

# Pacific Journal of Mathematics

**LIE ALGEBRAS OF TYPE  $D_4$  OVER ALGEBRAIC NUMBER  
FIELDS**

HARRY P. ALLEN

## LIE ALGEBRAS OF TYPE $D_4$ OVER ALGEBRAIC NUMBER FIELDS

H. P. ALLEN

If  $\tilde{\mathfrak{A}}$  is a nonassociative algebra over an algebraically closed field  $L$ , then the classification problem for  $\tilde{\mathfrak{A}}$  is the determination of all algebras  $\mathfrak{A}$  over  $\phi \subset L$  where  $\tilde{\mathfrak{A}} \cong \mathfrak{A} \otimes_{\phi} L$ . This brief note studies this problem for the case where  $\mathfrak{A}$  is the Lie algebra  $D_4$  and  $\phi$  is a (finite) algebraic number field. The main result is a type of Hasse principle which tells us that a Lie algebra  $\mathfrak{L}$  (over  $\phi$ ) of type  $D_4$  has known type if the algebra  $\mathfrak{L}_{\phi_p}$  has known type for every completion  $\phi_p$  of  $\phi$ . This is used in §3 to obtain canonical splitting fields for Lie algebras of type  $D_4$  over  $\phi$ . Although the results are inconclusive with regard to the existence or nonexistence of new algebras, it indicates a (twisted) construction, which if non-vacuous, would yield new exceptional algebras of type  $D_{4III}$ .<sup>1</sup>

The notation will be the same as that in the author's "Jordan Algebras and Lie Algebras of Type  $D_4$ " [2]. Throughout the present paper  $\phi, F, E, K, P$  will denote algebraic number fields and  $\Omega(X)$  will denote the complete set of primes on the algebraic number field  $X$ . Also, we shall adopt the following convention without further mention: if  $X$  is an algebraic number field,  $Y$  a subfield and  $p \in \Omega(X)$ , then we shall use  $p$  to represent  $p|Y$  and  $Y_p$  for the completion of  $Y$ , at  $p|Y$ , in  $X_p$ . We begin with a field theoretic preliminary.

1. PROPOSITION 1. Let  $P/\phi$  be a finite dimensional Galois extension with Galois group  $G$ , and let  $p \in \Omega(P)$ . Then  $P_p/\phi_p$  is Galois and  $G_p = g(P_p/\phi_p)$  is isomorphic to a subgroup of  $G$ .

*Proof.* If  $P$  is a splitting field for  $f(\lambda) \in \phi[\lambda]$  over  $\phi$ , then  $P_p$  is a splitting field for  $f(\lambda)$  over  $\phi_p$  and thus  $P_p/\phi_p$  is Galois. If  $P = \phi(\zeta)$ , then  $P_p = \phi_p(\zeta)$  and the correspondence  $s_p \rightarrow s_p|P = s'_p$  is an injection of  $G_p$  in  $G$ .

$G_p$  is called the local Galois group at  $p$  and we note that if  $E$  is the subfield of  $P/\phi$  of  $G'_p$ -invariants, then  $E_p = \phi_p$ , for  $E$  is contained in the  $P_p$  invariants of  $G_p$  so  $E_p \subseteq \phi_p$ .

To avoid unnecessary complication we let  $Q$  be the field of rational numbers,  $\mathbb{C}_0$  the split Cayley algebra over  $Q$ ,  $\mathfrak{J} = \mathfrak{h}(\mathbb{C}_0, 1)$  the split exceptional central simple Jordan algebra over  $Q$  and  $\mathfrak{D} = \mathfrak{D}(\mathfrak{J}/\Sigma Qe_i)$  the split Lie algebra of type  $D_4$  over  $Q$ . If  $X$  is any field of charac-

<sup>1</sup> The author has recently shown, in collaboration with J. Ferrar, that this construction can be carried out over algebraic number fields.

teristic 0, then  $\mathfrak{D}_X = \mathfrak{D}(\mathfrak{X}_X/\Sigma X e_i)$  will be taken as the split Lie algebra of type  $D_4$  over  $X$ .

Now let  $\mathfrak{L}$  be a Lie algebra of type  $D_4$  over  $\phi$  with  $P/\phi$  a finite dimensional Galois splitting extension. If  $p \in \Omega(P)$ , then  $\mathfrak{L}_{\phi_p}$  is a Lie algebra of type  $D_4$  over  $\phi_p$  split by  $P_p$ . We first determine the relationship between the pre-cocycle of  $G$  in  $\text{Aut}_{\phi}(\mathfrak{D}_P)$  corresponding to  $\mathfrak{L}$ , and the pre-cocycle of  $G_p$  in  $\text{Aut}_{\phi_p}(\mathfrak{D}_{P_p})$  corresponding to  $\mathfrak{L}_{\phi_p}$  ([2] § 2).

Thus let  $r \rightarrow \eta(r) \leftarrow C_r = [p(r), T(r)]$  be the pre-cocycle of  $G$  in  $\text{Aut}_{\phi}(\mathfrak{D}_P)$  corresponding to  $\mathfrak{L}$ . If  $h' = h_p | P \in G'_p$ , then  $h'$  has a unique extension to  $G_p$ , viz.,  $h_p$ . We let  $C_{h_p}$  be the  $h_p$ -semilinear extension of  $C_{h'}$  to  $\Gamma L_{\phi_p}(\mathfrak{X}_{P_p}/\Sigma P_p e_i)$ .  $C_{h_p} = [p(h_p | P), T(h_p)]$  where  $T(h_p)$  is the  $h_p$ -semilinear extension of  $T(h_p | P)$  ([3] p. 12).

We have  $C_{h_p} C_{r_p} = C_{h_p r_p} \delta_{h', r'}$  where  $C_{h'} C_{r'} = C_{h' r'} \delta_{h', r'}$ . Thus if  $\eta(h_p) \leftarrow C_{h_p}$ , then  $h_p \rightarrow \eta(h_p)$  is a pre-cocycle of  $G_p$  in  $\text{Aut}_{\phi_p}(\mathfrak{D}_{P_p})$ . The fixed  $\phi_p$ -form of  $\mathfrak{D}_{P_p}$  associated with this pre-cocycle clearly contains  $\mathfrak{L}_{\phi_p}$ , so it must be  $\mathfrak{L}_{\phi_p}$ .

**PROPOSITION 2.** Let  $\mathfrak{L}$  be a Lie algebra of type  $D_4$  over  $\phi$  with  $P/\phi$  a finite dimensional Galois splitting extension and  $F$  the canonical  $D_{4I}$ -field extension of  $\mathfrak{L}$ . If  $p \in \Omega(P)$  then

- (i) The  $D_4$  type of  $\mathfrak{L}_{\phi_p}$  is the  $D_4$  type of a canonical extension of  $\mathfrak{L}$  ([2] § 2).
- (ii) the canonical  $D_{4I}$ -field extension of  $\mathfrak{L}_{\phi_p}$  is  $F_p$ .
- (iii) if  $L$  is exceptional then  $\mathfrak{L}_{\phi_p}$  is exceptional if and only if  $[F_p; \phi_p] \geq 3$ .

*Proof.* (i) is a direct consequence of the preceding discussion. Let  $F(p)$  be the canonical  $D_{4I}$ -field extension of  $\mathfrak{L}_{\phi_p}$  and suppose that  $F(p)$  is the invariants of  $H_p \subset G_p$ . If  $F'$  is the invariants of  $H'_p$  then  $F \subset F'$  so  $F_p \subseteq F'(p)$ . But  $\mathfrak{L}_{F_p}$  is of type  $D_{4I}$  so  $F_p \cong F'(p)$ . If  $\mathfrak{L}$  is exceptional then this shows that  $\mathfrak{L}_{\phi_p}$  is exceptional if and only if  $[F_p; \phi_p] \geq 3$ .

2. The classical results on central simple associative algebras and quadratic forms over algebraic number fields are used to deduce the next two important results.

**THEOREM 1.** Let  $\mathfrak{L}$  be a Lie algebra of type  $D_4$  over an algebraic number field  $\phi$ . Then there exists a finite subset  $S$  of  $\Omega(\phi)$  such that  $\mathfrak{L}_{\phi_p}$  is a Jordan  $D_4$  for all  $p \in \Omega(\phi) - S$ .

*Proof.* First suppose that  $\mathfrak{L}$  is of type  $D_{4I}$  and let  $\mathfrak{L}^* = \mathfrak{A}_1 \oplus \mathfrak{A}_2 \oplus \mathfrak{A}_3$  be its  $\phi$ -enveloping algebra ([2] § 2). Let  $S$  be any finite subset of

$\Omega(\phi)$  such that  $\mathfrak{A}_{i\phi_p} \sim 1, i = 1, 2, 3$  for all  $p \in \Omega(\phi) - S$  ([1] Chap IX). Since  $\mathfrak{L}_{\phi_p}^*$  is clearly the  $\phi_p$ -enveloping algebra of  $\mathfrak{L}_{\phi_p}$  for any  $p \in \Omega(\phi)$ , we see that for  $p \in \Omega(\phi) - S$ ,  $\mathfrak{L}_{\phi_p}^*$  is a sum of matrix algebras over  $\phi_p$ . This implies that  $\mathfrak{L}_{\phi_p}$  is a Jordan  $D_4$  ([2] Th. 1).

Now let  $\mathfrak{L}$  be an arbitrary Lie algebra of type  $D_4$  and let  $F/\phi$  be its canonical  $D_{4I}$ -field extension. Let  $T$  be any finite subset of  $\Omega(F)$  such that  $(\mathfrak{L}_F)_{F_p}$  is a Jordan  $D_4$  for all  $p \in \Omega(F) - T$ , and choose  $S$  as the set of all traces of elements of  $T$  on  $\phi$ . If  $p \mid \phi \in \Omega(\phi) - S$  then  $p \in \Omega(F) - T$  and  $(\mathfrak{L}_{\phi_p})_{F_p} = \mathfrak{L}_{F_p} = (\mathfrak{L}_F)_{F_p}$  is a Jordan  $D_4$ . Since  $F_p$  is the canonical  $D_{4I}$ -field extension of  $\mathfrak{L}_{\phi_p}$ ,  $\mathfrak{L}_{\phi_p}$  is a Jordan  $D_4$  ([1] Th. 1).

**THEOREM 2.** *Let  $\mathfrak{L}$  be a Lie algebra of type  $D_4$  over an algebraic number field  $\phi$ . Then  $\mathfrak{L}$  is a Jordan  $D_4$  if and only if  $\mathfrak{L}_{\phi_p}$  is a Jordan  $D_4$  for every  $p \in \Omega(\phi)$ .*

*Proof.* One direction is clear. For the other let  $F$  be the canonical  $D_{4I}$ -field extension of  $\mathfrak{L}$  and let  $\mathfrak{L}_F^* = \mathfrak{A}_1 \oplus \mathfrak{A}_2 \oplus \mathfrak{A}_3$  be the  $F$ -enveloping algebra of  $\mathfrak{L}_F$ . Our hypothesis implies that  $\mathfrak{L}_{F_p} = (\mathfrak{L}_{\phi_p})_{F_p}$  is a Jordan  $D_4$  for every  $p \in \Omega(F)$ . Thus  $\mathfrak{A}_{iF_p} \sim 1, i = 1, 2, 3$  and all  $p \in \Omega(F)$ , so  $\mathfrak{A}_i \sim 1, i = 1, 2, 3$ . ([1] Chap IX). Thus  $\mathfrak{L}_F$  is a Jordan  $D_{4I}$  and  $\mathfrak{L}$  is a Jordan  $D_4$ .

**COROLLARY.**  *$\mathfrak{L}$  is split if and only if  $\mathfrak{L}_{\phi_p}$  is split for all  $p \in \Omega(\phi)$ .*

*Proof.* One direction is trivial. For the other, Theorem 1 and Theorem 2 imply that  $\mathfrak{L}$  is a Jordan  $D_{4I}$ . If  $\mathfrak{L} = \mathfrak{s}(\mathbb{C}, n(\cdot))$ ,  $\mathbb{C}$  a Cayley algebra over  $\phi$ , then  $\mathfrak{L}_{\phi_p}$  split for all  $p$  implies that  $\mathbb{C}_{\phi_p}$  is isotropic for all  $p$ . Thus  $\mathbb{C}$  is isotropic, ([4], Th. 66.1) hence split, and  $\mathfrak{L}$  is split.

3. This last section is devoted to a proof of Proposition 3. Using this proposition we are able to give a fairly explicit description of pre-cocycles arising from algebras of type  $D_{4III}$ .

**PROPOSITION 3.** *Let  $\mathfrak{L}$  be a Lie algebra of type  $D_4$  over an algebraic number field  $\phi$ , and let  $F$  be the canonical  $D_{4I}$ -field extension of  $\mathfrak{L}$ . Then  $\mathfrak{L}$  is split by a Galois extension of degree at most  $2[F:\phi]$ .*

*Proof.* We will only give the argument when  $\mathfrak{L}$  is of type  $D_{4I}$  or  $D_{4III}$ . The other cases are similar. Let  $S$  be a finite subset of  $\Omega(\phi)$  such that  $\mathfrak{L}_{\phi_p}$  is a Jordan  ${}_4\mathfrak{D}$  if  $p \in \Omega(\phi) - S$ . Without loss of generality we suppose that  $S$  contains all the real primes on  $\phi$ . If  $p \in S$ , then  $\mathfrak{L}_{\phi_p}$  is necessarily of type  $D_{4I}$ . By the local classification of  $D_4$ 's ([2] § 4),  $\mathfrak{L}_{\phi_p}$  is split by a quadratic extension  $K_{(p)}/\phi_p$ . Let  $K_{(p)}$  be a root field for  $\lambda^2 + \alpha_p \in \phi[\lambda]$ . By the approximation theorem,

since  $S$  consists of inequivalent primes, there exists an  $\alpha \in \phi$  with each  $|\alpha - \alpha_p|_p$  sufficiently small. Let  $K$  be a root field for  $\lambda^2 + \alpha$  over  $\phi$  and  $F$  the canonical  $D_{4I}$ -field extension of  $\mathfrak{L}$ . Note that  $K_p = K_{(p)}$  for all  $p \in S$ .

If  $\mathfrak{L}$  is of type  $D_{4I}$  then it is easy to see that  $(\mathfrak{L}_K)_{K_p}$  is split for all  $p \in \Omega(K)$ . Thus  $\mathfrak{L}_K$  is split. If  $\mathfrak{L}$  is of type  $D_{4III}$  then we claim that  $\mathfrak{L}$  is split by  $P = K \otimes_{\phi} F$ . Let  $p \in \Omega(P)$ . If  $p \mid \phi \in \Omega(\phi) - S$ , then  $\mathfrak{L}_{\phi_p}$  is a Jordan  $D_4$ . If  $p$  is complex,  $\mathfrak{L}_{\phi_p}$  is clearly split whereas if  $p$  is discrete,  $\mathfrak{L}_{\phi_p}$  is split by its canonical  $D_{4I}$  field extension ([2] § 4). In any event,  $(\mathfrak{L}_P)_{P_p} = (\mathfrak{L}_{\phi_p})_{P_p} = ((\mathfrak{L}_{\phi_p})_{F_p})_{P_p}$  is split. If  $p \mid \phi \in S$ , then  $\mathfrak{L}_{\phi_p}$  is split by  $K_{(p)} = K_p \subset P_p$  so  $(\mathfrak{L}_P)_{P_p}$  is split. Thus  $(\mathfrak{L}_P)_{P_p}$  is split at every  $p \in \Omega(P)$  and the corollary to Theorem 2 shows that  $\mathfrak{L}_P$  is split. Note that  $P$  is sixth degree cyclic.

Now let  $\mathfrak{L}$  be a Lie algebra of type  $D_{4III}$  over  $\phi$ , with  $P/\phi$  a cyclic sixth degree Galois splitting extension.  $P/\phi$  contains a unique quadratic subfield  $E/\phi$ .  $\mathfrak{L}_E$  is a  $D_{4III}$  split by  $P$ , so  $\mathfrak{L}_E$  is a Jordan  $D_4$ . If  $\mathfrak{L}_E = \mathfrak{D}(\mathfrak{Y}'/\mathfrak{x})$ , then since  $\mathfrak{Y}'$  is reduced and is split by  $P/E$ , a cubic extension,  $\mathfrak{Y}'$  is itself split.  $\mathfrak{k}$  of course is isomorphic to  $P$ . The isomorphism condition for Jordan  $D_4$ 's ([2] Th. II) implies that  $\mathfrak{L}_E$  is a Steinberg  $D_{4III}$  ([2] (10)).

Let  $r \rightarrow p(r)$  be the anti-homomorphism of  $g(P/\phi) = G$  onto  $A_s$  determined by  $\mathfrak{L}$ , and choose  $s$  as a generator for  $G$  with  $p(s) = (123)$ . Let  $\mathfrak{C}$  be any Cayley algebra over  $\phi$ , split by  $P$ , and let  $S$  be the  $s$ -semilinear automorphism of  $\mathfrak{C}_P$  which is one on  $\mathfrak{C}$ . Finally set  $D_s = [(123), S]$  (cf. [2] (10)), and let  $r \rightarrow \eta(r) \leftarrow C_r$  be the pre-cocycle of  $G$  in  $\text{Aut}_{\phi}(\mathfrak{D}_P)$  corresponding to  $\mathfrak{L}$ . The preceding observation about  $\mathfrak{L}_E$  enables us to assume that  $C_s^2 = D_s^2 \mu$ ,  $\mu \in K$ . By replacing  $C_s$  by  $C_s \lambda$ , for some suitable  $\lambda \in K$ , if necessary, we may assume that  $C_s$  and  $D_s^2$  commute. This implies that  $\mu^{s^2} = D_s^{-2} \mu D_s^2 = \mu$ . If  $C_s^6 = \delta \in K$ , then  $\delta^s = \mu^3$  and  $\delta^s = C_s^{-1} \delta C_s = D_s^{-1} \delta D_s = \delta$  so  $(\mu^3)^s = \mu^3$ . Applying  $\zeta(\cdot)$  to the relation  $C_s^2 = D_s^2 \mu$  we obtain

$$(1) \quad \zeta(C_s)^s \zeta(C_s) = \mu^2.$$

But  $\zeta(C_s)^s \zeta(C_s)$  is fixed under  $s$  since  $\zeta(C_s)$  is fixed  $s^2$ . Thus  $(\mu^2)^s = \mu^2$ . This, together with the previous relation shows that  $\mu = \mu^s$ .

For simplicity write  $\zeta(C_s) = (\rho, p^{s^2}, \rho^{s^4}) \mu = (\beta, \beta^{s^2}, \beta^s)$ ,  $\beta^{s^3} = \beta$ . (1) is now equivalent to  $\rho \rho^{s^3} = \beta^2$ . Since  $(\rho \beta^{-1})(\rho \beta^{-1})^{s^3} = 1$ ,  $\rho = \lambda^{-1} \lambda^{s^3} \beta$  and  $\rho \lambda^2 = \lambda \lambda^{s^3} \beta \in F$ , the canonical  $D_{4I}$ -field extension of  $\mathfrak{L}$ . Replacing  $C_s$  by  $\tilde{C}_s = C_s(\lambda, \lambda^{s^2} \lambda^{s^4})$  we again obtain  $\zeta(\tilde{C}_s)^{s^2} = \zeta(\tilde{C}_s)$ . But

$$\zeta(\tilde{C}_s) = (\lambda \lambda^{s^3} \beta, ((\lambda \lambda^{s^3} \beta)^{s^2}), (\lambda \lambda^{s^3} \beta)^s)$$

and is fixed under  $s^3$ . Thus  $\zeta(\tilde{C}_s)^s = \zeta(\tilde{C}_s)$ . We may affect a similar

alteration of  $C_s$  so that  $\zeta(C_s) = (\alpha_1, \alpha_1^2, \alpha_1^3) = \alpha$  where  $\alpha_1^3 = \alpha_1$  and  $\alpha_1 \alpha_1^2 \alpha_1^3 = 1$ , i.e.,  $\tilde{C}_s$ , in addition to the above is norm preserving. Calculating we see that  $\tilde{C}_s^2 = D_s^2 \alpha$ . Then  $\tilde{C}_s^3 = \alpha^3$  and  $\mathfrak{J}$  is a Jordan  $D_4$  if and only if  $\alpha_1 \in N_{P/F}(P^*)$ . Setting  $\tilde{C}_s = D_s E$  we see that  $D_s \alpha = E D_s E$ . The simplest form of this equation occurs where  $E$  and  $D_s$  commute and we obtain  $E^2 = \alpha$ . Thus we are led to the following (possibly vacuous) construction.

Let  $\mathfrak{J}$  be a reduced exceptional central simple Jordan algebra over a field  $\phi$ ,  $P$  a cyclic sixth degree extension of  $\phi$  and  $F$  a subfield of  $\mathfrak{J}$  isomorphic to the cubic subfield of  $P/\phi$ . Then if there exists an  $E \in GL(\mathfrak{J}/F)$  such that

- (i)  $E \in GL(\mathfrak{J}_P/\{Pe_i\}_i)$ ,
- (ii)  $\zeta(E) = (\alpha_1, \alpha_1^2, \alpha_1^3)$  where  $\alpha_1 \in N_{P/F}(P^*)$  and
- (iii)  $E^2 = \zeta(E)$ ,

then the  $s$ -semilinear extension of  $E$  to  $\mathfrak{J}_P$  induces a pre-cocycle corresponding to a non-Jordan  $D_{4III}$ .

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