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NILPOTENCY CLASS OF A MAP AND STASHEFF'S CRITERION

CHEONG SENG HOO

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Let $f: X \rightarrow Y$ be a map and let $e: \Sigma\Omega X \rightarrow X$ be the map whose adjoint is $1_{\Omega X}$. Then we prove the following results.

THEOREM 1. $\text{nil } f \leq 1$ if and only if $fe\mathcal{V}: \Sigma\Omega X \vee \Sigma\Omega X \rightarrow Y$ can be extended to $\Sigma\Omega X \times \Sigma\Omega X$.

THEOREM 2. Let X be an H' -space. Then $\text{nil } f \leq 1$ if and only if $f\mathcal{V}: X \vee X \rightarrow Y$ can be extended to $X \times X$.

THEOREM 3. $\text{nil } f = \text{nil } (fe)$.

Theorem 1 may be regarded as an extension of Stasheff's criterion for a loop space to be homotopy-commutative. These theorems may all be regarded as extensions of Stasheff's criterion in various ways. We also discuss the duals of these results. Theorem 3 dualises, but the others do not. A sample result in the dual situation is

THEOREM. $\text{conil } f \leq \Sigma w \text{ cat } (e'f)$ where $e': Y \rightarrow \Omega\Sigma Y$ is the adjoint of $1_{\Sigma Y}$.

In this paper we shall work in the category \mathcal{S} of spaces with base point and having the homotopy type of countable CW complexes. All maps and homotopies shall respect base points. The maps of our category \mathcal{S} shall be homotopy classes of maps, but for simplicity we shall use the same symbol for a map and its homotopy class. Given spaces X, Y , we denote the set of homotopy classes of maps from X to Y by $[X, Y]$. We have an isomorphism $\tau: [\Sigma X, Y] \rightarrow [X, \Omega Y]$ where Σ, Ω are the suspension and loop functors respectively. We denote $\tau(1_{\Sigma X})$ by e' and $\tau^{-1}(1_{\Omega X})$ by e .

1. For convenience let us recall some notions of Peterson's theory of structures [7]. We shall follow the definitions and notations of [4]. Let \mathcal{C} be a category. By a left structure system \mathcal{L} over \mathcal{C} we mean $\mathcal{L} = (L, W, S; d, j)$ where $L, W, S: \mathcal{C} \rightarrow \mathcal{S}$ are covariant functors and $d: W \rightarrow L, j: W \rightarrow S$ are natural transformations. Given an object X of \mathcal{C} we say that X is \mathcal{L} -structured if there exists a map $\varphi: SX \rightarrow LX$ such that $\varphi j(X) \simeq d(X)$. Given a category \mathcal{C} , we have a category \mathcal{C}^2 of pairs. An object of \mathcal{C}^2 is a map $f: X \rightarrow Y$ of \mathcal{C} , and given objects $f: X_1 \rightarrow X_2, g: Y_1 \rightarrow Y_2$ of \mathcal{C}^2 , a map $(u, v): f \rightarrow g$ is a pair of maps $u: X_1 \rightarrow Y_1, v: X_2 \rightarrow Y_2$ such that $gu = vf$. We have covariant functors $D_0, D_1: \mathcal{C}^2 \rightarrow \mathcal{C}$ given by $D_0(f) = Y, D_1(f) =$

X where $f: X \rightarrow Y$. Also given $(u, v): f \rightarrow g$, we have $D_0(u, v) = v$, $D_1(u, v) = u$. We have a natural transformation $G: D_1 \rightarrow D_0$ given by $G(f) = f$ for $f \in \mathcal{C}^2$. Given a left structure $\mathcal{L} = (L, W, S; d, j)$ over \mathcal{C} , we have a left structure $\mathcal{L}^2 = (LD_0, WD_1, SD_1; (dD_0)(WG), jD_1)$ over \mathcal{C}^2 . Given an object f of \mathcal{C}^2 , we shall say that f is \mathcal{L} -structured if it is \mathcal{L}^2 -structured. It is easily seen that if $f: X \rightarrow Y$ is an object of \mathcal{C}^2 , and X or Y is \mathcal{L} -structured, then f is \mathcal{L} -structured.

We have the left structure $H = (1, \mathbf{V}_{i=1}^2, \mathbf{II}_{i=1}^2, \nabla, j)$ over \mathcal{S} , where 1 is the identity functor of \mathcal{S} , $\mathbf{V}_{i=1}^2$ is the wedge product, $\mathbf{II}_{i=1}^2$ is the cartesian product and ∇, j are the folding and inclusion natural transformations respectively. We observe that a space X is H -structured precisely if it is an H -space. Also a map $f: X \rightarrow Y$ is H -structured if and only if $f\nabla: X \vee X \rightarrow Y$ extends to $X \times X$.

2. Let $\mathcal{L} = (L, W, S; d, j)$ be a left structure system over a category \mathcal{C} . Let $f: X \rightarrow Y, g: Y \rightarrow Z$ be maps. Then it is easily seen that if f is \mathcal{L} -structured or g is \mathcal{L} -structured, then gf is \mathcal{L} -structured.

We recall that in [1], there is defined a generalized Whitehead product $[\cdot, \cdot]: [\Sigma A, X] \times [\Sigma B, X] \rightarrow [\Sigma(A \wedge B), X]$ where A, B, X are spaces and $A \wedge B$ is the smashed product. Now suppose X is an H -space. Then we have a generalized Samelson product (see [2]) $\langle \cdot, \cdot \rangle: [A, X] \times [B, X] \rightarrow [A \wedge B, X]$. These homotopy operations are related in the following way. Suppose α is an element of $[\Sigma A, X], \beta$ is an element of $[\Sigma B, X]$ where A, B, X are spaces. Then

$$\tau[\alpha, \beta] = \langle \tau(\alpha), \tau(\beta) \rangle .$$

We shall also make the following convention. Let $f: X \rightarrow Y$ be a map. Then we have an H -map $\Omega f: \Omega X \rightarrow \Omega Y$. We shall write $\text{nil } f$ for $\text{nil } \Omega f$ (see [3] for definitions). Similarly, we have an H' -map $\Sigma f: \Sigma X \rightarrow \Sigma Y$. We shall write $\text{conil } f$ for $\text{conil } \Sigma f$.

THEOREM 1. *Let $f: X \rightarrow Y$ be a map. Then $\text{nil } f \leq 1$ if and only if $f\nabla: \Sigma \Omega X \vee \Sigma \Omega X \rightarrow Y$ can be extended to $\Sigma \Omega X \times \Sigma \Omega X$.*

Proof. Let $c: \Omega X \times \Omega X \rightarrow \Omega X$ be the basic commutator of ΩX . Then $\text{nil } f \leq 1$ if and only if $(\Omega f) c \simeq *$. Let $i_1, i_2: \Sigma \Omega X \rightarrow \Sigma \Omega X \vee \Sigma \Omega X$ be the inclusions in the first and second coordinates respectively. Then we have a generalized Whitehead product

$$[i_1, i_2] \in [\Sigma(\Omega X \wedge \Omega X), \Sigma \Omega X \vee \Sigma \Omega X] .$$

Now $\Sigma \Omega X \times \Sigma \Omega X$ is homotopically equivalent to

$$(\Sigma\Omega X \vee \Sigma\Omega X) \bigcup_{[i_1, i_2]} C\Sigma(\Omega X \times \Omega X)$$

(see [1]), so that $fe\mathcal{V}$ extends to $\Sigma\Omega X \times \Sigma\Omega X$ if and only if $fe\mathcal{V}[i_1, i_2]=0$, that is, $[fe, fe] = 0$. Now $\tau[fe, fe] = \langle \Omega f, \Omega f \rangle$ and

$$q^*\langle \Omega f, \Omega f \rangle = c(\Omega f \times \Omega f) \simeq (\Omega f)c$$

where the first c denotes the commutator $\Omega Y \times \Omega X \rightarrow \Omega Y$ and the second c denotes the commutator $\Omega X \times \Omega X \rightarrow \Omega X$ and $q: \Omega Y \times \Omega Y \rightarrow \Omega Y$ is the projection. Since τ is an isomorphism and q^* is a monomorphism, it follows that $fe\mathcal{V}$ extends to $\Sigma\Omega X \times \Sigma\Omega X$ if and only if $\text{nil } f \leq 1$.

REMARK. If we take f to be the identity map of X , then the theorem says that $\text{nil } X \leq 1$ if and only if $e\mathcal{V}: \Sigma\Omega X \vee \Sigma\Omega X \rightarrow X$ extends to $\Sigma\Omega X \times \Sigma\Omega X$, which is just Stasheff's criterion for the homotopy-commutativity of a loop space (see [8]). We also observe that the statement that $fe\mathcal{V}$ extends to $\Sigma\Omega X \times \Sigma\Omega X$ is just the statement that fe can be H -structured.

THEOREM 2. *Let $f: X \rightarrow Y$ be a map where X is an H' -space. Then $\text{nil } f \leq 1$ if and only if $f\mathcal{V}: X \vee X \rightarrow Y$ can be extended to $X \times X$.*

In view of the fact that $f\mathcal{V}$ can be extended if and only if f can be H structured, Theorem 2 will follow from Theorem 1 and the following lemma.

LEMMA. *Let $f: X \rightarrow Y$ be a map where X is an H' -space. Then f is H -structured if and only if $fe: \Sigma\Omega X \rightarrow Y$ is H -structured.*

Proof. We need only show that if fe is H -structured then f is H -structured. Suppose fe can be H -structured. Then we can find a map $\varphi: \Sigma\Omega X \times \Sigma\Omega X \rightarrow Y$ such that $\varphi j \simeq \mathcal{V}(fe \vee fe) = fe\mathcal{V}$. Since X is an H' -space we have a map $s: X \rightarrow \Sigma\Omega X$ such that $es \simeq 1_x$. Then $\varphi(s \times s): X \times X \rightarrow Y$ is an H -structure for f . In fact $\varphi(s \times s)j = \varphi j(s \vee s) \simeq fe\mathcal{V}(s \vee s) = fes\mathcal{V} \simeq f\mathcal{V}$.

REMARK. Theorems 1 and 2 imply that $\text{nil } e \leq 1$ if and only if ΩX is homotopy-commutative, that is, if and only if $\text{nil } X \leq 1$. In fact, we always have $\text{nil } X = \text{nil } e$. This fact follows from the next result.

THEOREM 3. *Let $f: X \rightarrow Y$ be a map. Then $\text{nil } f = \text{nil } (fe)$.*

Proof. Since we always have $\text{nil } (fe) \leq \text{nil } f$, it suffices to show that $\text{nil } f \leq \text{nil } (fe)$. Suppose $\text{nil } (fe) \leq n$. Then $(\Omega f)(\Omega e)c_{n+1} \simeq *$

where $c_{n+1}: (\Omega\Sigma\Omega X)^{n+1} \rightarrow \Omega\Sigma\Omega X$ is the commutator map of weight $(n + 1)$. Then we have

$$(\Omega f)c_{n+1}(\Omega e \times \dots \times \Omega e) \simeq *$$

where $c_{n+1}: (\Omega X)^{n+1} \rightarrow \Omega X$ is also the commutator map of weight $(n + 1)$. Consider the map $e': \Omega X \rightarrow \Omega\Sigma\Omega X$ such that $e' = \tau(1_{\Omega\Sigma X})$. Clearly $(\Omega e)e' = 1_{\Omega\Sigma}$. Hence we have $(\Omega f)c_{n+1} \simeq *$, that is, $\text{nil } f \leq n$. This proves the theorem.

3. We now consider the dual situation. It is clear that Theorem 3 dualises immediately to give the following result.

THEOREM 4. *Let $f: X \rightarrow Y$ be a map and let $e': Y \rightarrow \Omega\Sigma Y$ be the adjoint of $1_{\Sigma X}$. Then $\text{conil } f = \text{conil } (e'f)$.*

Let us first define a right structure system over a category \mathcal{C} . By this we shall mean $\mathcal{R} = (R, P, T; d, j)$ where $R, P, T: \mathcal{C} \rightarrow \mathcal{I}$ are covariant functors and $d: R \rightarrow P, j: T \rightarrow P$ are natural transformations. Given an object $X \in \mathcal{C}$, we say that X is \mathcal{R} -structured if there exists a map $\varphi: RX \rightarrow TX$ such that $j(X)\varphi \simeq d(X)$. Given a right structure $\mathcal{R} = (R, P, T; d, j)$ over \mathcal{C} , we can form a right structure $\mathcal{R}^2 = (RD_1, PD_0, TD_0; (dD_0)(RG), jD_0)$ over \mathcal{C}^2 . We shall say that an element $f: X \rightarrow Y$ of \mathcal{C}^2 is \mathcal{R} -structured if it is \mathcal{R}^2 -structured. It is easily checked that if X or Y is \mathcal{R} -structured, then f is \mathcal{R} -structured.

The dual of the H -structure is the H' -structure $(1, \prod_{i=1}^2, \mathbf{V}_{i=1}^2; \Delta, j)$, a right structure over \mathcal{I} . Clearly a space X is H' -structured if and only if it is an H' -space. Also a map $f: X \rightarrow Y$ is H' -structured if and only if $\Delta f: X \rightarrow Y^2$ can be compressed into $Y \vee Y$. The dual of Theorem 1 would read: $\text{conil } f \leq 1$ if and only if $\Delta e'f: X \rightarrow (\Omega\Sigma Y)^2$ can be compressed into $\Omega\Sigma Y \vee \Omega\Sigma Y$. This, however, is false (see [5]). But in this case, we can generalize the H' -structure to another familiar right structure, namely the n -cat structure $(1, \prod_{i=1}^{n+1}; T_1, \Delta, j)$ over \mathcal{I} , where T_1 is the fat wedge functor. Thus the 1-cat structure is precisely the H' -structure. Given a space X , we have $\text{cat } X \leq n$ if there exists a map $\varphi: X \rightarrow T_1(X, \dots, X)$ such that $j\varphi \simeq \Delta: X \rightarrow X^{n+1}$. Given a map $f: X \rightarrow Y$, we have $\text{cat } f \leq n$ if $\Delta f: X \rightarrow Y^{n+1}$ can be compressed into $T_1(Y, \dots, Y)$.

Given a right structure system $\mathcal{R} = (R, P, T; d, j)$ over \mathcal{C} , let us consider the cofibre of $j: T \rightarrow P$. Suppose the cofibre of j is $q: P \rightarrow Q$. Let $j_w \rightarrow P$ be the fibre of q . Then we obtain a right structure system $\mathcal{R}_w = (R, P, T_w; d, j_w)$ over \mathcal{C} , called the associated weak structure. We shall say that an object $X \in \mathcal{C}$ is weakly \mathcal{R} -

structured if it can be \mathcal{R}_w -structured. Clearly, given a map $f: X \rightarrow Y$ we have $w \text{ cat } f \leq n$ if $q_{\Delta} f \simeq *$ where $q: Y^{n+1} \rightarrow \bigwedge_{i=1}^{n+1} Y$ is the projection onto the smashed product. Given a right structure $\mathcal{R} = (R, P, T; d, j)$ over \mathcal{C} , we have a right structure $\Sigma\mathcal{R} = (\Sigma R, \Sigma P, \Sigma T; \Sigma d, \Sigma j)$ over \mathcal{C} , where Σ is the suspension functor. Clearly, if f is \mathcal{R} -structured, it is $\Sigma\mathcal{R}$ -structured and it is weakly \mathcal{R} -structured. Thus $\Sigma w \text{ cat } f \leq w \text{ cat } f \leq \text{cat } f$ for any map f .

Let $f: X \rightarrow Y, g: Y \rightarrow Z$ be maps. Then it is easily seen that $\text{cat}(gf) \leq \min\{\text{cat } f, \text{cat } g\}$ and $w \text{ cat}(gf) \leq \min\{w \text{ cat } f, w \text{ cat } g\}$.

THEOREM 5. *Let $f: X \rightarrow Y$ be a map and let $e': Y \rightarrow \Omega\Sigma Y$ be the adjoint of $1_{\Sigma Y}$. Then $\text{conil } f \leq \Sigma w \text{ cat}(e'f)$.*

Proof. Suppose $\Sigma w \text{ cat}(e'f) \leq n$. Then $\Sigma(q_{\Delta}e'f) \simeq *$ where $q: (\Omega\Sigma Y)^{n+1} \rightarrow \bigwedge_{i=1}^{n+1} \Omega\Sigma Y$ is the projection. Let $c: \Sigma Y \rightarrow \mathbf{V}_{i=1}^{n+1} \Sigma Y$ be the commutator map of weight $(n + 1)$ for ΣY . Then we can form a map $\bar{c}: Y^{n+1} \rightarrow \Omega(\mathbf{V}_{i=1}^{n+1} \Sigma Y)$ such that $\bar{c}_{\Delta} = \tau(c)$ (see [5]). Since $\Sigma(q_{\Delta}e'f) \simeq *$, applying τ we have $\Omega\Sigma(q_{\Delta}e'f) \simeq *$. Consider the following diagram where each square is homotopy-commutative.

$$\begin{array}{ccccc}
 X & & & & \\
 \downarrow f & & & & \\
 Y & \xrightarrow{\Delta} & Y^{n+1} & \xrightarrow{q} & \bigwedge_{i=1}^{n+1} Y \\
 \downarrow e' & & \downarrow e' & & \downarrow e' \\
 \Omega\Sigma Y & \xrightarrow{\Omega\Sigma\Delta} & \Omega\Sigma(Y^{n+1}) & \xrightarrow{\Omega\Sigma q} & \Omega\Sigma\left(\bigwedge_{i=1}^{n+1} Y\right)
 \end{array}$$

We have then that $e'q_{\Delta}f \simeq *$. Using Lemmas 4.1_k and 4.2_k of [5], it follows that $\bar{c}_{\Delta}f \simeq *$, that is, $\tau(c)f \simeq *$. Hence $c(\Sigma f) \simeq *$, and hence $\text{conil } f \leq n$. This proves that $\text{conil } f \leq \Sigma w \text{ cat}(e'f)$.

THEOREM 6. *Let $f: X \rightarrow Y$ be a map where Y is an H -space. Then $\text{cat } f = \text{cat}(e'f), w \text{ cat } f = w \text{ cat}(e'f)$ where $e': Y \rightarrow \Omega\Sigma Y$ is the adjoint of $1_{\Sigma Y}$.*

Proof. We need only show that $\text{cat } f \leq \text{cat}(e'f)$, and

$$w \text{ cat } f \leq w \text{ cat}(e'f) .$$

Since Y is an H -space, we have a map $r: \Omega\Sigma Y \rightarrow Y$ such that $re' \simeq 1_Y$. Then $\text{cat } f = \text{cat}(re'f) \leq \text{cat}(e'f)$ and $w \text{ cat } f = w \text{ cat}(re'f) \leq w \text{ cat}(e'f)$.

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