image of an ordered compactum. Then both X and Y are metrizable.*

The proof of Theorem 1 does not depend on Corollary 2 and, therefore provides a new proof of this important result (For another proof of Corollary 2 see [3]).

1. A metrization theorem for Hausdorff compacta. In this paper a compactum is a Hausdorff compact space and a continuum is a connected compactum. If Y is a compactum, $Z(Y)$ denotes the space of components of Y, i.e., $Z(Y)$ is the quotient space Y/R, where $y\,Ry'$, y, $y' \in Y$, means that both y and y' belong to the same (connected) component of Y. It is well-known that $Z(Y)$ is again a compactum and that the natural projection $\pi\colon Y \to Z(Y)$ ($\pi(y)$ is the component of y in Y) is a continuous mapping onto (see e.g. [8]).

We now consider, for compacta Y, the following two properties:

PROPERTY μ. For every closed subset $A \subset Y$ the space of components $Z(A)$ is metrizable.

PROPERTY σ. There exists a countable family $\mathfrak{S}$ of open sets S such that for any pair of disjoint closed sets $M, N \subset Y$ there exists an $S \in \mathfrak{S}$ which separates Y between M and N.

We say that S separates Y between M and N provided there exist disjoint sets $A, B \subset Y$ such that $M \subset A$, $N \subset B$, $A \cup B = Y\backslash S$, and A and B are both closed in $A \cup B$.

THEOREM 2. *In order that a compactum Y be metrizable it is necessary and sufficient that it has both properties μ and σ.*

Proof. If Y is metrizable, then so are its closed subsets $A \subset Y$. Therefore, their continuous images $Z(A) = \pi(A)$ are also metrizable, so that Y has property μ.

To prove that Y has property σ, consider a countable basis $\mathfrak{S}$ which is closed under finite unions. Given any pair of disjoined closed sets $M, N \subset Y$, one readily finds a closed set $F \subset Y\backslash(M \cup N)$ which separates Y between M and N. Now it suffices to cover F by a set $S \in \mathfrak{S}$ which does not meet $M \cup N$.

Suppose now that Y is a compactum with properties μ and σ. We construct a countable basis $\mathfrak{B}$ for the topology of Y as follows. Choose, by property σ, a countable family $\mathfrak{S}$ and consider for each $S \in \mathfrak{S}$ the closed set $Y\backslash S$. Next, choose a countable basis $\mathfrak{B}_S^*$ for the topology of the metric compactum $Z(Y\backslash S)$ (property μ). Let $\mathfrak{B}_S$ consist of all sets of the form

$$U = S \cup \pi^{-1}(V)\,, \tag{1}$$

where $V \in \mathfrak{B}_S^*$ and $\pi\colon Y\backslash S \to Z(Y\backslash S)$ is the natural projection. Clearly

$$\mathfrak{B} = \bigcup \mathfrak{B}_S,\ S \in \mathfrak{S}\ , \tag{2}$$

is a countable collection of open sets of Y.

To show that $\mathfrak{B}$ is a basis for Y, consider a point $y_0 \in Y$ and a closed set $M \subset Y$, $y_0 \notin M$. We shall exhibit a set $U \in \mathfrak{B}$ such that $y_0 \in U$ and $U \cap M = 0$.

First take an open set $S_0 \in \mathfrak{S}$ which separates Y between y_0 and M. Then choose a decomposition of $Y \backslash S_0$ in two disjoint closed sets A, B such that $y_0 \in A$, $M \subset B$. No component of $Y \backslash S_0$ meets simultaneously A and B. Hence,

$$\pi(A) \cap \pi(B) = 0\ , \tag{3}$$

where $\pi: Y \backslash S_0 \rightarrow Z(Y \backslash S_0)$ is the natural projection. We obtain thus a decomposition

$$Z(Y \backslash S_0) = \pi(A) \cup \pi(B) \tag{4}$$

of $Z(Y \backslash S_0)$ in two disjoint closed and open subsets $\pi(A)$, $\pi(B)$. Since,

$$\pi(y_0) \in \pi(A), \tag{5}$$

there exists an open set $V \in \mathfrak{B}^*_{S_0}$ such that

$$\pi(y_0) \in V \subset \pi(A)\ . \tag{6}$$

Consequently,

$$y_0 \in \pi^{-1}(V) \subset A \tag{7}$$

and we see that the set

$$U = S_0 \cup \pi^{-1}(V) \in \mathfrak{B}_{S_0} \subset \mathfrak{B} \tag{8}$$

fulfills the requirements

$$y_0 \in U\ , \qquad U \cap M = 0\ . \tag{9}$$

This completes the proof of Theorem 2.

REMARK. Property σ alone is not sufficient to imply metrizability of Y. E.g. every separable ordered compactum K has property σ (see Theorem 4 in §3), but K need not be metrizable. The corresponding question for property μ is discussed in §2.

2. Property μ and the Suslin problem.[1] A space Y is said to have the Suslin property if every family of nonempty disjoint open sets in Y is countable.

[1] The results of this section are not used in the sections that follow.

LEMMA 1. *If a compactum Y has property μ, it also has the Suslin property.*

Proof. Let $U = \{U_\lambda\}$, $\lambda \in L$, be a family of nonempty disjoint open sets in Y. Choose, for each $\lambda \in L$, a point $y_\lambda \in U_\lambda$. Let

$$A = \mathrm{Cl}\left[\bigcup_{\lambda \in L} \{y_\lambda\}\right]. \tag{1}$$

Clearly, the points y_λ are isolated in the set A and, therefore, $\pi(y_\lambda)$ are isolated points in $Z(A)$. Since, $Z(A)$ is a metrizable compactum, it can have only countably many isolated points. This proves that L is countable, i.e., that Y has the Suslin property.

LEMMA 2. *Let C be an ordered continuum with the Suslin property. Then C has property μ.*

Proof. An ordered continuum C is an ordered compactum which is connected. If A is a closed subset of C, then the open set $C \backslash A$ decomposes in a countable family of maximal disjoint open intervals U_n. Clearly, the space of components $Z(A)$ is a totally disconnected ordered compactum whose order is induced by the order $<$ in C.

By a gap in an ordered compactum $(K, <)$ we mean a pair of points $c_1, c_2 \in K$, such that the interval $(c_1, c_2)_K$ is empty. It is readily seen that a totally disconnected ordered compactum K with only countably many gaps is metrizable and is in fact a subset of the Cantor set (see e.g. Lemma 1 of [9]).

Thus, in order to show that $Z(A)$ is metrizable it suffices to show that $Z(A)$ has only countably many gaps. In fact, we can associate with every gap C_1, C_2 of $Z(A)$ the unique interval $U_n \subset C$ whose two end-points belong to the components C_1 and C_2 of A respectively. In this way we obtain a one-to-one mapping of the set of gaps of $Z(A)$ into the set of intervals U_n. This proves that $Z(A)$ has only countably many gaps and is, therefore, metrizable. Since $Z(A)$ is metrizable, for every closed set $A \subset C$, the continuum C has property μ.

The author does not know of any example of a compactum Y which has property μ but fails to be metrizable. However, if property μ alone would imply metrizability of compacta Y, then Lemma 2 would imply that every ordered continuum C with the Suslin property is metrizable and, therefore, homeomorphic to the real line segment I. In other words, we would have a positive answer to the Suslin problem (M. Ya. Suslin in Fund. Math. 1 (1920), p. 223).

THEOREM 3. *The following two statements are equivalent:*

(i) *In the class $\mathfrak{K}$ of images of ordered compacta every compactum*

$X \in \mathfrak{K}$ with property μ is metrizable,

(ii) *Every ordered continuum C with the Suslin property is homeomorphic to the real line segment I.*

Proof. (i) $\Rightarrow$ (ii) is an immediate consequence of Lemma 2.

In order to prove that (ii) $\Rightarrow$ (i), consider a compactum $X \in \mathfrak{K}$ which has property μ. It follows from Lemma 1 that X has the Suslin property. Using (ii), P. Papić and the author have proved that a compactum $X \in \mathfrak{K}$ with the Suslin property is separable (Corollary 6 of [6]), and in §3 of this paper we prove that every separable compactum $X \in \mathfrak{K}$ has property σ (Theorem 4). Hence, X has both properties μ and σ and is therefore metrizable, by Theorem 2.

3. Images of ordered compacta and property σ. In this section we prove

THEOREM 4. *Let X be a continuous image of an ordered compactum. If X is separable, it has property σ.*

We first recall that if $X \in \mathfrak{K}$ has the Suslin property, then every open subset of X is an F_σ-set (see Theorem 2 of [5] or Corollary 3, p. 13 of [6]). This holds a fortiori if X is separable so that we have

LEMMA 3 (*Mardešić-Papić*). *If $X \in \mathfrak{K}$ is separable, then every closed subset of X is a G_δ-set and every open subset of X is an F_σ-set.*

Proof of Theorem 4. The author has shown (Theorem 4 in [3]) that a separable $X \in \mathfrak{K}$ admits a countable family $\mathfrak{F}$ of closed sets F which separate X between any pair of disjoint closed sets $M, N \subset X$. We now choose such a family $\mathfrak{F}$.

Each $F \in \mathfrak{F}$ is a G_δ-set (Lemma 3) so that we can choose a countable collection $\mathfrak{S}_F$ of open sets $S \subset X$ such that $F \subset S$ and

$$(1) \qquad F = \cap (\mathrm{Cl}\, S), \qquad S \in \mathfrak{S}_F .$$

The family

$$(2) \qquad \mathfrak{S} = \cup \mathfrak{S}_F, \qquad F \in \mathfrak{F} ,$$

is a countable collection of open sets in X which has the required separation property σ.

Indeed, if M and N are disjoint closed subsets of X, then there exists a set $F \in \mathfrak{F}$ such that F separates X between M and N. Since

$$(3) \qquad F \subset X \backslash (M \cup N) ,$$

and (1) holds, we can find a set $S \in \mathfrak{S}_F \subset \mathfrak{S}$ such that

$$(4) \qquad F \subset S \subset \mathrm{Cl}\, S \subset X \backslash (M \cup N) .$$

Clearly, such a set $S \in \mathfrak{S}$ separates X between M and N, which concludes the proof.

4. The frontier of open $\mathbf{F}_\sigma$-sets and its space of components. In this section we prove the crucial

LEMMA 4. *Let $X \in \mathfrak{R}$ and let G be an open F_σ-set dense in X. If X is connected, the space of components $Z(\mathrm{Fr}\, G)$ is metrizable.*

Proof. Choose a sequence of open sets $H_n \subset G$, $n = 1, 2, \cdots$, such that

$$(1) \qquad \mathrm{Cl}\, H_n \subset H_{n+1} ,$$

$$(2) \qquad \bigcup_{n=1}^{\infty} \mathrm{Cl}\, H_n = G .$$

For each n, consider the compactum

$$(3) \qquad X \backslash H_n \supset X \backslash G .$$

Let

$$(4) \qquad Z_n = Z(X \backslash H_n) , \qquad Z = Z(X \backslash G) .$$

By (3), every component of $X \backslash G$ is contained in a unique component of $X \backslash H_n$. This inclusion defines a map

$$(5) \qquad p_n \colon Z \longrightarrow Z_n .$$

We shall now show that the maps p_n, $n = 1, 2, \cdots$, distinguish points of Z, i.e. that for any two distinct components C_1, C_2 of $X \backslash G$ there exists an n such that

$$(6) \qquad p_n(C_1) \neq p_n(C_2) .$$

The maps p_n, $n = 1, 2, \cdots$, will thus define an imbedding of Z in the direct product

$$(7) \qquad \prod_{n=1}^{\infty} p_n(Z) .$$

We first choose two disjoint closed sets F_1, F_2 in $\mathrm{Fr}\, G$ covering $\mathrm{Fr}\, G$ and such that $C_1 \subset F_1$ and $C_2 \subset F_2$. Since the sets F_i are at the same time closed in X, we can surround them by disjoint open sets U_1, U_2 of X. Thus

$$(8) \qquad C_i \subset U_i\ , \quad i = 1, 2\ ,$$

$$(9) \qquad U_1 \cup U_2 \supset \operatorname{Fr} G\ .$$

We now choose an n such that

$$(10) \qquad X \backslash (U_1 \cup U_2) \subset H_n\ .$$

The set $X \backslash H_n \subset U_1 \cup U_2$ splits in two disjoint open sets $U_i \cap (X \backslash H_n)$, $i = 1, 2$, which contain C_1 and C_2 respectively. This proves that C_1 and C_2 are included in different components of $X \backslash H_n$ so that (6) takes place.

In order to complete the proof of Lemma 4 it now suffices to show that the space $p_n(Z)$ is metrizable, for every n. In that case the direct product (7) will be metrizable and so will be Z itself, because Z is embeddable in this product.

To show that $p_n(Z)$ is metrizable, first notice that every component C of $X \backslash H_n$ meets $\operatorname{Cl} H_n$, because X is connected and compact. Moreover, if $C \in p_n(Z)$. Then C also meets Fr G.

Next, consider the natural projection

$$(11) \qquad \pi\colon X \backslash H_n \to Z(X \backslash H_n) = Z_n$$

and a map

$$(12) \qquad \varphi\colon X \backslash H_n \to I = [0, 1]\ ,$$

such that

$$(13) \qquad \varphi((X \backslash H_n) \cap \operatorname{Cl} H_n) = 0\ ,$$

$$(14) \qquad \varphi(\operatorname{Fr} G) = 1\ ;$$

φ exists by Urysohn's lemma.

Using π and φ we define the map

$$(15) \qquad \psi = \pi \times \varphi\colon X \backslash H_n \to Z_n \times I\ .$$

We now show that

$$(16) \qquad p_n(Z) \times I \subset \psi(X \backslash H_n)\ .$$

Indeed, if $C \in p_n(Z)$, then C meets Fr G and Cl H_n and so $\psi(C)$ meets both $C \times 1$ and $C \times 0$. Since, $\psi(C) \subset C \times I$ and $\psi(C)$ is connected, it follows that

$$(17) \qquad C \times I = \psi(C) \subset \psi(X \backslash H_n)$$

and (16) is established.

Since X belongs to $\mathfrak{K}$, we conclude that also $X \backslash H_n, \psi\ (X \backslash H_n)$ and $p_n(Z) \times$ I belong to $\mathfrak{K}$. Therefore, by the product theorem (see the introduction) $p_n(Z)$ is metrizable. This completes the proof of Lemma 4.

5. Proof of Theorem 1.

We first prove

LEMMA 5. *Let $X \in \mathfrak{K}$ and let G be an open F_σ-set in X. If* Cl *G is connected, then* Fr *G has property μ.*

Proof. Let A be a closed subset of Fr G and let

$$\Gamma = (\mathrm{Cl}\, G)\backslash A \,. \tag{1}$$

Clearly, Γ is an open set, dense in Cl G, and

$$\mathrm{Fr}\, \Gamma = A \,. \tag{2}$$

We now show that Γ is an F_σ-set in Cl $G = \mathrm{Cl}\, \Gamma$. In the first place, Fr G is a separable compactum from $\mathfrak{K}$, for the author has proved that the frontier of an open F_σ-set in a compactum $X \in \mathfrak{K}$ is always separable (Theorem 2 of [3]). It follows, by Lemma 3, that $(\mathrm{Fr}\, G)\backslash A$ is an F_σ-set.

On the other hand, G is by assumption an F_σ-set. Consequently,

$$\Gamma = (\mathrm{Fr}\, G\backslash A) \cup G \tag{3}$$

is also an F_σ-set in X.

Applying Lemma 4 to Cl G and Γ, and taking into account (2), we see that $Z(A) = Z(\mathrm{Fr}\, \Gamma)$ is metrizable. This concludes the proof of Lemma 5.

Proof of Theorem 1. To complete the proof, notice that Fr G is a separable compactum from $\mathfrak{K}$ and, therefore, has property σ (Theorem 4). On the other hand, by Lemma 5, Fr G has also property μ. Thus, by Theorem 2, Fr G is a metrizable compactum.

Proof of Corollary 1. Let $X \in \mathfrak{K}$ and let G be an open F_σ-set in X with the property that there is a finite collection of connected open sets $U_1, \cdots, U_n$ in Cl G such that

$$\mathrm{Fr}\, G \subset U_1 \cup \cdots \cup U_n \,. \tag{4}$$

Clearly, Cl U_i belongs to $\mathfrak{K}$ and is connected. On the other hand, $(\mathrm{Cl}\, U_i) \cap G$ is an open F_σ-set dense in Cl U_i, because $U_i \subset \mathrm{Cl}\, G$ implies

$$U_i \subset \mathrm{Cl}\, [U_i \cap G] \subset \mathrm{Cl}\, [\mathrm{Cl}\, (U_i) \cap G] \subset \mathrm{Cl}\, U_i \,, \tag{5}$$

so that

$$\mathrm{Cl}\, [\mathrm{Cl}\, (U_i) \cap G] = \mathrm{Cl}\, U_i \,. \tag{6}$$

It follows from (6) and Theorem 1 that

$$\mathrm{Fr}\,[\mathrm{Cl}\,(U_i) \cap G] = (\mathrm{Cl}\,U_i)\backslash G \tag{7}$$

is metrizable. Since, by (4), the sets $(\mathrm{Cl}\,U_i)\backslash G$, $i = 1, \cdots, n$, cover $\mathrm{Fr}\,G$, we conclude that $\mathrm{Fr}\,G$ itself is metrizable.

Proof of Corollary 2. Corollary 2 is an immediate consequence of Theorem 1 and this

LEMMA 6. *If $X \in \mathfrak{K}$ is separable, there exists a compactum $X' \in \mathfrak{K}$ and an open F_σ-set $G \subset X'$ dense in X' and such that $X = \mathrm{Fr}\,G$. Moreover, if X is connected, so is X'.*

Proof. Let $f\colon K \to X$ be a map of an ordered compactum K onto X and let $D = \{t_1, \cdots, t_n, \cdots\}$ be a countable subset of K such that $f(D)$ is dense in X. Let K' be a new ordered compactum obtained from K by replacing each point $t_n \in D$ by a copy I_n of the real line segment I. We denote the two end-points of I_n by t'_n and t''_n and its interior by I^0_n. $K\backslash D$ can be considered as a subset of K'.

We now define a map

$$f'\colon K' \to X \times I \tag{8}$$

as follows. For $t \in K\backslash D$, let

$$f'(t) = f(t) \times 0\,, \tag{9}$$

let

$$f'(t'_n) = f'(t''_n) = f(t_n)\,, \tag{10}$$

and let $f' \mid I_n$ be any map of I_n onto

$$f(t_n) \times \left[0, \frac{1}{n}\right], \tag{11}$$

such that the end-points t'_n, t''_n are the only points of I_n which are mapped into $f(t_n) \times 0$. It is easy to verify that $f'\colon K' \to X \times I$ is continuous.

We now define X' by

$$X' = f'(K') \subset X \times I\,. \tag{12}$$

$X' \in \mathfrak{K}$ and

$$X \times 0 = f'\Big(K'\backslash \bigcup_{n=1}^{\infty} I^0_n\Big) \subset X'\,. \tag{13}$$

Clearly, the set

$$G = X' \backslash (X \times 0) = \bigcup_{n=1}^{\infty} f'(I_n^0) \tag{14}$$

is an open F_σ-set in X' and

$$\operatorname{Fr} G = X \times 0 , \tag{15}$$

because $\operatorname{Cl} G \supset f(D) \times 0$ and, therefore,

$$\operatorname{Cl} G \supset \operatorname{Cl} [f(D) \times 0] = X' . \tag{16}$$

If X is connected, so is X', because it consists of $X \times 0$ and arcs (11) which meet $X \times 0$.

6. Local peripheral metrizablity.

LEMMA 7. *Let X be a continuous image of an ordered compactum. If X is locally connected, then it is locally peripherally metrizable.*

Proof. If $F \subset X$ is a closed connected set and $U \subset X$ is open and $F \subset U$, then one can easily find (using regularity and local connectedness of X) an open connected set V in X such that

$$F \subset V \subset \operatorname{Cl} V \subset U . \tag{1}$$

Using this argument repeatedly, one can find, for each point $x_0 \in X$ and each open neighborhood U of x_0, a sequence of connected open sets V_n, $n = 1, 2, \cdots$, such that

$$x_0 \in V_1 \subset \cdots \subset V_n \subset \operatorname{Cl} V_n \subset V_{n+1} \subset \cdots \subset U . \tag{2}$$

Clearly,

$$\mathrm{V} = \bigcup_{n=1}^{\infty} V_n = \bigcup_{n=1}^{\infty} \operatorname{Cl} V_n \tag{3}$$

is a connnected open F_σ-set in X such that

$$x_0 \in V \subset U . \tag{4}$$

By Theorem 1, $\operatorname{Fr} V$ is metrizable, which proves that X is locally peripherally metrizable.

THEOREM 5. *Every continuous image X of an ordered compactum K is locally peripherally metrizable.*

The result follows immediately from Lemma 7 and this

LEMMA 8. *Every continuous image X of an ordered compactum K can be embedded in a continuous image Y of an ordered continuum C.*

Proof. Insert between any two consecutive points of K a copy of the open real line interval filling thus all the gaps in K. Denote the obtained ordered continuum by C. Consider X as embedded in a cube I^κ. The map $f: K \to I^\kappa$ can be extended to a continuous map $g: C \to I^\kappa$, $g \mid K = f$. Clearly, $X \subset Y = g(C)$. Notice that Y is locally connected and thus Lemma 7 applies.

REMARK. Local peripheral metrizability together with local connectedness does not suffice for the conclusion that a compactum X belongs to $\mathfrak{K}$ as the following example shows.

EXAMPLE. Let $\Omega = \{\alpha \mid \alpha < \omega_1\}$ be the set of all countable ordinals. Let L be the ordered continuum obtained by ordering lexicographically the product $\Omega \times [0,1)$ and adjoining a last point ω_1. Let X be the quotient space

$$X = (L \times I)/\omega_1 \times \mathrm{I}\,. \tag{5}$$

X is a nonmetrizable locally connected continuum and is locally peripherally metric. However, X does not belong to $\mathfrak{K}$, because no two points separate X and every nonmetrizable continuum $X \in \mathfrak{K}$ has such a pair of points (see Theorem 2 of [10]).

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