

# Pacific Journal of Mathematics

**A NOTE ON SEMI-PRIMARY HEREDITARY RINGS**

ABRAHAM ZAKS

## A NOTE ON SEMI-PRIMARY HEREDITARY RINGS

ABRAHAM ZAKS

**We give an example of two nonisomorphic semi-primary hereditary rings,  $\Omega$  and  $\Sigma$  with radicals  $M$  and  $N'$  respectively, such that  $\Omega/M^2 = \Sigma/N'^2$ .**

Let  $A$  be a semi-primary ring i.e. its (Jacobson) radical  $N$  is nilpotent and  $\Gamma = A/N$  is an Artinian ring. The problem of characterizing a semi-primary ring  $A$  all of whose residue rings have finite global dimension—was dealt in several papers. It turns out that  $A$  is such a ring if and only if  $A$  is a residue ring of a semi-primary hereditary ring  $\Omega$ . It was suggested that  $\Omega$  is uniquely determined up to an isomorphism by the condition  $\Omega/M^2 \approx A/N^2$ , where  $M$  is the radical of  $\Omega$ .

One can prove that if  $A$  is an epimorphic image of a semi-primary hereditary ring  $\Omega$ , then  $\Omega$  is uniquely determined (up to an isomorphism) by the conditions (a)  $\Omega$  admits a (semi direct sum) splitting,  $\Omega = \Gamma + A + M^2$  and (b)  $\Omega/M^2 \approx A/N^2$ .

The following ring furnish a counter example to the uniqueness statement if we don't assume condition (a), even if  $A$  admits a splitting.

Let  $k$  be a field of characteristic  $p \neq 0$ , and let  $x$  be a transcendental element over  $k$ . Let  $R = k(x^{1/p}) \otimes_{k(x)} k(x^{1/p})$  and let  $V$  be the radical of  $R$ . Then  $V$  contains the nonzero element  $x^{1/p} \otimes 1 - 1 \otimes x^{1/p}$ . Let  $\Sigma$  be a subring of the  $3 \times 3$  matrix algebra over  $R$ , which consists of all matrices  $M$  for which:

$$\begin{array}{lll} M_{11} \in k(x^{1/p}) \otimes_{k(x)} 1 & M_{12} = 0 & M_{13} = 0 \\ M_{21} \in V & M_{22} \in 1 \otimes_{k(x)} k(x^{1/p}) & M_{23} = 0 \\ M_{31} \in R & M_{32} \in 1 \otimes_{k(x)} k(x^{1/p}) & M_{33} \in 1 \otimes_{k(x)} k(x^{1/p}) . \end{array}$$

It is obvious that  $\Sigma$  is an Artinian ring and its radical  $N'$  consists of all matrices  $M$  in  $\Sigma$  for which  $M_{11} = M_{22} = M_{33} = 0$ .

Let  $A$  be  $\Sigma/N'^2$ , then one easily verifies that:

- (a)  $\text{gl. dim } \Sigma = 1$
- (b)  $\text{gl. dim } A = 2$
- (c)  $A$  admits a splitting
- (d)  $\Sigma$  does not admit a splitting (since  $V$  is not an  $R$ -direct summand in  $R$ ).

From (b) and (c) it follows that  $\Omega = \Gamma + A + A \otimes_r A$ —with  $A = N'/N'^2$ —is a semi-primary hereditary ring ( $A \otimes_r A \otimes_r A = 0$ ) with radical  $M = A + A \otimes_r A$ . Also  $A = \Gamma + A$  and  $A^2 = 0$ . Therefore  $\text{gl. dim } \Omega = \text{gl. dim } \Sigma = 1$ ,  $\Omega/M^2 \approx \Sigma/N'^2 \approx A/N^2$  ( $N$  is the radical of  $A$ ).

Obviously  $\Omega$  admits a splitting, but  $\Sigma$  does not, thus  $\Omega$  and  $\Sigma$  are not isomorphic.

It is worth noticing that if  $A$  is a finite dimensional  $K$ -algebra ( $K$  a field) then the uniqueness of  $\Omega$  follows from condition (b), since condition (a) holds whenever  $\dim F = 0$ .

There still remains the problem of the existence of  $\Omega$  satisfying (a) and (b).  $\Omega$  is known to exist whenever  $A$  admits a splitting.  $A$  is known to admit a splitting whenever  $N^2 = 0$ .

#### REFERENCES

1. H. Cartan and S. Eilenberg, *Homological Algebra*, Princeton Univ. Press, Princeton, 1956.
2. J. P. Jans and T. Nakayama, *Algebras with finite dimensional residue algebras*. Nagoya Math. J. **11** (1957) pp. 67-76.
3. A. Zaks, *Residue rings of semi-primary hereditary rings*. Nagoya Math. J. (to appear)

Received December 29, 1966.

TECHNION, HAIFA  
ISRAEL

# PACIFIC JOURNAL OF MATHEMATICS

## EDITORS

H. ROYDEN  
Stanford University  
Stanford, California

J. DUGUNDJI  
Department of Mathematics  
Rice University  
Houston, Texas 77001

J. P. JANS  
University of Washington  
Seattle, Washington 98105

RICHARD ARENS  
University of California  
Los Angeles, California 90024

## ASSOCIATE EDITORS

E. F. BECKENBACH

B. H. NEUMANN

F. WOLF

K. YOSIDA

## SUPPORTING INSTITUTIONS

UNIVERSITY OF BRITISH COLUMBIA  
CALIFORNIA INSTITUTE OF TECHNOLOGY  
UNIVERSITY OF CALIFORNIA  
MONTANA STATE UNIVERSITY  
UNIVERSITY OF NEVADA  
NEW MEXICO STATE UNIVERSITY  
OREGON STATE UNIVERSITY  
UNIVERSITY OF OREGON  
OSAKA UNIVERSITY  
UNIVERSITY OF SOUTHERN CALIFORNIA

STANFORD UNIVERSITY  
UNIVERSITY OF TOKYO  
UNIVERSITY OF UTAH  
WASHINGTON STATE UNIVERSITY  
UNIVERSITY OF WASHINGTON  
\* \* \*  
AMERICAN MATHEMATICAL SOCIETY  
CHEVRON RESEARCH CORPORATION  
TRW SYSTEMS  
NAVAL ORDNANCE TEST STATION

---

Mathematical papers intended for publication in the *Pacific Journal of Mathematics* should be typewritten (double spaced). The first paragraph or two must be capable of being used separately as a synopsis of the entire paper. It should not contain references to the bibliography. Manuscripts may be sent to any one of the four editors. All other communications to the editors should be addressed to the managing editor, Richard Arens at the University of California, Los Angeles, California 90024.

50 reprints per author of each article are furnished free of charge; additional copies may be obtained at cost in multiples of 50.

---

The *Pacific Journal of Mathematics* is published monthly. Effective with Volume 16 the price per volume (3 numbers) is \$ 8.00; single issues, \$ 3.00. Special price for current issues to individual faculty members of supporting institutions and to individual members of the American Mathematical Society: \$ 4.00 per volume; single issues \$ 1.50. Back numbers are available.

Subscriptions, orders for back numbers, and changes of address should be sent to Pacific Journal of Mathematics, 103 Highland Boulevard, Berkeley 8, California.

Printed at Kokusai Bunken Insatsusha (International Academic Printing Co., Ltd.), 7-17, Fujimi 2-chome, Chiyoda-ku, Tokyo, Japan.

PUBLISHED BY PACIFIC JOURNAL OF MATHEMATICS, A NON-PROFIT CORPORATION

The Supporting Institutions listed above contribute to the cost of publication of this Journal, but they are not owners or publishers and have no responsibility for its content or policies.

A. A. Aucoin, <i>Diophantine systems</i> .....	419
Charles Ballantine, <i>Products of positive definite matrices. I</i> .....	427
David Wilmot Barnette, <i>A necessary condition for d-polyhedrality</i> .....	435
James Clark Beidleman and Tae Kun Seo, <i>Generalized Frattini subgroups of finite groups</i> .....	441
Carlos Jorge Do Rego Borges, <i>A study of multivalued functions</i> .....	451
William Edwin Clark, <i>Algebras of global dimension one with a finite ideal lattice</i> .....	463
Richard Brian Darst, <i>On a theorem of Nikodym with applications to weak convergence and von Neumann algebras</i> .....	473
George Wesley Day, <i>Superatomic Boolean algebras</i> .....	479
Lawrence Fearnley, <i>Characterization of the continuous images of all pseudocircles</i> .....	491
Neil Robert Gray, <i>Unstable points in the hyperspace of connected subsets</i> .....	515
Franklin Haimo, <i>Polynomials in central endomorphisms</i> .....	521
John Sollion Hsia, <i>Integral equivalence of vectors over local modular lattices</i> .....	527
Jim Humphreys, <i>Existence of Levi factors in certain algebraic groups</i> .....	543
E. Christopher Lance, <i>Automorphisms of postliminal C*-algebras</i> .....	547
Sibe Mardesic, <i>Images of ordered compacta are locally peripherally metric</i> .....	557
Albert W. Marshall, David William Walkup and Roger Jean-Baptiste Robert Wets, <i>Order-preserving functions: Applications to majorization and order statistics</i> .....	569
Wellington Ham Ow, <i>An extremal length criterion for the parabolicity of Riemannian spaces</i> .....	585
Wellington Ham Ow, <i>Criteria for zero capacity of ideal boundary components of Riemannian spaces</i> .....	591
J. H. Reed, <i>Inverse limits of indecomposable continua</i> .....	597
Joseph Gail Stampfli, <i>Minimal range theorems for operators with thin spectra</i> .....	601
Roy Westwick, <i>Transformations on tensor spaces</i> .....	613
Howard Henry Wicke, <i>The regular open continuous images of complete metric spaces</i> .....	621
Abraham Zaks, <i>A note on semi-primary hereditary rings</i> .....	627
Thomas William Hungerford, <i>Correction to: "A description of <math>\text{Mult}_r(A^1, \dots, A^n)</math> by generators and relations"</i> .....	629
Uppuluri V. Ramamohana Rao, <i>Correction to: "On a stronger version of Wallis' formula"</i> .....	629
Takesi Isiwata, <i>Correction: "Mappings and spaces"</i> .....	630
Henry B. Mann, Josephine Mitchell and Lowell Schoenfeld, <i>Correction to: "Properties of differential forms in n real variables"</i> .....	631
James Calvert, <i>Correction to: "An integral inequality with applications to the Dirichlet problem"</i> .....	631
K. Srinivasacharyulu, <i>Correction to: "Topology of some Kähler manifolds"</i> .....	632