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**ALGEBRAS SATISFYING THE DESCENDING CHAIN
CONDITION FOR SUBALGEBRAS**

THOMAS PAUL WHALEY

ALGEBRAS SATISFYING THE DESCENDING CHAIN CONDITION FOR SUBALGEBRAS

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In this paper we give a partial solution to the following problem of B. Jónsson:

(*) **For which cardinals m do there exist algebras of power m having finitely many operations and satisfying the descending chain condition for subalgebras?**

Of course a necessary condition for the existence of such an algebra is that there exist an algebra of power m having finitely many operations and having no proper subalgebra of power m . The first such construction was by F. Galvin who constructed an algebra of power ω_1 which satisfied the descending chain condition for subalgebras. It has been shown by Erdos and Hajnal [1] that for $n \in \omega$ there is an algebra of power ω_n which has finitely many operations and has no proper subalgebra of power ω_n . Actually C. C. Chang [3] has shown that if an algebra exists of power m having finitely many operations and having no proper subalgebra of power m , then such an algebra exists of power m^+ . In § 2 we modify this construction to show that if there is an algebra of power m with finitely many operations and satisfying the descending chain condition, then there is such an algebra of power m^+ .

Erdos and Hajnal [1] also showed, under the assumption of the generalized continuum hypothesis, that for any cardinal m there is a locally finite algebra of power m^+ having finitely many operations and having no proper subalgebra of power m^+ . In § 3 we show that for $n \in \omega$ there is a locally finite algebra of power ω_n having finitely many operations and satisfying the descending chain condition for subalgebras.

2. **General algebras.** Before beginning the construction of the algebras we note the following relevant theorem of W. Hanf.

THEOREM 2.1. (Hanf [2], [4]). *The lattice of subalgebras of an algebra with countably many operations is a compactly generated lattice in which each compact element contains at most countably many compact elements. Conversely, any such lattice can be realized as the lattice of subalgebras of a commutative loop in which each subalgebra is a subloop.*

COROLLARY 2.2. *The following are equivalent:*

(i) *There exists a compactly generated lattice having m compact elements in which each compact element contains at most countably*

many compact elements and which satisfies the descending chain condition (for elements).

(ii) There is an algebra of power m having countably many operations and satisfying the descending chain condition for subalgebras.

(iii) There is an algebra of power m having finitely many operations and satisfying the descending chain condition for subalgebras.

(iv) There is a commutative loop of power m satisfying the descending chain condition for subalgebras.

THEOREM 2.3. *If there is an algebra of power m having finitely many operations and satisfying the descending chain condition for subalgebras, then there is an algebra of power m^+ having finitely many operations and satisfying the descending chain condition for subalgebras.*

Proof. Suppose we have such an algebra of power m . Using Corollary 2.2 we assume our algebra is of the form $A = \langle m; f \rangle$ (identifying the cardinal m with the set of all ordinals of cardinality less than m). Actually we could take A to be a commutative loop, but these properties are not needed here. For each ordinal ξ with $m \leq \xi < m^+$, let ϕ_ξ be a one-to-one map of ξ onto m . We now define a binary operation \bar{f} on m^+ by

$$\bar{f}(\eta_0, \eta_1) = \begin{cases} f(\eta_0, \eta_1) & \text{if } \eta_0, \eta_1 < m, \\ \phi_{\eta_0}(\eta_1) & \text{if } m \leq \eta_0 \text{ and } \eta_1 < \eta_0, \\ \phi_{\eta_1}^{-1}(\eta_0) & \text{if } \eta_0 < m \leq \eta_1, \\ 0 & \text{otherwise.} \end{cases}$$

We show that $A' = \langle m^+; \bar{f} \rangle$ has the desired properties.

If B is a subalgebra of A' ($B \subseteq_s A'$) then it is clear that $B \cap m$ is a subalgebra of A . Furthermore, if $m \leq \xi \in B$ we can see that $m \cap B = \phi_\xi(\xi \cap B)$. To see this note that if $\eta \in \xi \cap B$ then $\phi_\xi(\eta) = \bar{f}(\xi, \eta) \in m \cap B$ while if $\eta' \in m \cap B$ then $\phi_\xi^{-1}(\eta') = \bar{f}(\eta', \xi) \in \xi \cap B$.

We now show that if $C \subset_s B \subset_s A'$, one of the following three conditions must hold:

- (i) $C \cap m \subset_s B \cap m$,
- (ii) $\Sigma C < \Sigma B$,
- (iii) $\Sigma B \in B - C$.

Assume that $\Sigma C = \Sigma B$ and $\Sigma B \in B - C$. Suppose first that B has a largest member, β . Then $\beta = \Sigma B \in B - C$ implying that $\beta \in C$. Thus $C \cap \beta \subset B \cap \beta$. We know that $C \cap m = \phi_\beta(C \cap \beta) \subset \phi_\beta(B \cap \beta) = B \cap m$. This leaves only the case where B has no largest member. Take $\xi \in B - C$. If $\xi < m$, we have $C \cap m \subset B \cap m$. Therefore we assume

that $m \leq \xi < m^+$. Since $\Sigma B = \Sigma C > \xi$, there is a $\xi' \in C$ with $\xi < \xi'$. Then $\xi' \cap C \subset \xi' \cap B$ so $m \cap C = \phi_{\xi'}(\xi' \cap C) \subset \phi_{\xi'}(\xi' \cap B) = m \cap B$.

Suppose we have $A' \cong_s B_0 \cong_s B_1 \cong_s \dots$. Clearly $\Sigma B_0 \geq \Sigma B_1 \geq \dots$. There is some $k_0 \in \omega$ so that $\Sigma B_{k_0} = \Sigma B_{k_0+1} = \dots$. Also we know that

$$A \cong_s B_{k_0} \cap m \cong_s B_{k_0+1} \cap m \cong_s \dots$$

Since A satisfies the descending chain condition for subalgebras, there is a $k_1 \geq k_0$ so that $B_{k_1} \cap m = B_{k_1+1} \cap m = \dots$. Assume now that $n_1 < n_2 < \dots$ and that $B_{k_1} \supset B_{k_1+n_1} \supset B_{k_1+n_2} \supset \dots$. Of the three conditions listed above, only (iii) applies to $B_{k_1+n_2} \subset_s B_{k_1+n_1} \subset_s A'$. Thus $\Sigma B_{k_0} \in B_{k_1+n_1} - B_{k_1+n_2}$. Similarly, we get $\Sigma B_{k_0} \in B_{k_1+n_2} - B_{k_1+n_3}$. This contradiction completes the proof.

COROLLARY 2.4. *For $n \in \omega$ there is a commutative loop of power ω_n satisfying the descending chain condition for subalgebras.*

3. Locally finite algebras. By a locally finite algebra we mean an algebra in which each finite subset generates a finite subalgebra. The following theorem characterizes the lattices of subalgebras of locally finite algebras in a manner somewhat analogous to Hanf's theorem.

THEOREM 3.1. *The lattice of subalgebras of a locally finite algebra is a compactly generated lattice in which each compact element contains only finitely many compact elements. Conversely, any such lattice may be realized as the lattice of subalgebras of a locally finite algebra having one commutative binary operation.*

Proof. Since the compact elements in the lattice of subalgebras of an algebra correspond to the finitely generated subalgebras and since each finitely generated subalgebra of a locally finite algebra is finite, it is clear that each compact element in the lattice of subalgebras of a locally finite algebra contains only finitely many compact elements.

Conversely, suppose $\langle L; +, \cdot \rangle$ is a compactly generated lattice in which each compact element contains only finitely many compact elements. Let L^c be the semilattice of compact elements of L . We know that L is isomorphic to the lattice of ideals of L^c . We now define a commutative binary operation, f , on L^c so that the subalgebras of $\langle L^c; f \rangle$ are precisely the ideals of $\langle L^c; + \rangle$ with the finitely generated subalgebras just the principal ideals. This will clearly complete the proof. For $a \in L^c$ let $\{a_0, a_1, \dots, a_{n(a)}\}$ be the principal ideal of $\langle L^c; + \rangle$ generated by a with $a = a_0$ and $a_i \neq a_j$ if $i \neq j$. Define f by

$$f(a, b) = \begin{cases} a_{j+1} & \text{if } b = a_j \text{ with } j < n(a) \\ b_{j+1} & \text{if } a = b_j \text{ with } j < n(b) \\ a + b & \text{otherwise.} \end{cases}$$

It is easy to check that the subalgebras of $\langle L^c; f \rangle$ are as described above.

COROLLARY 3.2. *For any m the following are equivalent:*

- (i) *There is a compactly generated lattice having m compact elements in which each compact element contains only finitely many compact elements and which satisfies the descending chain condition.*
- (ii) *There is a locally finite algebra of power m which satisfies the descending chain condition for subalgebras.*
- (iii) *There is a locally finite algebra of power m having one commutative binary operation and satisfying the descending chain condition for subalgebras.*

THEOREM 3.3. *For $n \in \omega$ there is a locally finite algebra of power ω_n which satisfies the descending chain condition for subalgebras.*

Proof. The proof will be by induction on n . First we construct A_0 of power ω . For each $m \in \omega$ define a unary operation $f_{m,0}$ on ω by

$$f_{m,0}(n) = \begin{cases} n - m & \text{if } m \leq n, \\ 0 & \text{otherwise.} \end{cases}$$

We then let $A_0 = \langle \omega; f_{m,0} \rangle_{m \in \omega}$.

As an induction hypothesis we assume that we have

$$A_n = \langle \omega_n; f_{m,n}, \omega_s \rangle_{\substack{m \in \omega \\ s < n}}$$

so that the following assertions are true of A_n :

- (1) $f_{m,n}$ is of rank $r(n)$ where $r(0) = 1$ and $r(l + 1) = 2r(l) + 1$;
- (2) A_n is locally finite;
- (3) For any $m \in \omega$ and for any $\eta_0, \eta_1, \dots, \eta_{r(n)-1} \in \omega_n$, we have

$$f_{m,n}(\eta_0, \dots, \eta_{r(n)-1}) \leq \cap (\{\eta_i \mid i \leq r(n) - 1\} - \{\omega_s \mid s < n\})$$

and $f_{0,n}(\eta_0, \eta_0, \dots, \eta_0) = \eta_0$;

- (4) Given $\{\xi_k \mid k \in \omega\}$ a sequence of distinct members of ω_n , there exist an $m \in \omega$ and $k_0, k_1, \dots, k_{r(n)} \in \omega$ so that $k_0 < \prod_{i=1}^{r(n)} k_i$ and

$$f_{m,n}(\eta_{k_1}, \dots, \eta_{k_{r(n)}}) = \xi_{k_0}$$

where either $\eta_{k_i} = \xi_{k_i}$ or else $\eta_{k_i} \in \{\omega_s \mid s < n\}$.

It is clear that A_0 satisfies these conditions with $n = 0$.

Condition (3) will be used to obtain local finiteness, and condition (4) will assure that we have the descending chain condition for subalgebras. To see this suppose

$$A_n \supset_s B_0 \supset_s B_1 \supset_s \dots$$

Take $\xi_i \in B_i - B_{i+1}$. Then applying (4) to $\{\xi_i \mid i \in \omega\}$ we find that there is a $k_0 \in \omega$ for which $\xi_{k_0} \in B_{k_0+1}$, a contradiction.

We now proceed to construct A_{n+1} which satisfies conditions (1)–(4) with n replaced by $n + 1$. For each ξ with $\omega_n \leq \xi < \omega_{n+1}$ we let ϕ_ξ map ξ onto ω_n in a one-to-one manner with ϕ_{ω_n} just the identity map on ω_n . For each $m \in \omega$ we define $f_{m,n+1}$ as follows: If $\omega_n \leq \prod_{i=0}^{r(n)-1} \xi_i$; if $\eta_i < \xi_i$ for $i = 0, 1, \dots, r(n) - 1$; if $\omega_n \leq \gamma$; and if

$$\begin{aligned} & \phi_r^{-1}(f_{m,n}(\phi_{\xi_0}(\eta_0), \dots, \phi_{\xi_{r(n)-1}}(\eta_{r(n)-1}))) \\ & \leq \cap \{ \{\eta_0, \dots, \eta_{r(n)-1}, \xi_0, \dots, \xi_{r(n)-1}\} \\ & \quad - \{ \omega_s \mid s \leq n \} \}; \end{aligned}$$

we define

$$\begin{aligned} & f_{m,n+1}(\xi_0, \dots, \xi_{r(n)-1}, \eta_0, \dots, \eta_{r(n)-1}, \gamma) \\ & = \phi_r^{-1}(f_{m,n}(\phi_{\xi_0}(\eta_0), \dots, \phi_{\xi_{r(n)-1}}(\eta_{r(n)-1}))). \end{aligned}$$

Otherwise we define

$$\begin{aligned} & f_{m,n+1}(\xi_0, \dots, \xi_{r(n)-1}, \eta_0, \dots, \eta_{r(n)-1}, \gamma) \\ & = \cap \{ \eta_0, \dots, \eta_{r(n)-1}, \xi_0, \dots, \xi_{r(n)-1}, \gamma \}. \end{aligned}$$

We let $A_{n+1} = \langle \omega_{n+1}; f_{m,n+1}, \omega_l \rangle_{\substack{m \in \omega \\ l \leq n}}$.

It is clear that A_{n+1} satisfies conditions (1) and (3) of the induction hypothesis.

We now show that A_{n+1} is locally finite. Suppose B is a finite subset of ω_{n+1} . Let

$$\begin{aligned} & B_0 = B \cup \{ \omega_s \mid s \leq n \}, \\ & \vdots \\ & B_{k+1} = \{ f_{m,n+1}(\xi_0, \dots, \xi_{r(n+1)-1}) \mid m \in \omega \text{ and } \xi_0, \dots, \xi_{r(n+1)-1} \in \bigcup_{i \leq k} B_i \}. \end{aligned}$$

Then $[B] = \bigcup_{k \in \omega} B_k$. In showing that $[B]$ is finite, we first show that each B_k is finite. This is true for $k = 0$. Assume that it is true for $k \leq l$. Then $\bigcup_{i \leq l} B_i$ is finite. Fix $\xi_0, \dots, \xi_{r(n+1)-1} \in \bigcup_{i \leq l} B_i$. Now we have

$$\begin{aligned} & \{ f_{m,n+1}(\xi_0, \dots, \xi_{r(n+1)-1}) \mid m \in \omega \} \\ & \subseteq \phi_{\xi_{r(n+1)-1}}^{-1} \{ f_{m,n}(\phi_{\xi_0}(\xi_{r(n)}), \dots, \phi_{\xi_{r(n)-1}}(\xi_{r(n+1)-1})) \mid m \in \omega \} \\ & \quad \cup \{ \xi_0 \cap \dots \cap \xi_{r(n+1)-1} \}. \end{aligned}$$

However, this set is finite since A_n is locally finite. Hence B_{l+1} is finite, and by induction each B_k is finite. Now let $C_0 = B_0$ and

$$C_{k+1} = B_{k+1} - B_k .$$

Then $[B] = \bigcup_{k \in \omega} C_k$, and each C_{k+1} is finite. If $1 \leq k < k'$ and if $C_k, C_{k'} \neq \emptyset$, then using (3) and the fact that $\{\omega_s \mid s \leq n\} \subseteq B_0$, we see that $\max C_{k'} < \max C_k$. Thus there are only finitely many $C_k \neq \emptyset$. Hence $[B]$ is finite.

Finally we show that A_{n+1} satisfies condition (4). Suppose we have $\{\xi_k \mid k \in \omega\}$ a sequence of distinct elements of ω_{n+1} . We consider two cases.

Case 1. There are infinitely many k 's for which $\xi_k \in \omega_n$: Without loss of generality we assume that $\{\xi_k \mid k \in \omega\} \subseteq \omega_n$. We then invoke the induction hypothesis to get an $m \in \omega$ and $k_0, k_1, \dots, k_{r(n)} \in \omega$ so that $k_0 < \prod_{i=1}^{r(n)} k_i$ and $f_{m,n}(\eta_{k_1}, \dots, \eta_{k_{r(n)}}) = \xi_{k_0}$ where either $\eta_{k_i} = \xi_{k_i}$ or else $\eta_{k_i} \in \{\omega_s \mid s < n\}$. But then we have

$$\begin{aligned} f_{m,n+1}(\omega_n, \dots, \omega_n, \eta_{k_1}, \dots, \eta_{k_{r(n)}}, \omega_n) \\ &= \phi_{\omega_n}^{-1}(f_{m,n}(\phi_{\omega_n}(\eta_{k_1}), \dots, \phi_{\omega_n}(\eta_{k_{r(n)}}))) \\ &= f_{m,n}(\eta_{k_1}, \dots, \eta_{k_{r(n)}}) \\ &= \xi_{k_0} . \end{aligned}$$

This completes the proof in this case.

Case 2. At most finitely many of the ξ_k 's are less than ω_n : Without loss of generality we assume that $\{\xi_k \mid k \in \omega\} \subseteq \omega_{n+1} - \omega_n$. We pick $k_0 < k_1 < \dots$ so that $\xi_{k_0} < \xi_{k_1} < \dots$. For each $i \in \omega$, we let $\pi_i = \phi_{\xi_{k_{i+1}}}(\xi_{k_i})$. Now consider $\{\pi_i \mid i \in \omega\}$. If for some $i, j \in \omega$ we have $i < j$ and $\pi_i = \pi_j$, then

$$\begin{aligned} f_{0,n+1}(\xi_{k_{j+1}}, \dots, \xi_{k_{j+1}}, \xi_{k_j}, \dots, \xi_{k_j}, \xi_{k_{i+1}}) \\ &= \phi_{\xi_{k_{i+1}}}^{-1}(f_{0,n}(\phi_{\xi_{k_{j+1}}}(\xi_{k_j}), \dots, \phi_{\xi_{k_{j+1}}}(\xi_{k_j}))) \\ &= \phi_{\xi_{k_{i+1}}}^{-1}(f_{0,n}(\pi_j, \dots, \pi_j)) \\ &= \phi_{\xi_{k_{i+1}}}^{-1}(\pi_j) \\ &= \phi_{\xi_{k_{i+1}}}^{-1}(\pi_i) \\ &= \phi_{\xi_{k_{i+1}}}^{-1} \phi_{\xi_{k_{i+1}}}(\xi_{k_i}) \\ &= \xi_{k_i} , \end{aligned}$$

and we're through. Thus we may assume that $\{\pi_i \mid i \in \omega\}$ is a sequence of distinct elements of ω_n . Applying the induction hypothesis again, we get an $m \in \omega$ and $i_0, i_1, \dots, i_{r(n)} \in \omega$ so that $i_0 < \prod_{j=1}^{r(n)} i_j$ and

$$f_{m,n}(\eta_{i_1}, \dots, \eta_{i_{r(n)}}) = \pi_{i_0}$$

where either $\eta_{i_j} = \pi_{i_j}$ or else $\eta_{i_j} \in \{\omega_s \mid s < n\}$.

Now let

$$\beta_{i_j} = \begin{cases} \xi_{k_{i_j}} & \text{if } \eta_{i_j} = \pi_{i_j}, \\ \eta_{i_j} & \text{otherwise,} \end{cases}$$

and let

$$\sigma_{i_j} = \begin{cases} \xi_{k_{i_j+1}} & \text{if } \beta_{i_j} = \xi_{k_{i_j}}, \\ \omega_n & \text{otherwise.} \end{cases}$$

Then $\phi_{\sigma_{i_j}}(\beta_{i_j}) = \eta_{i_j}$ in any case. This gives

$$\begin{aligned} & f_{m,n+1}(\sigma_{i_1}, \dots, \sigma_{i_{r(n)}}, \beta_{i_1}, \dots, \beta_{i_{r(n)}}, \xi_{k_{i_0+1}}) \\ &= \phi_{\xi_{k_{i_0+1}}}^{-1}(f_{m,n}(\phi_{\sigma_{i_1}}(\beta_{i_1}), \dots, \phi_{\sigma_{i_{r(n)}}}(\beta_{i_{r(n)}}))) \\ &= \phi_{\xi_{k_{i_0+1}}}^{-1}(f_{m,n}(\eta_{i_1}, \dots, \eta_{i_{r(n)}})) \\ &= \phi_{\xi_{k_{i_0+1}}}^{-1}(\pi_{i_0}) \\ &= \phi_{\xi_{k_{i_0+1}}}^{-1} \phi_{\xi_{k_{i_0+1}}}(\xi_{k_{i_0}}) \\ &= \xi_{k_{i_0}}. \end{aligned}$$

Since each $\sigma_{i_j}, \beta_{i_j}$ is a ξ_{k_i} with $i > i_0$ or is in $\{\omega_s \mid s \leq n\}$, this is the desired result. This completes the proof of Theorem 3.3.

COROLLARY 3.4. *For $n \in \omega$ there is a locally finite algebra of power ω_n which has one commutative binary operation and satisfies the descending chain condition for subalgebras.*

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Patrick Robert Ahern, <i>On the geometry of the unit ball in the space of real annihilating measures</i>	1
Kirby Alan Baker, <i>Equational classes of modular lattices</i>	9
E. F. Beckenbach and Gerald Andrew Hutchison, <i>Meromorphic minimal surfaces</i>	17
Tae Ho Choe, <i>Intrinsic topologies in a topological lattice</i>	49
John Bligh Conway, <i>A theorem on sequential convergence of measures and some applications</i>	53
Roger Cuppens, <i>On the decomposition of infinitely divisible probability laws without normal factor</i>	61
Lynn Harry Erbe, <i>Nonoscillatory solutions of second order nonlinear differential equations</i>	77
Burton I. Fein, <i>The Schur index for projective representations of finite groups</i>	87
Stanley P. Gudder, <i>A note on proposition observables</i>	101
Kenneth Kapp, <i>On Croisot's theory of decompositions</i>	105
Robert P. Kaufman, <i>Gap series and an example to Malliavin's theorem</i>	117
E. J. McShane, Robert Breckenridge Warfield, Jr. and V. M. Warfield, <i>Invariant extensions of linear functionals, with applications to measures and stochastic processes</i>	121
Marvin Victor Mielke, <i>Rearrangement of spherical modifications</i>	143
Akio Osada, <i>On unicity of capacity functions</i>	151
Donald Steven Passman, <i>Some 5/2 transitive permutation groups</i>	157
Harold L. Peterson, Jr., <i>Regular and irregular measures on groups and dyadic spaces</i>	173
Habib Salehi, <i>On interpolation of q-variate stationary stochastic processes</i>	183
Michael Samuel Skaff, <i>Vector valued Orlicz spaces generalized N-functions. I</i>	193
A. J. Ward, <i>On H-equivalence of uniformities. II</i>	207
Thomas Paul Whaley, <i>Algebras satisfying the descending chain condition for subalgebras</i>	217
G. K. White, <i>On subgroups of fixed index</i>	225
Martin Michael Zuckerman, <i>A unifying condition for implications among the axioms of choice for finite sets</i>	233