

# Pacific Journal of Mathematics

**MATRIX SUMMABILITY OVER CERTAIN CLASSES OF  
SEQUENCES ORDERED WITH RESPECT TO RATE OF  
CONVERGENCE**

DAVID FLEMING DAWSON

## MATRIX SUMMABILITY OVER CERTAIN CLASSES OF SEQUENCES ORDERED WITH RESPECT TO RATE OF CONVERGENCE

DAVID F. DAWSON

Let  $C_0$  denote the set of all complex null sequences, and let  $S_0$  denote the set of all sequences in  $C_0$  which have at most a finite number of zero terms. If  $a = \{a_p\} \in S_0$  and  $b = \{b_p\} \in S_0$ , we say that  $a$  converges faster than  $b$ ,  $a < b$ , provided  $\lim a_p/b_p = 0$ . We say that  $a$  and  $b$  converge at the same rate,  $a \sim b$ , provided  $0 < \liminf |a_p/b_p|$  and  $\limsup |a_p/b_p| < \infty$ . If  $a \in S_0$ , let  $[a] = \{x \in S_0: x \sim a\}$ . Let  $E_0 = \{[a]: a \in S_0\}$ . If  $[a], [b] \in E_0$ , then we say that  $[a]$  is less than  $[b]$ ,  $[a] <' [b]$ , provided  $a < b$ . We note that  $E_0$  is partially ordered with respect to  $\leq'$ . In this paper we study matrix summability over subsets of  $S_0$  and over elements of  $E_0$ . Open intervals in  $S_0$  will be denoted by  $(a, b)$ ,  $(a, -)$ , and  $(-, b)$ , where  $(a, -) = \{x \in S_0: a < x\}$  and  $(-, b) = \{x \in S_0: x < b\}$ . Some of our results characterize, for matrices, maximal summability intervals in  $S_0$ . Such intervals are of the form  $(-, b)$ , never of the form  $(-, b] = \{x \in S_0: \text{either } x < b \text{ or } x \sim b\}$ .

Notational conveniences used are as follows. If  $A = (a_{pq})$  is a matrix and  $b$  is a sequence such that for each positive integer  $p$ , the series  $\sum_{q=1}^{\infty} a_{pq}b_q$  converges, then  $A(b)$  will denote the sequence  $\{\sum_{q=1}^{\infty} a_{pq}b_q\}_{p=1}^{\infty}$ . We will use  $A_b$  to denote the matrix  $(a_{pq}b_q)$ . If each of  $a$  and  $b$  is a sequence, then  $ab$  will be used to denote the sequence  $\{a_p b_p\}$ .

Playing a basic role throughout the paper are the two classical Silverman-Toeplitz (abbreviated  $S - T$ ) conditions which are necessary and sufficient for a matrix  $A$  to be convergence preserving over (abbreviated c.p.o.)  $C_0$ . These conditions are

(1)  $\{a_{pq}\}_{p=1}^{\infty}$  converges,  $q = 1, 2, 3, \dots$ ,

and

(2) there exists  $K$  such that  $\sum_{q=1}^{\infty} |a_{pq}| < K$ ,  $p = 1, 2, 3, \dots$ .

We note that the  $S - T$  conditions are necessary and sufficient for a matrix  $A$  to be c.p.o.  $S_0$ .

REMARK 1. A matrix sums every sequence in some interval  $(-, b)$  if and only if it has convergent columns.

REMARK 2. If the matrix  $A$  is c.p.o.  $[b]$  and  $c$  is a sequence such that  $\lim c_p/b_p = 0$ , then  $A(c)$  is convergent.

REMARK 2'. If  $A$  is c.p.o.  $[b]$ , then  $A$  is c.p.o.  $(-, b]$ .

REMARK 3. If  $A$  is c.p.o.  $(a, -)$ , then  $A$  is c.p.o.  $C_0$ .

LEMMA. Suppose  $K$  and  $L$  are countable subsets of  $S_0$  such that if  $x \in K$  and  $y \in L$ , then  $x < y$ . Then there exists  $z \in S_0$  such that if  $x \in K$  and  $y \in L$ , then  $x < z < y$ .

*Proof.* Our proof will be for the case that both  $K$  and  $L$  are infinite sets. Let  $K = \{a^{(1)}, a^{(2)}, a^{(3)}, \dots\}$  and  $L = \{b^{(1)}, b^{(2)}, b^{(3)}, \dots\}$ . Let  $\{n_p\}_{p=1}^\infty$  be an increasing sequence of positive integers such that if  $i > n_p$ , then

$$\left| \frac{b_i^{(j)}}{a_i^{(t)}} \right| > 2^p, \quad j, t = 1, 2, \dots, p.$$

Define

$$\begin{aligned} c_i &= b_i^{(1)}, \quad i = 1, 2, \dots, n_2, \\ c_i &= (1/p) \min [ |b_i^{(1)}|, |b_i^{(2)}|, \dots, |b_i^{(p)}| ], \\ n_p &< i \leq n_{p+1}, \quad p = 2, 3, 4, \dots \end{aligned}$$

Let  $r$  be a positive integer. If  $p > r$  and  $q$  is a positive integer such that  $n_p < q \leq n_{p+1}$ , then we have  $|b_q^{(r)}/c_q| \geq p$ , and, since  $c_q = |b_q^{(t)}|/p$  for some  $t \in \{1, 2, \dots, p\}$ , we have  $|c_q/a_q^{(r)}| > 2^p/p$ . Thus  $a^{(r)} < c < b^{(r)}$ . This completes the proof.

THEOREM 1. If  $A$  is c.p.o.  $[b]$ , then there exists  $b' \in S_0$  such that  $b < b'$  and  $A$  is c.p.o.  $[b']$ .

*Proof.* Since  $A$  is c.p.o.  $[b]$ , then by Remarks 1 and 2',  $A$  has convergent columns. Let  $a_q = \lim_{p \rightarrow \infty} a_{pq}$ . By Remark 2,  $A$  sums every null sequence  $x$  such that  $\lim x_p/b_p = 0$ . Thus  $A_b$  sums every null sequence. Therefore from (2) of the  $S - T$  conditions there exists  $M$  such that if  $n$  is a positive integer, then  $\sum_{p=1}^\infty |a_{np}b_p| < M$ . Clearly  $\sum_{q=1}^\infty |a_qb_q| \leq M$ . Let  $C = (c_{pq})$  be the matrix defined by  $c_{pq} = a_{pq}b_q - a_qb_q$ . Let  $D = (d_{pq})$  be the matrix defined by  $d_{pq} = a_qb_q$ . Then  $A_b = C + D$ . We wish to show that the sequence

$$(*) \quad \left\{ \sum_{p=1}^\infty |a_{np}b_p - a_p b_p| \right\}_{n=1}^\infty$$

converges to zero. We note that (\*) is bounded. Suppose (\*) has a subsequence which converges to  $\mu > 0$ . Note that each column of  $C$  converges to zero. Let  $n_1$  be a positive integer such that

$$\left| \sum_{p=1}^\infty |c_{n_1 p}| - \mu \right| < \mu/8.$$

Let  $k_1$  be a positive integer such that  $\sum_{p=1}^{k_1} |c_{n_1 p}| > 7\mu/8$ . Let  $N_1 > n_1$  be an integer such that if  $q > N_1$ , then  $\sum_{p=1}^{k_1} |c_{qp}| < \mu/8$ . Let  $n_2 > N_1$  be an integer such that

$$\left| \sum_{p=1}^{\infty} |c_{n_2 p}| - \mu \right| < \mu/8 .$$

Let  $k_2 > k_1$  be an integer such that  $\sum_{p=1}^{k_2} |c_{n_2 p}| > 7\mu/8$ . Let  $N_2 > n_2$  be an integer such that if  $q > N_2$ , then  $\sum_{p=1}^{k_2} |c_{qp}| < \mu/8$ . Continue the process to obtain increasing sequences  $\{n_p\}_{p=1}^{\infty}$  and  $\{k_p\}_{p=1}^{\infty}$  of positive integers. Define  $t_{pq} = |c_{pq}|/c_{pq}$  if  $c_{pq} \neq 0$ ,  $t_{pq} = 1$  if  $c_{pq} = 0$ . Define

$$\begin{aligned} s_p &= 1, p = 1, 2, \dots, k_1, \\ s_p &= (-1)^{q+1} t_{n_q p}, k_{q-1} < p \leq k_q, q = 2, 3, 4, \dots . \end{aligned}$$

Suppose  $q$  is a positive even integer. Then

$$\begin{aligned} &\left| \sum_{p=1}^{\infty} c_{n_q p} s_p - (-\mu) \right| \\ &= \left| \sum_{p=1}^{k_{q-1}} c_{n_q p} s_p + \sum_{p=k_{q-1}+1}^{k_q} c_{n_q p} s_p + \sum_{p=k_q+1}^{\infty} c_{n_q p} s_p + \mu \right| \\ &\leq \sum_{p=1}^{k_{q-1}} |c_{n_q p}| + \sum_{p=k_q+1}^{\infty} |c_{n_q p}| + \left| \sum_{p=k_{q-1}+1}^{k_q} c_{n_q p} s_p + \mu \right| \\ &< \mu/8 + \mu/4 + \left| - \sum_{p=k_{q-1}+1}^{k_q} |c_{n_q p}| + \mu \right| \\ &< \mu/8 + \mu/4 + \mu/4 . \end{aligned}$$

Similarly, if  $q$  is a positive odd integer, then

$$\left| \sum_{p=1}^{\infty} c_{n_q p} s_p - \mu \right| < 5\mu/8 .$$

Thus  $C(s)$  is divergent. But  $A_b(s)$  is convergent since  $A_b(s) = A(bs)$  and  $bs \in [b]$ . Clearly  $D(s)$  is convergent. Hence  $C(s)$  is convergent since  $C(s) = A_b(s) - D(s)$ . Therefore we have a contradiction. Thus (\*) converges to zero since the assumption to the contrary leads to a contradiction.

Let  $j_1$  be a positive integer such that if  $q > j_1$ , then  $\sum_{p=1}^{\infty} |c_{pq}| < 1/4$ . Let  $K$  be a number such that  $\sum_{p=1}^{\infty} |c_{np}| < K, n = 1, 2, 3, \dots$ . Let  $i_1$  be a positive integer such that  $\sum_{p=i_1+1}^{\infty} |c_{np}| < 1/4, n = 1, 2, \dots, j_1$ . Let  $j_2 > j_1$  be an integer such that if  $q > j_2$ , then  $\sum_{p=1}^{\infty} |c_{qp}| < 1/4^2$ . Let  $i_2 > i_1$  be an integer such that  $\sum_{p=i_2+1}^{\infty} |c_{np}| < 1/4^2, n = 1, 2, \dots, j_2$ . Continue the process to obtain increasing sequences  $\{j_p\}_{p=1}^{\infty}$  and  $\{i_p\}_{p=1}^{\infty}$  of positive integers. Define

$$\begin{aligned} e_n &= 1, n = 1, 2, \dots, i_1, \\ e_n &= 2^t, i_t < n \leq i_{t+1}, t = 1, 2, 3, \dots \end{aligned}$$

Consider the matrix  $C_e$ . If  $q$  is a positive integer, then

$$\begin{aligned} \sum_{p=1}^{\infty} |c_{qp}e_p| &= \sum_{p=1}^{i_1} |c_{qp}e_p| + \sum_{t=1}^{\infty} \left( \sum_{p=i_t+1}^{i_{t+1}} |c_{qp}e_p| \right) \\ &< K + \sum_{t=1}^{\infty} \left( 2^t \cdot \sum_{p=i_t+1}^{i_{t+1}} |c_{qp}| \right) \\ &\leq K + \sum_{t=1}^{\infty} 2^t/4^t \\ &= K + 1. \end{aligned}$$

Let  $\{r_p\}$  be an increasing sequence of positive integers such that

$$\sum_{p=r_n+1}^{\infty} |a_p b_p| < 1/4^n.$$

Define

$$\begin{aligned} f_p &= 1, p = 1, 2, \dots, r_1, \\ f_p &= 2^q, r_q < p \leq r_{q+1}, q = 1, 2, 3, \dots \end{aligned}$$

Then

$$\begin{aligned} \sum_{p=1}^{\infty} |a_p b_p f_p| &= \sum_{p=1}^{r_1} |a_p b_p f_p| + \sum_{q=1}^{\infty} \left( \sum_{p=r_q+1}^{r_{q+1}} |a_p b_p f_p| \right) \\ &\leq M + \sum_{q=1}^{\infty} \left( 2^q \cdot \sum_{p=r_q+1}^{r_{q+1}} |a_p b_p| \right) \\ &< M + \sum_{q=1}^{\infty} 2^q/4^q. \end{aligned}$$

Let  $g_p = \min [e_p, f_p]$ ,  $p = 1, 2, 3, \dots$ . Then  $g_p \rightarrow \infty$  as  $p \rightarrow \infty$ . Thus  $b < bg$ . If  $n$  is a positive integer, then

$$\begin{aligned} \sum_{p=1}^{\infty} |a_{np} b_p g_p| &\leq \sum_{p=1}^{\infty} |c_{np} g_p| + \sum_{p=1}^{\infty} |a_p b_p g_p| \\ &\leq \sum_{p=1}^{\infty} |c_{np} e_p| + \sum_{p=1}^{\infty} |a_p b_p f_p| \\ &< K + 1 + M + 1. \end{aligned}$$

Therefore the matrix  $A_{bg}$  sums every null sequence. Thus if  $b < b' < bg$ , then  $A$  is c.p.o.  $[b']$ . The existence of a sequence  $b'$  such that  $b < b' < bg$  follows from the lemma. This completes the proof of the theorem.

REMARK 4. We note that the matrix  $A$ , defined by  $a_{pq} = 1$  if  $p \neq q$ ,  $a_{pq} = 2^{p-1}$  if  $p = q$ , has a maximal interval  $(-, b)$  over which it is convergence preserving. For example  $b = \{1/2^{p-1}\}$ .

On the other hand, the matrix  $A$ , defined by  $a_{pq} = 0$  if  $q > p$ ,  $a_{pq} = 1$  if  $p \geq q$ , has no such maximal interval. This is easily shown by supposing that  $(-, b)$  is a maximal summability interval for  $A$ . Then  $A_b$  is c.p.o.  $C_0$  and hence satisfies the  $S - T$  conditions. Thus  $\sum_{p=1}^{\infty} |b_p|$  converges. It is easy to find  $c \in S_0$  such that  $b < c$  and  $\sum_{p=1}^{\infty} |c_p|$  converges. Thus  $A_c$  satisfies the  $S - T$  conditions and hence is c.p.o.  $C_0$ . Therefore  $A$  is c.p.o.  $(-, c)$ .

It is easy to show that if there exist numbers  $r$  and  $R$  such that  $0 < r < |a_{pq}| < R$ ,  $p, q = 1, 2, 3, \dots$ , then  $A = (a_{pq})$  has no maximal summability interval. The proof will be omitted.

REMARK 5. Let  $\mathcal{A}$  be a chain in  $S_0$  unbounded above. If  $a \in \mathcal{A}$ , let  $a' = \{a_1, 1/2, a_2, 1/4, a_3, 1/8, \dots\}$ . Let  $\mathcal{A}' = \{a' : a \in \mathcal{A}\}$ . Then  $\mathcal{A}'$  is a chain in  $S_0$  which is unbounded above. Let  $A = (a_{pq})$  be defined by  $a_{pq} = 1/2^n$  if  $q = 2n - 1$ ,  $a_{pq} = 1$  if  $q$  is an even integer. Clearly if  $a' \in \mathcal{A}'$ , then  $A$  is c.p.o.  $[a']$ . But  $A$  is not c.p.o.  $C_0$ .

THEOREM 2. If  $A$  is c.p.o. each of the sets  $[b^{(1)}], [b^{(2)}], [b^{(3)}], \dots$ , then there exists  $d \in S_0$  such that  $b^{(p)} < d$ ,  $p = 1, 2, 3, \dots$ , and  $A$  is c.p.o.  $[d]$ .

*Proof.* By Theorem 1 we can find  $t^{(n)}$  in  $S_0$  such that  $t^{(n)} > b^{(n)}$  and  $A$  is c.p.o.  $[t^{(n)}]$ ,  $n = 1, 2, 3, \dots$ . If  $n$  is a positive integer, let  $\alpha^{(n)} \in [t^{(n)}]$  such that  $0 < \alpha_p^{(n)} < 1$ ,  $p = 1, 2, 3, \dots$ . If  $n$  is a positive integer, let  $M_n$  be a number which exceeds  $\sum_{q=1}^{\infty} |a_{pq} \alpha_q^{(n)}|$ ,  $p = 1, 2, 3, \dots$ . If  $n$  is a positive integer, let

$$\beta_p^{(n)} = \frac{\alpha_p^{(n)}}{2^n [M_n + 1]}, \quad p = 1, 2, 3, \dots$$

If  $p$  is a positive integer, let  $c_p = \sum_{n=1}^{\infty} \beta_p^{(n)}$ . We wish to show that  $c \in S_0$ . Let  $\mu > 0$ , and let  $k$  be a positive integer such that  $2^{-k} < \mu/2$ . Let  $R$  be a positive integer such that if  $q > R$ , then  $\beta_q^{(p)} < \mu/2^{k+1}$ ,  $p = 1, 2, \dots, k$ . Then if  $n > R$ , we have

$$c_n = \sum_{p=1}^{\infty} \beta_n^{(p)} = \sum_{p=1}^k \beta_n^{(p)} + \sum_{p=k+1}^{\infty} \beta_n^{(p)} < \mu/2 + 2^{-k} < \mu.$$

Thus  $c \in S_0$ . If  $q$  is a positive integer, then, using the double sum theorem, we have

$$\begin{aligned}
\sum_{p=1}^{\infty} |a_{qp}c_p| &= \sum_{p=1}^{\infty} |a_{qp}| \left( \sum_{n=1}^{\infty} \beta_p^{(n)} \right) \\
&= \sum_{p=1}^{\infty} \sum_{n=1}^{\infty} |a_{qp}| \beta_p^{(n)} \\
&= \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} |a_{qp}| \beta_p^{(n)} \\
&= \sum_{n=1}^{\infty} \sum_{p=1}^{\infty} |a_{qp}| \cdot \frac{\alpha_p^{(n)}}{2^n [M_n + 1]} \\
&= \sum_{n=1}^{\infty} \frac{1}{2^n [M_n + 1]} \sum_{p=1}^{\infty} |a_{qp}| \alpha_p^{(n)} \\
&< \sum_{n=1}^{\infty} 2^{-n} .
\end{aligned}$$

Thus  $A_c$  sums every null sequence. Therefore  $A$  sums every sequence  $x \in S_0$  such that  $x < c$ . We note that if  $n$  is a positive integer, then  $c_p/\beta_p^{(n)} > 1$ ,  $p = 1, 2, 3, \dots$ . Thus if  $n$  is a positive integer, then

$$\lim_{p \rightarrow \infty} \left| \frac{c_p}{b_p^{(n)}} \right| = \lim_{p \rightarrow \infty} \frac{c_p}{\beta_p^{(n)}} \cdot \frac{\beta_p^{(n)}}{t_p^{(n)}} \cdot \frac{t_p^{(n)}}{|b_p^{(n)}|} = \infty .$$

Hence  $b^{(n)} < c$ ,  $n = 1, 2, 3, \dots$ . By the lemma, there exists  $d \in S_0$  such that  $b^{(n)} < d < c$ ,  $n = 1, 2, 3, \dots$ .  $A$  is c.p.o.  $[d]$  since  $d < c$  and  $A$  sums every sequence  $x \in S_0$  such that  $x < c$ . This completes the proof of the theorem.

**COROLLARY.** *Suppose  $M$  is a countable set of matrices and  $L$  is a countable subset of  $E_0$  such that if  $A \in M$  and  $[b] \in L$ , then  $A$  is c.p.o.  $[b]$ . Then there exists  $[c] \in E_0$  such that if  $A \in M$  and  $[b] \in L$ , then  $[b] < [c]$  and  $A$  is c.p.o.  $[c]$ .*

*Proof.* The proof will be for the case that both  $M$  and  $L$  are infinite sets. Let  $M = \{A^{(1)}, A^{(2)}, A^{(3)}, \dots\}$  and  $L = \{[b^{(1)}], [b^{(2)}], [b^{(3)}], \dots\}$ . By Theorem 2, if  $p$  is a positive integer, there exists  $c^{(p)} \in S_0$  such that  $b^{(n)} < c^{(p)}$ ,  $n = 1, 2, 3, \dots$ , and  $A^{(p)}$  is c.p.o.  $[c^{(p)}]$ . Let  $L' = \{[c^{(1)}], [c^{(2)}], [c^{(3)}], \dots\}$ . By the lemma, there exists  $c \in S_0$  such that if  $b^{(s)} \in L$  and  $c^{(t)} \in L'$ , then  $b^{(s)} < c < c^{(t)}$ . If  $j$  is a positive integer, then by Remark 2',  $A^{(j)}$  is c.p.o.  $[c]$  since  $A^{(j)}$  is c.p.o.  $[c^{(j)}]$  and  $[c] \subset (-, c^{(j)})$ . This completes the proof.

Received January 31, 1967.

NORTH TEXAS STATE UNIVERSITY

# PACIFIC JOURNAL OF MATHEMATICS

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# Pacific Journal of Mathematics

Vol. 24, No. 1

May, 1968

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